DIOPTRICAE
PARS PRIMA
CONTINENS
LIBRVM PRIMVM,
DE
EXPLICATIONE
PRINCIPIORVM,
EX QVIBVS
CONSTRVCTIO TAM TELESCOPIORVM
QVAM
MICROSCOPIORVM
EST PETENDA.

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LIBER PRIMVS

CONTINENS

EXPLICATIONEM PRINCIPIORVM,

EX QVIBVS CONSTRVCTIO

TAM TELESCOPIORVM

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EST PETENDA.

CAPVT I.

DE DIFFVSIONE IMAGINIS PER VNICAM LENTEM REPRAESENTATAE.

Definitio I.

I.

Imago principalis vocatur ea, quae a radiis axi lentis proximis per lentem refractis repraesentatur. Tab. I. Fig. 1.

Scilicet ſi lentis PP axis ſit EF in eaque tantum ſpatium minimum *a*A*a* ſit apertum, per quod radiis tranſitus concedatur, radii a puncto lucido E emiſſi in vno puncto F colligentur, quod punctum imago principalis vocatur.

Coroll. 1.

2. Cum nempe apertura *aAa* ſit minima, omnes radii, qui a puncto quocunque in eam incidunt, ita aequabilem patiuntur refractionem, vt omnes iterum in vnum punctum colligantur, id quod non ſolum de puncto lucido E in axe lentis ſito, ſed etiam de quibusuis aliis extra axem poſitis eſt intelligendum.

Coroll. 2.

3. Quodſi ergo lentis apertura fuerit minima, ſingula cuiusuis obiecti puncta poſt refractionem iterum ſingulis punctis referentur, ſicque imago principalis erit diſtincta et confuſione carebit: ſiquidem confuſio tum demum oritur, quando radii ex vno puncto emiſſi non iterum in vno puncto colliguntur.

Coroll. 3.

4. Et quamdiu apertura lentis *aAa* eſt minima, nihil intereſt, ſecundum quamnam figuram facies lentis ſint elaboratae; quaecumque enim earum figura fuerit, quoniam tantum portiuncula minima in computum venit, ea ſemper vt ſphaerica ſpectari poterit.

Coroll. 4.

5. Pendet ergo locus imaginis principalis primum a loco puncti lucidi E, ſiue id in axe ſiue extra axem fuerit ſitum; deinde a ſphaericitate

vtrius-

vtriusque faciei *aAa* et *bBb* refringentis; tertio ab earum diſtantia AB ſeu lentis craſſitie, et quarto a ratione refractionis, quam radii in tranſitu per lentem patiuntur.

Scholion 1.

6. Refractio radiorum per huiusmodi lentium aperturas minimas transmiſſorum, ideoque determinatio imaginum principalium ſatis accurate in elementis Dioptricis tradi ſolet, quod igitur negotium hic fuſius non proſequar; ſed potius in eam refractionis rationem hic inquirere conſtitui, quando lentium apertura eſt modicae quantitatis, in quo imprimis ad vtriusque faciei figuram eſt ſpectandum. Hic autem perpetuo lentium figuras ſphaericas aſſumo, propterea quod haec figura vulgo lentibus induci, vel ſaltem, niſi forte accurate ſucceſſerit, intendi ſolet. In praxi certe nulla adhuc alia figura lentibus commode et accurate tribui poterit, atque adeo a ſphaerica figura, etſi ad praxin maxime eſt accommodata, ab artificibus frequenter aberrari ſolet. A ſollertioribus autem talia vitia iam plerumque ſatis feliciter euitantur, vnde non adeo erit verendum, ne ea, quae per calculum ex hypotheſi ſphaericae figurae elicientur, experientiae non conſentanea ſint futura. Hanc ob rem hic perpetuo poſtulo, vt ambae facies lentium exactiſſime ſecundum ſphaericam figuram ſint elaboratae.

Scholion 2.

7. Sphaerica autem figura hoc laborat incommodo, quod statim ac lenti maior apertura tribuatur, non amplius omnes radii, qui quidem ab vno obiecti puncto sunt profecti, post refractionem ad vnum punctum dirigantur radiique EM longius ab axe lentis transmissi non amplius in puncto F concurrant. Vnde confusionen eo maiorem nasci necesse est, quo magis hi radii remotiores ab iis, qui prope axem transeunt, declinauerint; et quoniam talis declinatio a figura sphaerica originem ducit, eo maior euadet, quo maior lenti apertura tribuatur. Quanta igitur quouis casu futura sit haec confusio, hoc capite definire constitui; quae cum ceteris paribus a quantitate aperturae pendeat, hic in perpetuum moneo, me cuiusuis lentis aperturam circularem assumere, per cuius centrum axis lentis transeat: ita vt semidiameter huius circuli simul mensuram aperturae exhibeat. Ita si in superficie lentis PP spatium MAM apertum relinquatur, reliqua parte MP velamine opaco obducta, punctorum extremorum MM distantia diametrum aperturae eiusque semissis semidiametrum aperturae praebebit.

Definitio 2.

8. *Imago extrema est ea, quam radii per extremitatem aperturae transmissi exhibent.*

Ita

Ita si MM sit apertura lentis, radiique EM, EM a puncto lucido E circa oram aperturae transmissi conueniant in puncto f, in hoc ipso puncto f erit imago extrema.

Coroll. 1.

9. Si punctum lucidum E est in ipso axe lentis, nullum est dubium, quin radii inde per marginem circularem MM transeuntes iterum in vno axis puncto f concurrant, imaginemque distinctam repraesentent, quae imago extrema vocatur.

Coroll. 2.

10. Verum si punctum lucidum non esset in axe lentis, hoc neutiquam eueniet, radiique per marginem illum circularem transmissi non amplius in vno puncto colligentur; vnde hoc casu imago extrema eo magis erit confusa, quo magis punctum lucidum ab axe fuerit remotum.

Scholion.

11. Quomodo se habeat refractio radiorum, quando punctum lucidum extra axem lentis fuerit constitutum, quaestio est non solum difficillima, sed etiam ita prolixis calculis inuoluitur, vt inde vix quicquam concludi possit. Ceterum in vsu, ad quem lentes accommodantur, nunquam obiecta ab axe remotiora spectari solent, atque contentos esse nos oportet, dummodo obiecta in ipso axe lentis sita distincte

cte repraesententur, neque etiam confusio, qua obiecta axi proxima afficiuntur, sensibilis esse potest; nam cum imago extrema puncti E in ipso lentis axe siti sit punctum f, nulla confusione inquinatum; etiamsi id parumper esset remotum ab axe, vix sensibilis confusio se immiscere poterit. Quam ob causam investigationes sequentes tantum ad obiecta in ipso lentis axe sita adstringam.

Definitio 3.

12. *Spatium diffusionis vocatur interuallum, inter imaginem principalem et extremam interceptum.*

Ita si imago principalis sit in F, extrema vero in f, interuallum Ff appellatur spatium diffusionis.

Coroll. 1.

13. Si ergo apertura lentis MM euanescat, spatium simul diffusionis euanescit, tum enim tantum radii axi proximi transmittuntur, quibus imago distincta in F effingitur. Ex quo intelligi licet, quo maior fuerit apertura lentis, eo maius fore spatium diffusionis Ff.

Coroll. 2.

14. Cum in F imago a radiis axi proximis, in f autem imago a radiis circa marginem circularem MM transmissis formetur; si totam aperturam lentis infinitis circulis concentricis diuisam concipiamus, radii

radii per singulos circulos transmissi imagines intermedias exhibebunt, quibus interuallum Ff replebitur.

Coroll. 3.

15. Si enim apertura primum nulla, tum vero continuo increscens statuatur, imago extrema primum cum principali congruet; tum vero continuo magis ab ea discedet, sicque cum vsque ad MM fuerit aucta, omnes illae imagines etiam nunc subsistent, spatiumque Ff implebunt.

Scholion 1.

16. Spatium hoc diffusionis caussam continet confusionis, qua repraesentatio imaginis perturbatur; cum enim eiusdem puncti lucidi E infinitae imagines per interuallum Ff dispositae exhibeantur; earum commistio confusionem pariat necesse est, quae eo maior erit, quo maius fuerit spatium diffusionis Ff. Quemadmodum enim ad repraesentationem distinctam requiritur, vt omnes radii ex eodem obiecti puncto emissi iterum in vnico puncto colligantur; ita si hi radii in plura puncta coeant, pluresque eiusdem puncti imagines referant, prout hae magis minusue inter se discrepant, maior inde minorue confusio nascetur. Quemadmodum autem hanc confusionem aestimari oporteat, deinceps demum explicare licebit, cum ante spatium diffusionis accurate definire docuerimus: quo circa in hoc capite, cum proposita fuerit lens quaecunque vitrea faciebus sphaericis terminata

pro quauis puncti lucidi E ab ea distantia et quauis apertura spatium diffusionis Ff inuestigare constitui. Quo facto eandem inuestigationem pro duabus pluribusue lentibus inter se coniunctis suscipi conueniet, vt tandem inde confusionem in quibusuis instrumentis dioptricis assignare valeamus.

Scholion 2.

17. Quaestio igitur principalis huius capitis in hoc versatur, vt proposita lente PP in eiusque axe puncto lucido E, radius quicunque incidens EM consideretur, eiusque per lentem refractio definiatur, vnde punctum f, vbi iterum in axem incidat, assignari queat. Cum enim in eodem puncto f omnes radii per totam peripheriam circularem MM transmissi concurrant; ab his imago quaepiam puncti E in f exprimetur, quae erit extrema, si circulus MM in lente eius aperturam determinet; sin autem capiatur minor, imago quaedam intermedia habebitur. Cum autem hic duplex refractio eueniat, altera in ingressu radii EM in vitrum, altera in egressu eiusdem e vitro, quemadmodum eius directio in vtraque inflectatur, seorsim est inuestigandum; vnde duo nascuntur problemata quasi praeliminaria, ex quorum combinatione deinceps negotium conficietur. Verum quo haec problemata commodius calculo expediri queant, quaedam lemmata ex doctrina angulorum petita praemitti oportet.

Lemma I.

18. *Si angulus Φ triginta gradus non excedat eius sinus satis accurate erit* sin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, *si quidem in circulo, cuius radius est* $= 1$, *arcus* Φ, *qui pro illius anguli mensura habetur, in partibus radii exprimatur.*

Demonstratio.

Quantuscunque fuerit angulus Φ, notum est, eius sinum hac serie infinita exprimi:

$$\text{sin. } \Phi = \Phi - \tfrac{1}{6}\Phi^3 + \tfrac{1}{120}\Phi^5 - \tfrac{1}{5040}\Phi^7 + \text{etc.}$$

sumtis igitur tantum binis primis terminis, error committitur reliquis neglectis aequalis, si ergo statuamus sin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, hincque pro variis angulis Φ sinus colligamus, eorum comparatio cum tabulis sinuum errores manifestabit. Ita si sumatur $\Phi = 30^\circ$, quia arcus 180° valet 3, 14159265, erit in partibus radii $\Phi = 0, 52359877$, et $\frac{1}{6}\Phi^3 = 0, 0239246$, hincque

$\Phi - \frac{1}{6}\Phi^3 = 0, 4996741$. at est reuera

sin. $\Phi = 0, 5000000$ vnde habetur

error $= 0, 0003259$, qui ergo ne ad $\frac{1}{3000}$ partem radii quidem assurgit. At si angulus Φ caperetur duplo minor, scilicet $\Phi = 15^\circ$, reperiretur

$\Phi - \frac{1}{6}\Phi^3 = 0, 2588088$

cum tamen sit sin. $\Phi = 0, 2588190$

errore existente $= 0, 0000102$

qui tricies bis minor est quam casu praecedente. Cum ergo in praxi error partem adeo termillesimam radii adaequans facile tolerari possit, multo magis, si an-

 gulus

gulus ϕ fuerit 30° minor, expreſſio $\phi - \frac{1}{6}\phi^3$ verum eius ſinum exhibere cenſenda erit.

Lemma 2.

19. *Viciſſim ſi anguli triginta gradibus minoris detur ſinus* $= s$, *ex eo ipſe angulus ita proxime definitur, vt ſit in circulo, cuius radius* $= 1$, *arcus eum metiens* $= s + \frac{1}{6}s^3$.

Demonſtratio.

Si enim ϕ deſignet iſtum angulum, cuius ſinus proponitur $= s$; modo vidimus, eſſe ſatis exacte $s = \phi - \frac{1}{6}\phi^3$, hinc autem per conuerſionem oritur proxime $\phi = s + \frac{1}{6}s^3$, quae expreſſio quantum peccet in angulo triginta graduum, vt videamus, ſit $s = \frac{1}{2}$ eritque $s + \frac{1}{6}s^3 = 0,5208333$. Cum autem numerus $3,14159265$ reſpondeat angulo 180°, hic numerus $0,5208333$ praebet angulum $29^\circ\ 50'\ 30''$: verus autem angulus eſt 30°, ita vt error ſit $9'\ 30''$. Sit autem $s = \frac{1}{4}$ cui ſinui reſpondet angulus $= 14^\circ\ 28'\ 39''$, erit $s + \frac{1}{6}s^3 = 0,2526041 = 14^\circ\ 28'\ 23''$, ita vt hoc caſu error non excedat $16''$. Patet ergo dum angulus minor ſit 30°, hoc modo ſatis exacte ex dato ſinu reperiri angulum.

Problema 1.

Tab. I. Fig. 2. 20. Si ex puncto lucido E in ſuperficiem ſphaericam reuolutione arcus circularis AM circa axem EC genitam incidat radius EM, definire eius, poſtquam fuerit refractus, concurſum cum axe EC.

Solutio.

Solutio.

Sit C centrum ſuperficiei ſphaericae, eiusque radius $CA = CM = f$, et diſtantia puncti lucidi E a ſuperficie refringente $EA = a$, tum vero ſit ratio refractionis radiorum ex aere in vitrum $= n : 1$. Vocetur pro radio incidente CM angulus $ACM = \Phi$, eritque huius anguli ſinus $= \Phi - \frac{1}{6}\Phi^3$. Ponatur iam breuitatis gratia diſtantia $EC = a + f = c$, et quia in triangulo ECM dantur latera $CM = f$, $EC = c$ cum angulo $CEM = \Phi$, fiet producta CM in c

$$CM\,(f) : \text{ſin. } CEM = CE\,(c) : \text{ſin. } EMc$$

ideoque

$$\text{ſin. } EMc = \frac{c}{f}\,\text{ſin. }\Phi = \frac{c}{f}\left(\Phi - \frac{1}{6}\Phi^3\right);$$

vnde ipſe angulus EMc elicitur, neglectis tertia altioribus poteſtatibus ipſius Φ:

$$EMc = \frac{c}{f}\Phi - \frac{c}{6f}\Phi^3 + \frac{c^3}{6f^3}\Phi^3 = \frac{c}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

qui cum ſit $= ECM + CEM$, erit

$$ECM = \frac{c-f}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

Verum angulus EMc eſt angulus incidentiae, ac ſi MO referat radium refractum, erit CMO angulus refractionis, ſicque per hypotheſin

$$\text{ſin. } EMc : \text{ſin. } CMO = n : 1.$$

vnde colligitur

$$\text{ſin. } CMO = \frac{c}{nf}\left(\Phi - \frac{1}{6}\Phi^3\right)$$

hincque ipſe angulus

$$CMO = \frac{c}{nf}\Phi + \frac{c(cc-nnff)}{6n^3f^3}\Phi^3$$

quo ablato ab angulo ECM relinquitur angulus

$$COM = \frac{(n-1)c - nf}{nf}\Phi + \frac{c((n^3-1)cc - nn(n-1)ff)}{6n^3f^3}\Phi^3$$

cuius ſinus propterea erit

$$\text{ſin. } COM = \frac{(n-1)c - nf}{nf}\Phi + \frac{3(n-1)c^3 + 3(n-1)^2ccf - 4n(n-1)cff + nnf^3}{6nnf^3}\Phi^3$$

Iamuero ob

$$\text{ſin. } COM : CM\,(f) = \text{ſin. } CMO : CO, \text{ obtinebitur}$$

$$CO = \frac{\frac{c}{n} - \frac{c}{6n}\Phi\Phi.}{\frac{(n-1)c - nf}{nf} + \frac{3(n-1)c^3 + 3(n-1)^2ccf - 4n(n-1)cff + nnf^3}{6nnf^3}\Phi^2}$$

cuius formulae euolutio praebet:

$$CO = \frac{cf}{(n-1)c - nf} - \frac{cf\Phi\Phi}{6((n-1)c - nf)} - \frac{c(3(n-1)c^3 + 3(n-1)^2ccf - 4n(n-1)cff + nnf^3}{6nf((n-1)c - nf)^2}\Phi\Phi$$

quae porro reducitur ad hanc formam:

$$CO = \frac{cf}{(n-1)c - af} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c - nf)^2}\Phi\Phi$$

cui ſi addamus $CA = f$ prodibit

$$AO = \frac{nf(c-f)}{(n-1)c - nf} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c - nf)^2}\Phi\Phi$$

atque hinc poſitio radii refracti MO primo per interuallum AO modo inuentum definitur, tum vero inſuper ex angulo AOM, qui erit:

$$AOM = \frac{(n-1)c - nf}{nf}\Phi + \frac{c((n^3-1)cc - nn(n-1)ff)}{6n^3f^3}\Phi^3$$

ſiue

$$AOM = \frac{(n-1)c - nf}{nf}\Phi + \frac{(n-1)c((nn+n+1)cc - nnff)}{6n^3f^3}\Phi^3$$

Coroll. I.

21. Cum poſuerimus $c = a + f$ erit $c - f = a$, et $(n-1)c - nf = (n-1)a - f$,

atque

atque

$$(nn+n+1)cc-nnff=(nn+n+1)aa+2(nn+n+1)af+(n+1)ff$$

quibus valoribus ſubſtitutis habebimus

$$AO=\frac{naf}{(n-1)a-f}-\frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi \text{ et}$$

$$AOM=\frac{(n-1)a-f}{nfa}\Phi+\frac{(n-1)(a+f)((nn+n+1)a(a+2f)+(n+1)ff)}{6n^3f^3}\Phi^3$$

Coroll. 2.

22. Si M ſit in extremitate aperturae, erit ſemidiameter aperturae $=f$ ſin. ECM $=a\Phi$ ſatis exacte: neque enim neceſſe eſt, aperturam tam exacte noſſe. Vnde ſi ſemidiameter aperturae ponatur $=x$, erit prope $x=a\Phi$, ideoque $\Phi=\frac{x}{a}$.

Coroll. 3.

23. Si accuratius rem definire veſimus, quia eſt

$$\text{ſin. } ECM=\frac{c-f}{f}\Phi+\frac{(c-f)(3c-f)}{6ff}\Phi^3=\frac{a}{f}\Phi+\frac{a(3a+2f)}{6ff}\Phi^3$$

erit $x=a\Phi+\frac{a(3a+2f)}{6f}\Phi^3$

ideoque $\Phi=\frac{x}{a}-\frac{(3a+2f)x^3}{6a^3f}$

ſed quia in valore ipſius AO non vltra ſecundam dimenſionem ipſius Φ aſcendimus, hac expreſſione non indigemus.

Coroll. 4.

24. Etſi hae expreſſiones tantum ſunt prope verae; tamen in praxi ſine errore adhiberi poterunt, dummodo anguli, qui in calculum ſunt ingreſſi, infra

30°

30° ſubſiſtant. Non ſolum ergo neceſſe eſt, vt angulus $AEM = \Phi$ ſed etiam angulus EMc ſeu $\frac{c}{f}\Phi = \Phi + \frac{a}{f}\Phi$ minor ſit 30 gradibus.

Scholion.

25. Summo quidem rigore geometrico diſtantiam EO definire potuiſſemus, neque opus fuiſſet ad approximationes confugere: ſcilicet ſi poſuiſſemus angulum $ECM = \omega$, et diſtantiam $CO = u$; deueniſſemus ad hanc determinationem.

$$\frac{cf}{u} = -c\cos.\omega + \sqrt{(nn(c\cos.\omega - f)^2 + (nn-1)cc\sin.\omega^2)}$$

foretque

$$\cos.\omega = \frac{c}{f}\sin.\Phi^2 + \cos.\Phi\sqrt{(1 - \frac{cc}{ff}\sin.\Phi^2)} \text{ et}$$

$$\sin.\omega = \frac{c}{f}\sin.\Phi\cos.\Phi - \sin.\Phi\sqrt{(1 - \frac{cc}{ff}\sin.\Phi^2)}$$

hisque valoribus ſubſtitutis

$$\frac{cf}{u} = \frac{-cc\sin.\Phi^2}{f} - c\cos.\Phi\sqrt{(1 - \frac{cc}{ff}\sin.\Phi^2)}$$
$$+ (c\cos.\Phi - \sqrt{(ff - cc\sin.\Phi^2)})\sqrt{(nn - \frac{cc}{ff}\sin.\Phi^2)}$$

vnde ponendo angulo Φ valde paruo per aproximationem ſuperior expreſſio eliceretur. Verum praecedens analyſis magis ad praeſens inſtitutum videtur accommodata. Interim ſi quis propius ad veritatem accedere voluerit, ex formula vera hic exhibita adipiſcetur.

$$\frac{cf}{u} = (n-1)c\cos.\Phi - nf + \frac{(n-\cdot)cc}{2nff}((n-1)f + c\cos.\Phi)\sin.\Phi^2$$
$$+ \frac{c^4}{8n^3f^4}((nn-1)^2 f + (n^3-1)c\cos.\Phi)\sin.\Phi^4$$

ſeu

feu

$$\frac{cf}{u} = (n-1)c - nf + \frac{(n-1)c(c-f)(c+nf)}{2nff}\Phi\Phi$$
$$+ \frac{(n-1)(c-f)((nn+n+1)c^3 + 3nn(n+1)ccf + nn(3n-4)cff - n^3f^3)}{24n^3f^4}\Phi^4.$$

Problema 2.

26. Si prout in problemate praecedente inuenimus, radius MO in vitrum miſſus per ſuperficiem ſphaericam BN iterum in aerem erumpat, definire punctum V, vbi is cum axe concurret.

Solutio.

Ponatur interuallum $AB = d$; ſitque ſuperficiei ſphaericae BN centrum in D eiusque radius $DB = DN = g$. Quoniam igitur poſitio radii incidentis MNO datur, ponamus $BO = b$, et angulum $BON = \psi$, et breuitatis ergo interuallum $DO = b + g = e$. Cum nunc in triangulo DON dentur latera $DN = g$, $DO = e$ cum angulo $BON = \psi$, reperitur

$$\text{ſin. } DNM = \frac{e}{g}\text{ ſin. }\psi = \frac{e}{g}\psi - \frac{e}{6g}\psi^3, \text{ hincque}$$
$$DNM = \frac{e}{g}\psi + \frac{e(ee-gg)}{6g^3}\psi^3 \text{ et}$$
$$ODN = \frac{e-g}{g}\psi + \frac{e(ee-gg)}{6g^3}\psi^3$$

Sed hic ſin. DNM eſt ſinus incidentiae, cui reſpondet angulus refractionis VNd, cuius ſinus propterea eſt ad illum vt $n : 1$, vnde fit

$$\text{ſin. } VNd = \frac{ne}{g}\psi - \frac{ne}{6g}\psi^3$$

 hinc-

hincque

$$VNd = \frac{ne}{g}\psi + \frac{ne(nnee - gg)}{6g^3}\psi^3$$

a quo ablato angulo ODN relinquitur angulus

$$DVN = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3$$

ergo

$$\text{fin. } DVN = \frac{(n-1)e+g}{g}\psi + \frac{3n(n-1)e^3 - 3(n-1)^2eeg - 4(n-1)egg - g^3}{6g^3}\psi^3$$

Cum nunc fit

$$\text{fin. } DVN : g = \text{fin. } VNd : DV \text{ erit}$$

$$\frac{g}{DV} = \frac{(n-1)e+g+\frac{1}{6gg}(3n(n-1)e^3-3(n-1)^2eeg-4(n-1)egg-g^3)\psi^2}{ne-\frac{1}{6}ne\psi^2}$$

quae expreffio reducitur ad hanc formam:

$$\frac{g}{DV} = \frac{(n-1)e+g}{ne} + \frac{(n-1)(e-g)(ne+g)\psi^2}{2ngg}$$

vnde reciproce oritur

$$DV = \frac{neg}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

et

$$BV = \frac{g(e-g)}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

Puncto autem V inuentó notandum eft effe angulum

$$BVN = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3;$$

Coroll. I.

27. Cum fit

$e = b+g$ erit $(n-1)e+g = (n-1)b+ng$,

quo

quo valore reſtituto habebimus.

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^2}\psi^2 \quad \text{et}$$

$$BVN = \frac{(n-1)b+ng}{g}\psi + \frac{(b+g)((n^3-1)(b+g)^2-(n-1)gg)}{6g^3}\psi^3$$

Coroll. 2.

28. Hae formulae etiam ex praecedentibus erui poſſunt, ſi pro litteris n, a, f ſcribantur $\frac{1}{n}$, $-b$, et $-g$, quoniam hoc modo caſus praecedentis problematis ad hunc reducitur. Praeterea autem, qui angulus ibi erat Φ hic eſt ψ.

Coroll. 3.

29. Quia in praecedente problemate inuenimus tam lineam AO quam angulum AOM erit hoc problemate ad radium illum refractum MO accommodato

$$BO = b = \frac{naf}{(n-1)a-f} - d - \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi$$

et

$$\psi = \frac{(n-1)a-f}{nf}\Phi + \frac{(n-1)(a+f)((nn+n+1)a(a+2f)+(n+1)ff)}{6n^3f^3}\Phi^3$$

Problema 3.

30. Propoſita lente vitrea MABN faciebus ſphaericis AM et BN terminata, ſi a puncto quocunque E in eius axe poſito incidat in eam radius EM, definire punctum V, in quo is poſt geminam refractionem iterum cum axe lentis ſit concurſurus. Tab. I. Fig. 2.

 Solutio.

Solutio.

Confideremus lentem vt vtrinque conuexam, fitque faciei anterioris AM radius $AC = f$, pofterioris vero BN radius $BD = g$; ipfius lentis autem craffities $AB = d$. Lens porro fit vitrea, ita vt fi in eam radius lucis ex aere incidat, fit finus incidentiae ad finum refractionis vt n ad 1. Iam recta iungens centra vtriusque faciei C et D erit axis lentis, in quo reperiatur punctum lucidum E ante lentem in diftantia $AE = a$, vnde fub angulo $AEM = \Phi$ in lentem incidat radius EM, qui prima refractione ita inflectatur, vt productus cum axe concurrat in O. Quodfi iam ex iis, quae problemate primo funt inuenta, ponamus

$$BO = \frac{naf}{(n-1)a-f} - d - \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi = b$$

$$\text{et ang. } BON = \frac{(n-1)a-f}{nf}\Phi = \Psi$$

in valore enim anguli Ψ negligere licet terminum Φ^3 inuoluentem, quoniam calculum tantum ad fecundam poteftatem ipfius Ψ extendimus; his pofitis in problemate fecundo inuenimus fore.

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^2}\Psi\Psi$$

et

$$BVN = \frac{(n-1)b+ng}{g}\Psi.$$

Totum ergo negotium huc redit, vt ifthic pro b et Ψ valo-

valores affignatos fubftituamus, quod quo facilius fieri poffit, ftatuamus

$$b = P - Q\Phi\Phi \text{ et } \psi = R\Phi \text{ vt fit}$$

$$P = \frac{naf}{(n-1)a-f} - d = \frac{naf-(n-1)ad+df}{(n-1)a-f}$$

$$Q = \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2} \text{ et } R = \frac{(n-1)a-f}{nf}$$

Hinc erit:

$$\frac{bg}{(n-1)b+ng} = \frac{Pg-Qg\Phi\Phi}{(n-1)P+ng-(n-1)Q\Phi\Phi} = \frac{Pg}{(n-1)P+ng} - \frac{nQgg\Phi\Phi}{((n-1)P+ng)^2}$$

at in altero membro fufficit pro b fcribere P: ex quo obtinebimus

$$BV = \frac{Pg}{(n-1)P+ng} - \frac{nQgg\Phi\Phi}{((n-1)P+ng)^2} - \frac{n(n-1)PRR(P+g)^2(nP+(n+1)g)}{2g((n-1)P+ng)^2}\Phi\Phi$$

$$\text{et } BVN = \frac{(n-1)P+ng}{g}R\Phi$$

Pro his autem fubftitutionibus notandum eft fore:

$$(n-1)P+ng = \frac{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}{(n-1)a-f}$$

$$P+g = \frac{naf+(n-1)ag-fg-(n-1)ad+df}{(n-1)a-f}$$

$$nP+(n+1)g = \frac{nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+ndf}{(n-1)a-f}$$

vnde concluditur:

$$BV = \frac{nafg-(n-1)adg+dfg}{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}$$

$$- \frac{(n-1)agg(a+f)^2(a+(n+1)f)}{2f(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^2}\Phi\Phi$$

$$- \frac{(n-1)(naf-(n-1)ad+df)(naf+(n-1)ag-fg-(n-1)ad+df)^2(nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+ndf)}{2nffg(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^2}\Phi\Phi$$

$$\text{et } BVN = \frac{n(n-1)a(f+g)-nfg+(n-1)df-(n-1)^2ad}{nfg}\Phi.$$

Coroll. 1.

31. Si angulus Φ prorſus euaneſcat, punctum V cadet in imaginem principalem, cuius ſi a lente diſtantia dicatur $= \alpha$ erit

$$\alpha = \frac{nafg - (n-1)adg + dfg}{n(n-1)a(f+g) - nfg - (n-1)^2 ad + (n-1)df}$$

quae ſi vt data ſpectetur, hac aequatione relatio inter f et g definitur, vt haec diſtantia principalis oriatur.

Coroll. 2.

32. Quare ſi diſtantia obiecti ante lentem ſit $= a$, eiusque imago principalis poſt lentem ad diſtantiam $= \alpha$ proiici debeat huic aequationi ſatisfieri oportet.

$$\begin{aligned} n(n-1)a\alpha(f+g) - nafg + (n-1)adg - (n-1)^2 a\alpha d &= 0 \\ - n\alpha fg + (n-1)\alpha df \quad - dfg & \end{aligned}$$

Coroll. 3.

33. Hanc autem imaginis principalis diſtantiam α in calculum introducendo expreſſiones noſtrae inuentae haud mediocriter contrahentur. Cum enim ſit $\alpha = \frac{Pg}{(n-1)P + ng}$, erit hinc $P = \frac{n\alpha g}{g - (n-1)\alpha}$

et porro

$$(n-1)P + ng = \frac{ngg}{g-(n-1)\alpha}; \quad P + g = \frac{gg + \alpha g}{g - (n-1)\alpha} = \frac{g(\alpha+g)}{g-(n-1)\alpha}$$

$$nP + (n+1)g = \frac{g(\alpha + (n+1)g)}{g - (n-1)\alpha}$$

vnde

vnde facta ſubſtitutione ſequentes determinationes ſimpliciores aſſequemur.

$$BV = \alpha - \frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2}\Phi\Phi$$
$$-\frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2}\Phi\Phi$$

et

$$BVN = \frac{((n-1)a-f)g}{(g-(n-1)\alpha)f}\Phi.$$

Scholion.

34. Non ſolum hae formulae multo ſunt breuiores et concinniores, quam primo inuentae, ſed etiam perſpicuus ordo in iis obſeruatur quo litterae α et g cum litteris a et f ita connexae ſunt, vt permutationem admittant. Nullum igitur eſt dubium, quin ſi ſtatim diſtantiam α in calculum introduxiſſemus, via breuiori ad eas peruenire licuiſſet. Ceterum quia hae formulae poſteriores non amplius craſſitiem lentis $AB = d$ inuoluunt, euidens eſt eas ex primo inventis naſci, ſi introducta diſtantia α craſſities d eliminetur, ſeu eius loco hic valor ſurrogetur.

$$d = \frac{n(n-1)a\alpha(f+g) - n(a+\alpha)fg}{(n-1)^2 a\alpha - (n-1)(ag+\alpha f) + fg} = n\frac{(n-1)a\alpha(f+g)-(a+\alpha)fg}{((n-1)a-f)((n-1)\alpha-g)}$$

qui labor autem ne ſuſcipi quidem mereretur, niſi iam ante de eximio eius vſu certiores eſſemus facti. In ſequentibus igitur his elegantioribus formulis vtamur, quoad negotium adhuc ſuccinctius expedire didicerimus.

Proble-

Problema 4.

35. Propoſita lente quacunque faciebus ſphaericis terminata, ſi obiectum in data ab ea diſtantia ſit conſtitutum, pro data lentis apertura ſpatium diffuſionis aſſignare.

Solutio.

Tab. I. Fig. 1. Concipiamus lentem vtrinque conuexam, ſitque faciei anterioris MAM radius $= f$, poſterioris $NBN = g$, lentisque craſſities $AB = d$. Sit porro MM lentis huius apertura, cuius ſemidiameter ſit $= x$; atque in axe lentis expoſitum ſit obiectum vel ſaltem punctum lucidum E, cuius a lente ponatur diſtantia $AE = a$. Iam primo quaeramus eius imaginem principalem, quae cadat in F atque ſupra (31) invenimus fore

$$BF = \frac{nafg - (n-1)adg + dfg}{n(n-1)a(f+g) - nfg - (n-1)^2 ad + (n-1)df}$$

Vocetur ergo haec diſtantia $= \alpha$, et ſi radii EM circa oram aperturae tranſeant, erit angulus $AEM = \Phi = \frac{x}{a}$, quo valore in ſuperioribus formulis ſubſtituto prodibit diſtantia imaginis extremae f a lente ſcilicet:

$$Bf = \alpha - \frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2} \cdot \frac{xx}{aa}$$
$$- \frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2} \cdot \frac{xx}{aa}$$

ex quo colligitur ſpatium diffuſionis quaeſitum:

$$Ff = + \frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2} \cdot \frac{xx}{aa}$$
$$+ \frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2} \cdot \frac{xx}{aa}$$

ac

ac praeterea angulus $Bf N$ erit

$$Bf N = \frac{((n-1)a-f)g}{(g-(n-1)\alpha)f} \cdot \frac{x}{a}.$$

Coroll. 1.

36. Quoties ergo ſpatium diffuſionis hoc modo expreſſum eſt poſitiuum, imago extrema propius ad lentem cadit quam principalis ſeu eſt $Bf < BF$. Contra autem ſi iſta expreſſio valorem obtineat n gatiuum, imago extrema a lente longius erit remota principali.

Coroll. 2.

37. Patet hinc etiam ſpatium diffuſionis cum apertura ita creſcere, vt ſit quadrato ſemidiametri aperturae proportionale, ſequetur ergo ipſam aperturae rationem.

Coroll. 3.

38. Spatium diffuſionis etiam hoc modo exprimi poteſt

$$Ff = \begin{cases} + \dfrac{(n-1)a(1+\frac{a}{f})^2(1-\frac{(n-1)\alpha}{g})^2(n+1+\frac{a}{f})}{2nn(\frac{(n-1)a}{f}-1)^2} \cdot \dfrac{xx}{aa} \\ + \dfrac{(n-1)\alpha(1+\frac{\alpha}{g})^2(\frac{(n-1)a}{f}-1)^2(n+1+\frac{\alpha}{g})}{2nn(1-\frac{(n-1)\alpha}{g})^2} \cdot \dfrac{xx}{aa} \end{cases}$$

et angulus

$$Bf N = \frac{\frac{(n-1)a}{f}-1}{1-\frac{(n-1)\alpha}{g}} \cdot \frac{x}{a};$$

Coroll. 4.

39. Quia hae formulae introducendis litterarum valoribus reciprocis redditae ſunt ſimpliciores, in hunc modum etiam aequatio (§. 32) exhibita tractetur quae per $a\alpha dfg$ diuiſa abit in hanc formam.

$$n(n-1)\frac{1}{d}\left(\frac{1}{f}+\frac{1}{g}\right)-n\frac{1}{d}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+(n-1)\left(\frac{1}{\alpha f}+\frac{1}{ag}\right)-(n-1)^2\frac{1}{fg}-\frac{1}{a\alpha}=0$$

ſeu

$$\frac{n}{d}\left((n-1)a\alpha\left(\frac{1}{f}+\frac{1}{g}\right)-a-\alpha\right)=\left(\frac{(n-1)a}{f}-1\right)\left(\frac{(n-1)\alpha}{g}-1\right)$$

quae commodior erit tam ad relationem inter a et α, quam inter f et g definiendam.

Scholion.

40. Scilicet ſi propoſita lente obiectum in variis diſtantiis exponatur ac pro ſingulis diſtantiam imaginis principalis definire velimus, aequatio hoc modo referatur:

$$\frac{1}{a\alpha}-(n-1)\left(\frac{1}{\alpha f}+\frac{1}{ag}\right)+n\left(\frac{1}{da}+\frac{1}{d\alpha}\right)-n(n-1)\left(\frac{1}{df}+\frac{1}{dg}\right)+(n-1)^2\frac{1}{fg}=0$$

quae per factores ita adornari poterit

$$\left(\frac{1}{a}-\frac{(n-1)}{f}+\frac{n}{d}\right)\left(\frac{1}{\alpha}-\frac{(n-1)}{g}+\frac{n}{d}\right)=\frac{nn}{dd}$$

Vnde patet productum ex his duobus factoribus ſemper eſſe idem. Cuius reſolutio quo facilius inſtituatur, capiantur numeri μ et ν ita, vt eorum ſumma ſit $=1$, ſcilicet $\mu+\nu=1$ ac ſtatuatur

$$\frac{1}{a}-\frac{(n-1)}{f}+\frac{n}{d}=\frac{-\mu n}{\nu d} \text{ erit } \frac{1}{a}=\frac{n-1}{f}-\frac{n}{\nu d}$$

$$\frac{1}{\alpha}-\frac{(n-1)}{g}+\frac{n}{d}=\frac{-\nu n}{\mu d} \text{ erit } \frac{1}{\alpha}=\frac{n-1}{g}-\frac{n}{\mu d}$$

Quare

Quare si distantia obiecti $AE = a$ ita capiatur, vt sit $\frac{1}{a} = \frac{n-1}{f} - \frac{n}{\nu d}$, distantia imaginis principalis $BF = \alpha$ ita se habebit, vt sit $\frac{1}{\alpha} = \frac{n-1}{g} - \frac{n}{\mu d}$.

Eodem modo si dentur binae distantiae $AE = a$ et $BF = \alpha$ cum crassitie lentis $AB = d$, radii facierum f et g ita debent esse comparati, vt sit

$$\frac{1}{f} = \frac{1}{(n-1)a} + \frac{n}{\nu(n-1)d} \text{ et } \frac{1}{g} = \frac{1}{(n-1)\alpha} + \frac{n}{\mu(n-1)d}$$

id quod infinitis modis praestari potest, cum numeri μ et ν pro arbitrio accipi queant, dummodo eorum summa $\mu + \nu$ aequetur vnitati. Si hoc modo etiam in formulis pro spatio diffusionis inuentis omnes litterae in denominatores detrudantur, reperietur:

$$Ff = \frac{(n-1)\alpha\alpha xx}{2nn}\left\{\frac{\left(\frac{1}{a}+\frac{1}{f}\right)^2\left(\frac{1}{\alpha}-\frac{(n-1)}{g}\right)^2\left(\frac{n+1}{a}+\frac{1}{f}\right)}{\left(\frac{n-1}{f}-\frac{1}{a}\right)^2} + \frac{\left(\frac{1}{\alpha}+\frac{1}{g}\right)^2\left(\frac{n-1}{f}-\frac{1}{a}\right)^2\left(\frac{n+1}{\alpha}+\frac{1}{g}\right)}{\left(\frac{1}{\alpha}-\frac{(n-1)}{g}\right)^2}\right\}$$

et

$$\text{angulus } BfN = \frac{\frac{n-1}{f}-\frac{1}{a}}{\frac{1}{\alpha}-\frac{(n-1)}{g}} \cdot \frac{x}{\alpha} = \frac{\frac{n}{\nu d}}{\frac{-n}{\mu d}} \cdot \frac{x}{\alpha} = -\frac{\mu}{\nu} \cdot \frac{x}{\alpha}$$

Cum autem sit

$$\frac{n-1}{f} - \frac{1}{a} = \frac{n}{\nu d} \text{ et } \frac{1}{\alpha} - \frac{(n-1)}{g} = \frac{-n}{\mu d} \text{ erit}$$

$$Ff = \frac{(n-1)\alpha\alpha xx}{2nn}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{1}{a}+\frac{1}{f}\right)^2\left(\frac{n+1}{a}+\frac{1}{f}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{1}{\alpha}+\frac{1}{g}\right)^2\left(\frac{n+1}{\alpha}+\frac{1}{g}\right)\right)$$

et

$$\text{angulus } BfN = -\frac{\mu x}{\nu\alpha} = \frac{\mu x}{(\mu-1)\alpha}.$$

 Proble-

Problema 5.

Tab. I. Fig. 1. 41. Datis diſtantiis obiecti ante lentem $AE = a$ et imaginis principalis poſt lentem $BF = \alpha$ vna cum lentis craſſitie $AB = d$, definire omnes lentes ſatisfacientes, ſimulque pro ſingulis ſpatium diffuſionis Ff.

Solutio.

Modo vidimus ſi lentis faciei anterioris AM radius ponatur $= f$, poſterioris $BN = g$, lente vt conuexa vtrinque ſpectata, hos duos radios ita comparatos eſſe debere, vt ſit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{\nu d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} + \frac{n}{\mu d}$$

ſumtis pro μ et ν numeris quibuscunque, vt ſit $\mu + \nu = 1$; vnde infinitae lentes quaeſito ſatisfacientes obtinentur. Deinde ſi huius lentis ſemidiameter aperturae ponatur $= x$, ſpatium diffuſionis Ff ita exprimi poteſt vt ſit

$$Ff = \frac{\alpha\alpha xx}{2nn(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{n-1}{a} + \frac{n-1}{f}\right)^2\left(\frac{nn-1}{a} + \frac{n-1}{f}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{n-1}{\alpha} + \frac{n-1}{g}\right)^2\left(\frac{nn-1}{\alpha} + \frac{n-1}{g}\right)\right)$$

et angulus $BfN = \frac{\mu x}{(\mu - 1)\alpha}$.

Quod ſi iam hic pro $\frac{n-1}{f}$ et $\frac{n-1}{g}$ valores aſſignati ſubſtituantur ſpatium diffuſionis erit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{1}{a} + \frac{1}{\nu d}\right)^2\left(\frac{n}{a} + \frac{1}{\nu d}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{1}{\alpha} + \frac{1}{\mu d}\right)^2\left(\frac{n}{\alpha} + \frac{1}{\mu d}\right)\right)$$

ſeu

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left\{\begin{aligned}&+\frac{\nu\nu}{\mu\mu}\left(\frac{n}{a^3} + \frac{2n+1}{\nu aad} + \frac{n+2}{\nu\nu a dd} + \frac{1}{\nu^3 d^3}\right)\\&+\frac{\mu\mu}{\nu\nu}\left(\frac{n}{\alpha^3} + \frac{2n+1}{\mu\alpha\alpha d} + \frac{n+2}{\mu\mu\alpha dd} + \frac{1}{\mu^3 d^3}\right)\end{aligned}\right\}$$

Si

Si craſſities lentis fuerit valde parua, ne confuſio fiat enormis, neceſſe eſt pro μ et ν ſumi numeros vehementer magnos, alterum ſcilicet poſitiuum alterum negatiuum. Cum igitur ſit $\mu d + \nu d = d$, ſtatuatur $\mu d = \frac{d-k}{2}$ et $\nu d = \frac{d+k}{2}$, vt ſit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d} \text{ ſeu}$$

$$f = \frac{(n-1)a(k+d)}{2na+k+d} \text{ et } g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

hincque obtinetur ſpatium diffuſionis

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left\{\begin{array}{l}+\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2\\ +\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2\end{array}\right\}$$

in quibus formulis paruitas craſſitiei lentis $AB = d$ nullum negotium faceſſit: tum vero eſt

angulus $BfN = \frac{k-d}{k+d} \cdot \frac{x}{\alpha}$.

Coroll. 1.

42. Propoſitis ergo binis diſtantiis $AE = a$ et $BF = \alpha$ vna cum craſſitie lentis $AB = d$, infinitis modis lentes idoneae parari poſſunt cum pro k quantitates pro lubitu ſiue poſitiuae ſiue negatiuae aſſumi queant.

Coroll. 2.

43. Cum ſemidiameter aperturae x faciem anteriorem reſpiciat, et angulus BfN ſeu BFN ſit $= \frac{k-d}{k+d} \cdot \frac{x}{\alpha}$, exiſtente diſtantia $BF = \alpha$, manifeſtum eſt ſemidiame-

 trum

trum aperturae poſterioris faciei eſſe debere $= \frac{k-d}{k+d} \cdot x$ vel ſaltem non minorem.

Coroll. 3.

44. Quodſi lentis craſſities tam ſit parua, vt ea prae k contemni queat, formulae noſtrae fient ſimpliciores

$$f = \frac{(n-1)ak}{k+2na} \text{ et } g = \frac{(n-1)\alpha k}{k-2n\alpha}$$

et ſpatium diffuſionis

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k}\right)\left(\frac{1}{a}+\frac{2}{k}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k}\right)\left(\frac{1}{\alpha}-\frac{2}{k}\right)^2\right)$$

angulusque $BfN = \frac{x}{\alpha}$.

Scholion.

45. Sufficiat haec de ſpatio diffuſionis in genere quaecunque fuerit lentis craſſities, tradidiſſe, cum non obſtante his ſatis concinnis transformationibus calculus nimium fieret moleſtus, ſi in determinatione ſpatii diffuſionis rationem craſſitiei lentis habere vellemus. Licebit autem, vt modo vidimus, craſſitiem lentis negligere non ſolum, quando ipſa eſt per ſe valde exigua verum etiam dummodo prae quantitate k fuerit perparua. Atque hinc etiam in ſequentibus facile iudicare poterimus, vtrum craſſitiem lentis recte in calculo contemſerimus, nec ne? quouis enim caſu conſideretur quantitas pro k aſſumta, quae ſi multoties fuerit maior, quam d error nullus erit pertimeſcendus;

ſcendus; contra vero ſi k non multum ſuperet d multum aberrabitur, quantumuis exigua fuerit craſſities ipſa per ſe.

Vt fiat Ff minimum, definiri poteſt conueniens valor ipſius k hoc modo. Ponatur $\frac{k-d}{k+d} = z$ et pro minimo peruenitur ad hanc aequationem:

$$\begin{aligned} 0 = {} & 2z^4\left(\frac{n}{\alpha}+\frac{1}{d}\right)\left(\frac{1}{\alpha}+\frac{1}{d}\right)^2 \\ & -\frac{2nz^3}{\alpha d}\left(\frac{1}{\alpha}+\frac{1}{d}\right)-\frac{z^3}{d}\left(\frac{1}{\alpha}+\frac{2}{d}\right)^2 \\ & +\frac{2nz}{ad}\left(\frac{1}{a}+\frac{1}{d}\right)+\frac{z}{d}\left(\frac{1}{a}+\frac{2}{d}\right)^2 \\ & -2\left(\frac{n}{a}+\frac{1}{d}\right)\left(\frac{1}{a}+\frac{1}{d}\right)^2 \end{aligned}$$

vnde valor ipſius z erui debet.

Problema 6.

46. Neglecta lentis craſſitie, ſi detur cum obiecti ante lentem diſtantia $AE = a$, tum imaginis principalis poſt lentem diſtantia $BF = \alpha$, eam definire lentem quae pro data apertura minimam pariat diffuſionem.

Solutio.

Poſitis radiis faciei anterioris $AM = f$ et poſterioris $BN = g$ vtraque vt conuexa ſpectata, vidimus omnes lentes datis diſtantiis a et α conuenientes his formulis determinari: ſi ſcilicet pro k ſcribamus $2k$

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{k} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{n}{k}$$

deno-

denotante k quantitatem quamcunque. Tum autem ſpatium diffuſionis ita exprimi eſt repertum

$$Ff = \frac{n a \alpha x x}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{1}{k}\right)\left(\frac{1}{a}+\frac{1}{k}\right)^2 + \left(\frac{n}{\alpha}-\frac{1}{k}\right)\left(\frac{1}{\alpha}-\frac{1}{k}\right)^2\right)$$

quae expreſſio ad hanc reducitur formam:

$$Ff = \frac{n a \alpha x x}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{2n+1}{k}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{n+2}{kk}\right).$$

Quaeſtio igitur huc reducitur, vt definiatur quantitas k qua huic expreſſioni valor minimus concilietur; cui quidem requiſito ſatisfacit valor

$$\frac{1}{k} = \frac{-(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right) \text{ ſeu } \frac{1}{k} = \frac{2n+1}{2(n+2)}\left(\frac{1}{\alpha}-\frac{1}{a}\right)$$

quo ſubſtituto habebitur:

$$\frac{n-1}{f} = \frac{4+n-2nn}{2(n+2)a} + \frac{n(2n+1)}{2(n+2)\alpha} \text{ et } \frac{n-1}{g} = \frac{4+n-2nn}{2(n+2)\alpha} + \frac{n(2n+1)}{2(n+2)a}.$$

et ſpatium diffuſionis quod eſt minimum:

$$Ff = \frac{n a \alpha x x}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)-\frac{(2n+1)^2}{4(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2\right)$$

At eſt

$$n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right) = n\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{n}{a\alpha} \text{ ideoque}$$

$$Ff = \frac{n a \alpha x x}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{n}{a\alpha}\right)$$

quae expreſſio etiam in hanc formam transfunditur

$$Ff = \frac{n a \alpha x x}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{(n-1)^2}{(n+2)a\alpha}\right)$$

ſeu

$$Ff = \frac{n(4n-1)a\alpha x x}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

Coroll.

Coroll. 1.

47. Vt igitur lens minimum ſpatium diffuſionis producat, eam ita formari neceſſe eſt vt ſit

$$f = \frac{2(n-1)(n+2)a\alpha}{n(2n+1)a + (4+n-2nn)\alpha} \text{ et}$$

$$g = \frac{2(n-1)(n+2)a\alpha}{n(2n+1)\alpha + (4+n-2nn)a}$$

neglecta ſcilicet lentis craſſitie, quae quidem recte negligitur, ſi modo fuerit vehementer parua prae quantitate $k = \frac{2(n+2)a\alpha}{(2n+1)(a-\alpha)}$.

Coroll. 2.

48. Si igitur ſit $a = \alpha$, ſeu BF $=$ AE craſſities lentis, quantacunque fuerit, nihil turbat in ſpatio diffuſionis. Pro hoc autem caſu erit $f = g = (n-1)a$ et ſpatium diffuſionis ipſum $Ff = \frac{nnxx}{(n-1)^3 a}$. At quo magis diſtantiae a et α a ſe inuicem diſcrepant, ſeu minor fuerit quantitas $\frac{a\alpha}{a-\alpha}$, eo magis haec determinatio ob lentis craſſitiem fiet erronea.

Coroll. 3.

49. Spatium autem hoc confuſionis minimum Ff pluribus modis exhiberi poteſt, inter quos commodiſſimum eligi conuenit; ſunt autem praecipui:

$$\text{I. } Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

$$\text{II. } Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2 + \frac{4n(n+2)}{(4n-1)a\alpha}\right)$$

$$\text{III. } Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{aa}+\frac{1}{\alpha\alpha}\right) + \frac{2(2nn+1)}{(4n-1)a\alpha}\right)$$

$$\text{IV. } Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right) + \frac{(2n+1)^2}{(4n-1)a\alpha}\right)$$

Coroll. 4.

50. Cum ergo hoc ſpatium diffuſionis ſit minimum, ſi lenti alia quaecunque figura tribuatur, ita tamen, vt obiecti ad diſtantiam $AE = a$ remoti imaginem principalem in diſtantia $BF = \alpha$ exhibeat, ſpatium diffuſionis erit maius, quam hic inuenimus.

Scholion.

51. Quia $n:1$ denotat rationem refractionis ex aere in vitrum, haec autem pro radiorum natura eſt variabilis, conueniet pro n medium valorem aſſumi, qui eſt $n = \frac{31}{20}$; hinc ergo erit

$$n-1 = \tfrac{11}{20};\ n+2 = \tfrac{71}{20};\ 2n+1 = \tfrac{41}{10};\ 4+n-2nn = \tfrac{149}{200},$$

hincque

$$\frac{n(2n+1)}{2(n-1)(n+2)} = \frac{1271}{781} = 1,627401$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = \frac{149}{781} = 0,190781$$

vnde lens minimam confuſionem pariens ita definietur

$$\frac{1}{f} = \frac{149}{781a} + \frac{1271}{781\alpha} = \frac{0,190781}{a} + \frac{1,627401}{\alpha}$$

$$\frac{1}{g} = \frac{149}{781\alpha} + \frac{1271}{781a} = \frac{0,190781}{\alpha} + \frac{1,627401}{a}$$

ſeu

$$f = \frac{781a\alpha}{149\alpha + 1271a} = \frac{a\alpha}{0,190781\,\alpha + 1,627401\,a}$$

et

$$g = \frac{781a\alpha}{149a + 1271\alpha} = \frac{a\alpha}{0,190781\,a + 1,627401\,\alpha}$$

Pro

Pro ſpatio autem diffuſionis ipſo definiendo ob $4n-1=\frac{26}{6}$

erit $\frac{n(4n-1)}{8(n-1)^2(n+2)}=\frac{8060}{8591}=0,938191$

et $\frac{4(n-1)^2}{4n-1}=\frac{121}{520}=0,232692$

hincque

$$Ff=0,938191\,\alpha\alpha xx(\tfrac{1}{a}+\tfrac{1}{\alpha})((\tfrac{1}{a}+\tfrac{1}{\alpha})^2+\tfrac{0,232692}{\alpha\alpha})$$

vnde reliquae formulae facile deducuntur. Ceterum hic monendum eſt, quoniam valor $n=\frac{31}{20}$ ex experimentis eſt concluſus, neque ipſe pro omnibus radiorum generibus valet, ſuperfluum fore in praxi hos numeros inuentos nimis ſtudioſe obſeruare; quin etiam ipſa natura minimi aliquam aberrationem permittit. Nihilo vero minus has fractiones decimales ad tot figuras producere viſum eſt, quo facilius quantum ab hac hypotheſi aberretur, dignoſci queat.

Problema 7.

52. Neglecta lentis craſſitie, ſi detur cum obiecti ante lentem diſtantia $AE=a$, tum imaginis principalis poſt lentem diſtantia $BF=\alpha$, eam definire lentem, quae pro data apertura non minimum, ſed datum pariat ſpatium diffuſionis Ff.

Solutio.

Lente vtrinque vt conuexa ſpectata, ſit faciei anterioris radius $=f$, poſterioris $=g$; atque vt ex data

 diſtan-

diſtantia obiecti $AE = a$, data oriatur imaginis principalis diſtantia $= \alpha$ neceſſe eſt ſit in genere

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{k} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{n}{k}$$

vnde ſpatium diffuſionis fit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa} - \frac{1}{a\alpha} + \frac{1}{\alpha\alpha}\right) + \frac{2n+1}{k}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{n+2}{kk}\right)$$

Cum autem ſpatium diffuſionis minimum repertum ſit

$$\frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

neceſſe eſt vt illud ſit maius: ſtatuatur ergo:

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha} + S\right)$$

et habebitur iſta aequatio:

$$4n(n+2)\left(\frac{1}{aa} - \frac{1}{a\alpha} + \frac{1}{\alpha\alpha}\right) + \frac{4(n+2)(2n+1)}{k}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{4(n+2)^2}{kk}$$
$$= (4n-1)\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{a\alpha} + (4n-1)S$$

quae in hanc formam redigitur.

$$(2n+1)^2\left(\frac{1}{a} - \frac{1}{\alpha}\right)^2 + \frac{4(n+2)(2n+1)}{k}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{4(n+2)^2}{kk} = (4n-1)S$$

vnde radice extracta fit

$$(2n+1)\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{2(n+2)}{k} = \sqrt{(4n-1)S}$$

$$\text{et } \frac{1}{k} = \frac{-(2n+1)}{2(n+2)}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{\sqrt{(4n-1)S}}{2(n+2)}$$

Quare erit

$$\frac{n-1}{f} = \frac{1}{a} - \frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{n}{2(n+2)}\sqrt{(4n-1)S}$$

$$\frac{n-1}{g} = \frac{1}{\alpha} + \frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a} - \frac{1}{\alpha}\right) - \frac{n}{2(n+2)}\sqrt{(4n-1)S}$$

ſiue

fiue

$$f = \frac{2(n-1)(n+2)a\alpha}{(4+n-2nn)\alpha + n(2n+1)a + na\alpha\sqrt{(4n-1)S}}$$

$$g = \frac{2(n-1)(n+2)a\alpha}{(4+n-2nn)a + n(2n+1)\alpha - na\alpha\sqrt{(4n-1)S}}$$

Oportet ergo pro S fumi quantitatem pofitiuam, atque vt cum reliqua parte, cui jungitur, fit homogenea, ponamus

$$S = (\lambda - 1)\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2$$

vbi λ denotat numerum vnitate maiorem; atque effici poterit, vt fpatium diffufionis ita exprimatur

$$Ff = \frac{n(4n-1)a\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

quod cum vt datum fpectetur, numerus λ pro dato affumi poterit, ac lens hunc effectum producens ita determinabitur.

$$f = \frac{2(n-1)(n+2)a\alpha}{(4+n-2nn)\alpha + n(2n+1)a + n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

$$g = \frac{2(n-1)(n+2)a\alpha}{(4+n-2nn)a + n(2n+1)\alpha - n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

quod ergo cum fignum radicale ambiguo illatum inuoluat, duplici modo fieri poterit.

Coroll. 1.

53. Omnes igitur lentes, quae obiecti ad diftantiam $AE = a$ remoti, imaginem principalem in diftantia $BF = \alpha$ repraefentant, hanc habent proprietatem vt fit:

$$\frac{n-1}{f} + \frac{n-1}{g} = \frac{1}{a} + \frac{1}{\alpha} \quad \text{feu} \quad \frac{fg}{f+g} = \frac{(n-1)a\alpha}{a+\alpha}.$$

et cum ſit

$$n = \frac{31}{20} \text{ erit } \frac{fg}{f+g} = \frac{11\,a\alpha}{20(a+\alpha)} = \frac{0,55\,a\alpha}{a+\alpha}.$$

Coroll. 2.

54. Si ergo lens ſit vtrinque aeque conuexa ſeu $f = g$, oportet eſſe $f = g = \frac{2(n-1)a\alpha}{a+\alpha} = \frac{11\,a\alpha}{10(a+\alpha)}$.

ſin autem lens deſideretur plano-conuexa vt ſit $f = \infty$, capi debet

$$g = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11\,a\alpha}{20(a+\alpha)} = \frac{0,55\,a\alpha}{a+\alpha}$$

At ſi lens debeat eſſe conuexo-plana ſeu $g = \infty$, capi oportet

$$f = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11\,a\alpha}{20(a+\alpha)} = \frac{0,55\,a\alpha}{a+\alpha}$$

Scholion I.

55. Subſtituamus pro n valorem ipſi conuenientem $\frac{31}{20}$, et iam vidimus fore:

$$\frac{n(4n-1)}{8(n-1)^2(n+2)} = \frac{8060}{8591} = 0,938191$$

$$\frac{4(n-1)^2}{4n-1} = \frac{121}{520} = 0,232692$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = \frac{149}{781} = 0,190781$$

$$\frac{n(2n+1)}{2(n-1)(n+2)} = \frac{1271}{781} = 1,627401$$

nunc vero notari oportet eſſe:

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)} = \frac{62\sqrt{130}}{781} = 0,905133$$

Quare

Quare ſi lens ita conſtruatur vt ſit

$$f = \frac{a\alpha}{0,190781\,\alpha + 1,627401\,a \pm 0,905133(a+\alpha)\sqrt{(\lambda - 1)}}$$

$$g = \frac{a\alpha}{0,190781\,a + 1,627401\,\alpha \mp 0,905133(a+\alpha)\sqrt{(\lambda - 1)}}$$

erit pro eius apertura, cuius ſemidiameter $= x$, ſpatium diffuſionis

$$Ff = 0,938191\,a\alpha xx\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)^2 + \tfrac{0,232692}{a\alpha}\right)$$

Cum autem in poſterum hi numeri frequentiſſime occurrant eorum loco ad abbreuiandum certis characteribus vtamur, ponamus ergo

$$\frac{n(4n-1)}{8(n-1)^2(n+2)} = 0,938191 = \mu$$

$$\frac{4(n-1)^2}{4n-1} = 0,232692 = \nu$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = 0,190781 = \varrho$$

$$\frac{n(2n+1)}{2(n-1)(n+2)} = 1,627401 = \sigma$$

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)} = 0,905133 = \tau$$

ſeu

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n+1)^2}$$

$$\varrho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1$$

$$\sigma = 1 + \frac{1}{2(n-1)} - \frac{1}{n+2}$$

$$\tau = \tfrac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right)\sqrt{(4n-1)}$$

vnde pro quouis alio medio pellucido hi valores ſupputari poſſunt ſicque

ſicque vt prodeat ſpatium diffuſionis

$$Ff = \mu a\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

lentis conſtructio erit

$$f = \frac{a\alpha}{\varrho\alpha+\sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

$$g = \frac{a\alpha}{\varrho a+\sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

Dummodo igitur λ fuerit numerus poſitiuus vnitate non minor talis lens duplici modo effici poterit. Caſu autem $\lambda = 1$, quo ſpatium diffuſionis eſt minimum, vnica lens propoſito ſatisfaciens conſtrui poteſt.

Coroll. 3.

56. Si igitur lens ſit vtrinque aequaliter conuexa, ideoque $f = g = \frac{2(n-1)a\alpha}{a+\alpha} = \frac{11}{10}\cdot\frac{a\alpha}{a+\alpha}$, in expreſſione noſtra pro ſpatio diffuſionis inuenta valor ipſius λ ex aequalitate inter f et g ſtatuta definietur, vnde fit

$$(\sigma-\varrho)(a-\alpha) = 2\tau(a+\alpha)\sqrt{(\lambda-1)} \text{ hincque}$$

$$\lambda = 1 + \frac{0{,}6297950\alpha\alpha - 1{,}2595890 a\alpha + 0{,}629795 aa}{(a+\alpha)^2}.$$

Coroll. 4.

57. Si lens capiatur plano conuexa vt ſit:

$$f = \infty \text{ et } g = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11}{20}\cdot\frac{a\alpha}{a+\alpha}$$

pro ſpatio diffuſionis habebitur

$$\varrho\alpha+\sigma a = \pm\tau(a+\alpha)\sqrt{(\lambda-1)}$$

vnde in numeris colligitur

$$\lambda = 1 + \frac{0{,}044427\alpha\alpha + 0{,}757940 a\alpha + 3{,}232692 aa}{(a+\alpha)^2}$$

Coroll.

Coroll. 5.

58. Si denique lens adhibeatur conuexo-plana, vt sit:

$$g = \infty \text{ et } f = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11}{20} \cdot \frac{a\alpha}{a+\alpha}$$

pro ſpatio diffuſionis inueniendo poni oportet

$$\varrho a + \sigma\alpha = \pm \tau(a+\alpha)\sqrt{(\lambda-1)}$$

vnde in numeris colligimus:

$$\lambda = 1 + \frac{3,232692\alpha\alpha + 0,757940a\alpha + 0,044427aa}{(a+\alpha)^2}$$

Scholion 2.

59. Quod ad aperturam attinet iam initio animaduertimus, in ea maiores arcus comprehendi non debere, quam qui ſint principiis ſtabilitis conformes. Scilicet vt nullus angulus ſupra 30° gradus occurrat, anguli ACM et BDN certe minores 30 gradibus eſſe debent, cum anguli EMc et VNd ipſis ſint maiores, quorum alter cum quandoque ad duplum aſſurgere poſſit, poterimus hanc regulam ſtatuere, vt aperturae ſemidiameter x neque $\frac{1}{4}f$ neque $\frac{1}{4}g$ ſuperet. Verum quouis caſu ad ipſos angulos EMc et VNd, qui ſunt maximi, attendi conueniet, atque tanta apertura admitti poterit, vnde neuter horum angulorum 30 gradus ſuperans oriatur, ſi cautius procedere velimus, etiam angulos 20 gradibus minores euitare poterimus, quo pacto apertura magis reſtringetur. Tab. I. Fig. 2.

Problema 8.

Tab. I. Fig. 3. 60. Non neglecta lentis craſſitie AB ſi pro diſtantia obiecti $AE = a$ detur diſtantia imaginis principalis $BF = \alpha$, obiectum autem parumper longius in e remoueatur, definire locum imaginis principalis f.

Solutio.

Poſita lentis craſſitie $AB = d$, radiisque faciei anterioris $AM = f$ et poſterioris $BN = g$, ſupra vidimus hos radios ita a binis diſtantiis a et α atque craſſitie lentis d pendere debere, vt ſit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d}$$

denotante k quantitatem quamcunque. Hinc ergo cum ſit

$$\frac{k+d}{2n} = \frac{af}{(n-1)a-f} \text{ et } \frac{k-d}{2n} = \frac{\alpha g}{g-(n-1)\alpha}$$

erit eliminando k.

$$\frac{d}{n} = \frac{af}{(n-1)a-f} - \frac{\alpha g}{g-(n-1)\alpha}.$$

Ponamus iam diſtantiam $AE = a$ creſcere particula $Ee = da$ ac per differentiationem inueniemus, quantum inde diſtantia imaginis $BF = \alpha$ immutetur: habebimus ſcilicet:

$$\frac{-ffda}{((n-1)a-f)^2} - \frac{ggd\alpha}{(g-(n-1)\alpha)^2} = 0$$

vnde elicimus

$$d\alpha = \frac{-ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2}da = \frac{-\alpha\alpha}{aa}da\left(\frac{k+d}{k-d}\right)^2$$

Quare

Quare obiecto E per spatiolum minimum Ee longius a lente remoto, imago principalis ex F propius ad lentem admouebitur per spatiolum minimum Ff, ita vt sit

$$Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee = \frac{ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2} . Ee.$$

Coroll. 1.

61. Quia quantitas $\frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2$ necessario est positiua, euidens est si obiectum longius a lente remoueatur, imaginem semper propius ad lentem admoueri. Contra ergo etiam si obiectum propius ad lentem accedat, imago longius ab ea recedet.

Coroll. 2.

62. Si crassities lentis d euanescat erit $Ff = \frac{\alpha\alpha}{aa} . Ee$, sin autem ea non euanescat fieri potest vt sit vel $Ff > \frac{\alpha\alpha}{aa} Ee$ vel $Ff < \frac{\alpha\alpha}{aa} . Ee$, prius eueniet si k sit quantitas positiua, posterius si negatiua. At si sit vel $k = \infty$ vel $k = 0$, vtroque casu erit $Ff = \frac{\alpha\alpha}{aa} . Ee$ etiamsi crassities lentis non euanescat.

Coroll. 3.

63. Cum in distantiis a et α mutatio minima fieri concipiatur, spatium diffusionis nullam inde variationem subire censendum est: siue ergo obiectum in E siue e reperiatur, ac semidiameter aperturae

 lentis

lentis in facie anteriore fuerit $= x$, erit ſpatium diffuſionis:

$$\frac{n\alpha a x x}{2(n-1)^2}\left\{\begin{array}{l}+\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2\\ +\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2\end{array}\right\}$$

in facie autem poſteriori ſemidiameter aperturae debet eſſe $= \frac{k-d}{k+d} \cdot x$.

Problema 9.

Tab. I. Fig. 3. 64. Rationem definire, quam habet magnitudo imaginis ad magnitudinem obiecti non neglecta lentis craſſitie.

Solutio.

Sit obiecti ante lentem diſtantia $AE = a$, imaginis vero $BF = \alpha$ exiſtente lentis craſſitie $= d$; vbi quidem tantum imaginem principalem ſpectamus neglecto ſpatio diffuſionis. Sit iam $E\varepsilon$ obiectum, cui tribuatur magnitudo quam minima $E\varepsilon = z$, normaliter axi lentis inſiſtens; eiusque imago in $F\zeta$ exhibebitur, cuius magnitudo $F\zeta$ quaeritur. Cum igitur punctum ζ a puncto ε oriatur, ita vt radii ab ε emiſſi in ζ colligantur, conſideretur radius quicunque εM, qui productus cum axe in e occurrat: et perinde eſt, ac ſi hic radius ex axis puncto e emanaret. Quare is poſt refractionem cum axe in f concurret, vt ſit $Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 \cdot Ee$, indeque ad ζ perget: ex quo magnitudinem $F\zeta$ definire licebit. Statuatur in

in hunc finem $AM = x$, erit $BN = \frac{k-d}{k+d} \cdot x$; hincque colligemus has proportiones:

$$Ee : E\varepsilon = eA : AM = \alpha : x$$

$$Ff : F\zeta = fB : BN = \alpha : \frac{k-d}{k+d} x$$

ob spatiola Ee et Ff quam minima: vnde habebimus

$$\frac{Ff}{Ee} : \frac{F\zeta}{E\varepsilon} = \frac{\alpha}{a} : \frac{k-d}{k+d} = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 : \frac{F\zeta}{E\varepsilon}$$

Concluditur ergo $\frac{F\zeta}{E\varepsilon} = \frac{\alpha}{a} \cdot \frac{k+d}{k-d}$, ex quo cum magnitudo obiecti $E\varepsilon$ posita fit $= z$, erit magnitudo imaginis $F\zeta = \frac{\alpha(k+d)}{a(k-d)} z$.

Coroll. 1.

65. Secundum hanc ergo rationem diameter obiecti immutatur. Vbi notandum eft, fi expreffio $\frac{\alpha(k+d)}{a(k-d)}$ habeat pofitiuum valorem obiecti imaginem fitu inuerfo repraefentari, contra autem fitu erecto, fi $\frac{\alpha(k+d)}{a(k-d)}$ negatiuum valorem adipifcatur.

Coroll. 2.

66. Si craffities lentis euanefcat, fit $F\zeta = \frac{\alpha}{a} z$: quo ergo cafu recta iungens puncta extrema ε et ζ tranfit per centrum lentis. At fi craffities in computum ducatur, recta $\varepsilon\zeta$ modo intra lentem modo extra, axem lentis fecare poterit.

Scholion.

67. Sic igitur omnia, quae circa vnam lentem quamcunque noffe oportet, expediuimus, vt etiam craffi-

craſſitiei rationem habuerimus. Ac primo quidem ad binas diſtantias lentis quaſi determinatrices ſpectari conuenit, quae ſunt obiecti diſtantia ante lentem $AE = a$ et imaginis principalis poſt lentem diſtantia $BF = \alpha$, quibus addi debet lentis craſſities $AB = d$. His autem conditionibus infinitae lentes ſatisfaciunt; ſi enim radius faciei anterioris AM dicatur $= f$, et poſterioris $BN = g$, vtraque tanquam conuexa conſiderata, conſtructio lentis his continetur formulis:

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \quad \text{et} \quad \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d}$$

ſeu

$$f = \frac{(n-1)a(k+d)}{k+d+2na} \quad \text{et} \quad g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

exiſtente $n = \frac{31}{20}$; vbi k eſt quantitas arbitraria, hincque lentes innumerabiles quaeſito ſatisfacientes obtinentur.

Quod ſi iam obiecti diameter ponatur $= z$, erit imaginis principalis repraeſentatae diameter $= \frac{\alpha(k+d)}{a(k-d)} z$, quatenus imago ſitu inuerſo exhibita conſideratur.

Deinde ſi aperturae in facie anterioris lentis ſemidiameter ſit $= x$, ſpatium diffuſionis, quatenus ab imagine principali ad lentem porrigitur, ita exprimetur vt ſit:

$$\frac{na\alpha xx}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2 \left(\frac{n}{a} + \frac{2}{k+d}\right)\left(\frac{1}{a} + \frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2 \left(\frac{n}{\alpha} - \frac{2}{k-d}\right)\left(\frac{1}{\alpha} - \frac{2}{k-d}\right)^2 \end{array} \right\}$$

in facie autem poſteriori aperturae ſemidiameter debet eſſe $= \frac{k-d}{k+d} \cdot x$ vel ſaltem non minor. Quia ſpatium diffu-

diffusionis factorem habet $\alpha\alpha xx$, breuitatis gratia id ita $P\alpha\alpha xx$ indicabimus. Haecque in genere teneantur etiam crassitiei lentis ratione habita. At si crassities lentis euanescat, formulas magis euoluere licuit, scilicet si breuitatis gratia ponatur:

$\mu = 0,938191$; $\varrho = 0,190781$; $\tau = 0,905133$
$\nu = 0,232692$; $\sigma = 1,627401$; $\lambda > 1$ arbitr:

sumaturque pro formatione lentis:

$$f = \frac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

$$g = \frac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

spatium diffusionis erit pro aperturae semidiametro x

$$\mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

et obiecti diametro existente $= z$ imaginis diameter erit $= \frac{\alpha}{a}z$. His ergo pro vna lente determinatis, videamus quomodo in combinatione duarum pluriumve lentium spatium diffusionis definiatur: vt deinceps in omnis generis instrumentis dioptricis siue Telescopiis siue microscopiis confusionem assignare, modumque eam diminuendi inuestigare possimus.

CAPVT II.

CAPVT II.

DE DIFFVSIONE IMAGINIS PER PLVRES LENTES REPRAESENTATAE.

Problema I.

68.

Tab. I. Fig. 4. Si loco obiecti adsit imago per spatium Ee diffusa, indeque radii per lentem PP aperturae indefinitae transmittantur, determinare spatium diffusionis Ff per hanc lentem productum.

Solutio.

Loco obiecti veri hic consideramus imaginem iam per aliam lentem repraesentatam, quae sit diffusa per spatium Ee, ita vt in E sit imago principalis per radios axi proximos formata in e autem imago extrema, radiis scilicet per marginem aperturae lentis praecedentis transmissis formata; qui radii cum axe constituant angulum $= \Phi$. Quare a puncto E nonnisi radii axi proximi in lentem PP emittuntur, a puncto e autem eiusmodi tantum radii, qui ad axem eA sub angulo $MeA = \Phi$ sint inclinati. Ponatur iam

iam diſtantia $EA = a$, prae qua ſpatium diffuſionis Ee vt valde paruum ſpectetur, lens autem PP ſit eiusmodi vt obiecti in E exiſtentis imaginem principalem referat in diſtantia $BF = \alpha$, exiſtente lentis craſſitie $AB = d$. Hanc ob rem ſi ponatur faciei anterioris AM radius $= f$, poſterioris $BN = g$, lente vt conuexa vtrinque ſpectata oportet eſſe:

$$f = \frac{(n-1)a(k+d)}{k+d+2na} \text{ et } g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

vbi eſt $n = \frac{31}{20}$ et k quantitas arbitraria. Hinc autem vti demonſtrauimus, pro aperturae ſemidiametro $= x$ naſceretur ſpatium diffuſionis

$$\frac{n\alpha\alpha xx}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2 \end{array} \right\}$$

pro quo ſcribamus breuitatis gratia $P\alpha\alpha xx$. Iam puncti E, quia inde nonniſi radii axi proximi ad lentem emittuntur imago exhibebitur in F, vt ſit diſtantia $BF = \alpha$: videamus ergo quorſum imago puncti e debeat cadere. Ac ſi radii ex puncto e emisſi eſſent axi proximi, quia id a lente magis eſt remotum quam E, eius imago propius ad lentem caderet puta in Φ, vt eſſet, ſicut §. 60 definiuimus

$$F\Phi = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 \cdot Ee.$$

in Φ ſcilicet caderet imago principalis, ſi obiectum eſſet in e. Sed quia ab e tantum radii eM ad axem angulo $AeM = \Phi$ inclinati emittuntur, hi lenti in

punctis M ab A interuallo $AO = eA . \Phi = a\Phi$ remotis occurrent, quandoquidem interuallum Ee prae distantia $Ae = a$ contemnimus, perinde igitur est ac si lenti apertura tribueretur, cuius semidiameter esset $= a\Phi$, et obiecti in e existentis imago extrema definiri deberet, quae cadat in f ita vt Φf sit spatium diffusionis obiecto in e existenti, et aperturae lentis, cuius semidiameter $= a\Phi$, conueniens. Hinc ergo erit interuallum $\Phi f = P a a \alpha \alpha \Phi \Phi$; et quia puncti E imago in F, puncti e autem imago in f exhibetur, erit spatium diffusionis per lentem PP productum

$$Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee + P a a \alpha \alpha \Phi \Phi.$$

In imagine autem extrema f radii Nf ita cum axe concurrent, vt sit:

$$\text{angulus } BfN = \frac{k-d}{k+d} \cdot \frac{a\Phi}{\alpha}.$$

Coroll. 1.

69. Si ergo imago iam diffusa per spatium Ee locum obiecti teneat respectu lentis PP, per hanc spatium diffusionis nouum producitur Ff, ita vt imago principalis cadat in F extrema vero in f: fierique poterit, vt hoc nouum spatium diffusionis Ff maius sit vel minus proposito Ee.

Coroll. 2.

70. Aperturam lentis PP vt indefinitam assumsi, manifestum autem est sufficere, dum eius semidiameter non sit minor quam $a\Phi$. Si enim esset minor

minor, radii ex puncto e emissi plane non per lentem transitum inuenirent, neque imago puncti e exprimeretur, sed imaginis Ff punctum extremum responderet imagini cuipiam intermediae spatii Ee.

Coroll. 3.

71. Si diameter obiecti seu potius imaginis in E sitae sit $= z$, tum imaginis inde per lentem PP in F formatae diameter erit $= \frac{\alpha(k+d)}{a(k-d)} z$ quae expressio si sit positiua simul declarat situm imaginis in F esse inuersum respectu eius, quae est in E.

Scholion.

72. Cum in hoc capite plures lentes sim considaturus, pro cuiuslibet determinatione, spectandae sunt primo binae distantiae determinatrices, altera obiecti seu imaginis, a qua lens radios accipit ante lentem, altera imaginis inde per lentem repraesentatae post lentem: quae distantiae ex imaginibus principalibus aestimentur. Deinde cuiusque lentis crassities in computum est ducenda. Tertio cum his lens nondum prorsus determinetur, insuper pro quaque lente accedit distantia quaedam arbitraria hactenus per litteram k indicata. Quarto vero imprimis ratio haberi deberet, spatii diffusionis, quod cuique lenti pro data apertura conueniat. Quemadmodum autem ex binis distantiis determinatricibus, crassitie lentis et quantitate illa arbitraria k, cum binae lentis facies tum

etiam spatium diffusionis pro apertura, cuius semidiameter ponitur $= x$ definiatur, in praecedente capite fusius est expositum. Hic igitur cum plures lentes sint considerandae, pro singulis haec elementa sequentibus litteris indicabo.

Pro Lente	Distantiae determinatrices obiecti	imaginis	Crassities lentis	Quantitas arbitraria	Spatium diffusionis pro aperturae semidiametro x
Prima	a	α	v	k	$P\alpha\alpha xx$
Secunda	b	β	v'	k'	$Q\beta\beta xx$
Tertia	c	γ	v''	k''	$R\gamma\gamma xx$
Quarta	d	δ	v'''	k'''	$S\delta\delta xx$
Quinta	e	ε	v''''	k''''	$T\varepsilon\varepsilon xx$

Sin autem ratio refractionis in singulis lentibus discrepet, pro prima lente eam ponamus $= n$; pro secunda $= n'$; pro tertia $= n''$ etc.

Prima autem lens hic mihi perpetuo erit ea, quae obiecto est proxima indeque recedendo reliquas lentes ordine numero: ex quo simul habebuntur distantiae lentium, scilicet secundae a prima $= \alpha + b$, tertiae a secunda $= \beta + c$; quartae a tertia $= \gamma + d$; quintae a quarta $= \delta + e$ etc. quae distantiae ex sua natura debent esse positiuae etiamsi singulae distantiae determinatrices praeter primam a, quippe quae ad ipsum obiectum necessario ante lentem primam constituendum refertur, sint quandoque negatiuae. Crassitiem lentium hic

hic littera v designo, quia littera d inter distantias determinatrices, si numerus lentium vltra ternarium assurgat, reperitur. Pro crassitie autem et quantitate arbitraria iisdem litteris vtor, commatibus inscriptis distinguendis ob penuriam litterarum diuersarum. Ceterum obseruandum est me omnes lentes tanquam super communi axe dispositas assumere.

Problema 2.

73. Si radii ab obiecto $E\varepsilon$ emissi per duas lentes PP et QQ transmittantur definire spatium diffusionis Gg, a data apertura primae lentis oriundum; vt et magnitudinem imaginis principalis in G exhibitae. Tab. I. Fig. 5.

Solutio.

Sit obiecti magnitudo $E\varepsilon = z$, eiusque imago principalis per primam lentem PP proiiciatur in $F\zeta$, at per ambas in $G\gamma$ erunt distantiae determinatrices
pro lente prima PP, obiecti $EA = a$, imaginis $aF = \alpha$
pro lente secunda QQ, obiecti $FB = b$, imaginis $bG = \beta$
Deinde sit
pro lente PP, crassities $Aa = v$, quantitas arbitr: $= k$
pro lente QQ, crassities $Bb = v'$, quantitas arbitr: $= k'$
Denique pro apertura in anteriori facie vtriusque lentis cuius semidiameter sit $= x$

spatium diffusionis primae lentis $PP = \mathfrak{P}\alpha\alpha xx$
- - - - secundae lentis $QQ = \mathfrak{Q}\beta\beta xx$.

His poſitis pro imagine per primam lentem proiecta $F\zeta$ erit eius magnitudo $F\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ pro inuerſa habenda, ſi haec expreſſio fuerit poſitiua. Deinde ſi lentis primae PP ſemidiameter aperturae in anteriori facie ponatur $= x$, erit ſpatium diffuſionis $Ff = P\alpha\alpha xx$ et inclinatio radiorum in f ad axem $= \frac{k-v}{k+v} \cdot \frac{x}{\alpha}$. tum vero in facie lentis PP poſteriori ſemidiameter aperturae non minor eſſe debet quam $\frac{k-v}{k+v}x$. Tota iam haec imago diffuſa per ſpatium Ff reſpectu alterius lentis QQ tanquam obiectum ſpectari debet, cuius proinde repraeſentatio Gg per propoſitionem praecedentem determinabitur. Erit autem hic $\Phi = -\frac{k-v}{k+v} \cdot \frac{x}{\alpha}$, et ſpatium ibi expreſſum $Ee = P\alpha\alpha xx$; tum vero pro a, α, k, d et P hic ſcribi oportet b, β, k', v' et Q, vnde fiet ſpatium diffuſionis quaeſitum:

$$Gg = \frac{\beta\beta}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb\beta\beta}{\alpha\alpha}\left(\frac{k-v}{k+v}\right)^2 Qxx$$

ſiue

$$Gg = \frac{\beta\beta}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb}{\alpha\alpha}\left(\frac{k-v}{k+v}\right)^2 Q\beta\beta xx$$

Radiorum porro in g incidentium inclinatio ad axem eſt $\left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{\alpha\beta}$. Denique cum ſit $F\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ erit magnitudo imaginis in G vt erutae conſideratae:

$$G\eta = \frac{\alpha\beta}{ab}\left(\frac{k+v}{k-v}\right)\left(\frac{k'+v'}{k'-v'}\right)z.$$

Coroll. I.

74. Quod ad aperturam facierum lentis QQ attinet, ea maior eſſe debet ſpatio tranſitus radiorum; hinc ergo erit ſemidiameter aperturae

faciei

faciei anterioris $> \left(\frac{k-v}{k+v}\right)\frac{bx}{\alpha}$

faciei posterioris $> \left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{\alpha}$.

Coroll. 2.

75. Si ponatur pro lente prima PP radius faciei anterioris $= f$, et posterioris $= g$ erit posito $n = \frac{31}{20}$.

$$f = \frac{(n-1)a(k+v)}{k+v+2na} \text{ et } g = \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}.$$

Similique modo si pro lente altera QQ radius faciei anterioris ponatur $= f'$, et posterioris $= g'$ erit

$$f' = \frac{(n-1)b(k'+v')}{k'+v'+2nb} \text{ et } g' = \frac{(n-1)b(k'-v')}{k'-v'-2nb}$$

omnibus scilicet faciebus vt conuexis consideratis.

Coroll. 3.

76. Pro spatio autem diffusionis ab vtraque lente productae erit quemadmodum inuenimus:

$$P = \frac{n}{2(n-1)^2}.\left\{\begin{array}{l} +\left(\frac{k+v}{k-v}\right)^2\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2 \\ +\left(\frac{k-v}{k+v}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2 \end{array}\right\}$$

similique modo

$$Q = \frac{n}{2(n-1)^2}.\left\{\begin{array}{l} +\left(\frac{k'+v'}{k'-v'}\right)^2\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2 \\ +\left(\frac{k'-v'}{k'+v'}\right)^2\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2 \end{array}\right\}$$

Non opus est vt omnibus lentibus eadem refractionis ratio $n:1$ tribuatur, sed simili modo pro pluribus lentibus poni potest n, n', n'', n''', etc. vt iam supra monuimus vnde nullum aliud discrimen nascitur, nisi

nisi quod in formulis pro f' et g' inuentis loco n scribi debeat n', et Q statui debeat.

$$=\frac{n'}{2(n'-1)^2}\left\{\begin{array}{l}+\left(\frac{k'+v'}{k'-v'}\right)^2\left(\frac{n'}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2\\+\left(\frac{k'-v'}{k'+v'}\right)^2\left(\frac{n'}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\end{array}\right\}$$

Coroll. 4.

77. Sin autem crassities lentium euanescat, et pro quantitate arbitraria k, k' introducamus numerum λ, λ', erit

pro lente PP

$$f=\frac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\surd(\lambda-1)};\ g=\frac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\surd(\lambda-1)}$$

et

$$P=\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

at pro lente QQ

$$f'=\frac{b\beta}{\varrho\beta+\sigma b\pm\tau(b+\beta)\surd(\lambda'-1)};\ g'=\frac{b\beta}{\varrho b+\sigma\beta\mp\tau(b+\beta)\surd(\lambda'-1)}$$

$$Q=\mu\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right).$$

In expressionibus autem inuentis formulae $\left(\frac{k-v}{k+v}\right)$ et $\left(\frac{k'-v'}{k'+v'}\right)$ abeunt in vnitatem.

Numerorum vero $\mu, \nu, \varrho, \sigma, \tau$ indoles in §. 55 exposita est.

Si refractio lentium differat etiam litterae $\mu, \nu, \varrho, \sigma, \tau$ diuersos valores pro singulis lentibus sortientur, quae litte-

litterae si etiam commatibus a prioribus distinguantur, vt sit

$$\varrho' = \frac{1}{2(n'-1)} + \frac{1}{n'+2} - 1$$

$$\sigma' = 1 + \frac{1}{2(n'-1)} - \frac{1}{n'+2}$$

$$\tau' = \tfrac{1}{3}\left(\frac{1}{2(n'-1)} + \frac{1}{n'+2}\right)\sqrt{(4n'-1)}$$

pro secunda lente erit

$$f' = \frac{b\mathfrak{b}}{\varrho'\mathfrak{b} + \sigma' b \pm \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}}$$

$$g' = \frac{b\mathfrak{b}}{\varrho' b + \sigma'\mathfrak{b} \mp \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}}$$

et ob eandem rationem erit

$$\mu' = \frac{1}{4(n'+2)} + \frac{1}{4(n'-1)} + \frac{1}{8(n'+1)^2}$$

$$\text{et } \nu' = \frac{4(n'-1)^2}{4n'-1}$$

quod et de sequentibus lentibus est intelligendum, si forte diuersa refractionis lege fuerint praeditae.

Scholion.

78. Quo formulas in problemate inuentas magis contrahamus, vt cum ad plures lentes processerimus, succinctiores euadant, ponamus

$$\frac{k-v}{k+v} = i \text{ et } \frac{k'-v'}{k'+v'} = i'$$

ita vt hi numeri i et i' abeant in vnitatem euanescente lentium crassitie v et v'. Tum autem erit spatium diffusionis

$$Gg = \frac{1}{i'i'} \cdot \frac{\mathfrak{b}\mathfrak{b}}{bb} \cdot P\alpha\alpha xx + ii \cdot \frac{bb}{\alpha\alpha} \cdot Q\mathfrak{b}\mathfrak{b}xx$$

et radiorum in g incidentium inclinatio ad axem $= ii' . \frac{bx}{\alpha\beta}$: porro magnitudo imaginis $G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z$. Ac pro apertura singularum facierum erit vt sequitur

Semidiam. aperturae lentis	faciei anterioris	faciei posterioris
primae PP	x	ix
secundae QQ	$i . \frac{bx}{\alpha}$	$ii' . \frac{bx}{\alpha}$

aperturae autem hae praeter primam maiores esse debent assignatis: valores enim assignati sufficerent, si obiectum esset merum punctum in axe positum; quando autem habet magnitudinem, radii ab eius terminis per primam faciem ingressi latius diuagantur, et ad sequentes facies maiorem aperturam exigunt.

Problema 3.

Tab. II. Fig. 6. 79. Si radii ab obiecto $E\varepsilon$ emissi per tres lentes PP, QQ et RR refringantur definire spatium diffusionis Hh ob datam aperturam lentis primae oriundum, vt et magnitudinem imaginis principalis in H exhibitae.

Solutio.

Posita obiecti magnitudine $E\varepsilon = z$, cadat eius imago principalis per lentem primam PP in $F\zeta$, dehinc per secundam QQ in $G\eta$, tum vero per tertiam RR in $H\theta$. Erunt ergo distantiae determinatrices.

pro lente PP, obiecti $EA = a$, imaginis $aF = \alpha$
pro lente QQ, obiecti $FB = b$, imaginis $bG = \beta$
pro lente RR, obiecti $GC = c$; imaginis $cH = \gamma$

Deinde

Deinde ſit

pro lente PP, craſſities $Aa = v$, quantitas arbitr: $= k$
pro lente QQ, craſſities $Bb = v'$, quantitas arbitr: $= k'$
pro lente RR craſſities $Cc = v''$, quantitas arbitr: $= k''$
ac ponatur breuitatis gratia:

$$\frac{k-v}{k+v} = i;\ \frac{k'-v'}{k'+v'} = i';\ \frac{k''-v''}{k''+v''} = i''$$

Denique pro quauis lente, ſi eſſet ſingularis, eiusque aperturae ſemidiameter eſſet $= x$, ſit

ſpatium diffuſionis primae lentis $PP = P\alpha\alpha xx$
- - - - ſecundae lentis $QQ = Q\beta\beta xx$
- - - - tertiae lentis $RR = R\gamma\gamma xx$

Iam in praecedente problemate inuenimus fore ſpatium diffuſionis per duas lentes priores productum

$$Gg = \frac{1}{i'i'} \cdot \frac{\beta\beta}{bb} P\alpha\alpha xx + ii. \frac{bb}{\alpha\alpha} \cdot Q\beta\beta xx$$

et radiorum in g concurrentium inclinationem ad axem $= ii'. \frac{bx}{\alpha\beta}$ imaginisque in G magnitudinem $G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z$. Haec igitur ſi ad problema 1. transferantur, ibi loco

$Ee, \frac{\alpha\alpha}{aa}, \left(\frac{k+d}{k-d}\right)^2, Paa\alpha\alpha$ et Φ ſcribi debebit

$Gg;\ \frac{\gamma\gamma}{cc};\ \frac{1}{i''i''},\ Rcc\gamma\gamma$ et $ii'. \frac{bx}{\alpha\beta}$,

hincque ſpatium diffuſionis per tres lentes productum orietur.

$$Hh = \frac{1}{i'i'.i''i''} \cdot \frac{\beta\beta\gamma\gamma}{bb\,cc} P\alpha\alpha xx + \frac{ii}{i''i''} \cdot \frac{bb\gamma\gamma}{\alpha\alpha cc} \cdot Q\beta\beta xx$$
$$+ ii.\, i'i'. \frac{bbcc}{\alpha\alpha\beta\beta} \cdot R\gamma\gamma xx$$

at radiorum in h concurrentium inclinatio ad axem

 erit

erit $= i i' i''. \frac{bcx}{\alpha\beta\gamma}$. Denique imaginis in H formatae magnitudo erit $H\theta = \frac{1}{i i' i''}.\frac{\alpha\beta\gamma}{abc}z$ ad ſitum inuerſum relata.

Coroll. 1.

80. Quod ad aperturas ſingularum facierum attinet, eas ſequenti modo comparatas eſſe conuenit.

Semid. aperturae Lentis	faciei anterioris	faciei poſterioris
Primae PP	x	ix
Secundae QQ	$i.\frac{bx}{\alpha}$	$ii'.\frac{bx}{\alpha}$
Tertiae RR	$ii'.\frac{bcx}{\alpha\beta}$	$ii'i''.\frac{bcx}{\alpha\beta}$

his ſcilicet valoribus non debent eſſe minores.

Coroll. 2.

81. Si inclinatio radiorum in h concurrentium ad axem vocetur $= \Phi$. Cum ſit $\Phi = i i' i''. \frac{bcx}{\alpha\beta\gamma}$ et $H\theta = \frac{1}{i i' i''}.\frac{\alpha\beta\gamma}{abc}z$ erit $\Phi.\ H\theta = \frac{xz}{a}$: quae proprietas pro lentium numero quantumuis magno locum habet.

Coroll. 3.

82. Quemadmodum radii ſingularum facierum determinandi ſint, ex praecedentibus facile liquet:

Erit nempe pro	Radius faciei anterioris	Radius faciei poſterioris
Lente prima PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente ſecunda QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Lente tertia RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$

exiſtente pro vitro $n = \frac{31}{20}$.

Coroll.

Coroll. 4.

83. Tum vero valores literarum P, Q, R ita ſe habebunt

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2+ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

$$Q = \frac{n}{2(n-1)^2}\left(\frac{1}{i'i'}\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2+i'i'\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\right)$$

$$R = \frac{n}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{c}+\frac{2}{k''+v''}\right)^2+i''i''\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{\gamma}-\frac{2}{k''-v''}\right)^2\right)$$

quibus valoribus ſpatia diffuſionis definiuntur.

Si refractio differat:

litteris	tribuatur refr.
P	n
Q	n'
R	n''.

Coroll. 5.

84. Iuuabit etiam ſpatia diffuſionis, prout per vnam, duas ac tres lentes producuntur, inter ſe comparaſſe, quod ita commodiſſime fieri videtur.

$$Ff = \alpha\alpha xx.\ P$$

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}.\frac{\alpha\alpha}{bb}P+ii.\frac{bb}{\alpha\alpha}Q\right)$$

$$Hh = \gamma\gamma xx\left(\frac{1}{i'i'.i''i''}.\frac{\alpha\alpha\beta\beta}{bbcc}P+\frac{ii}{i''i''}.\frac{bb\beta\beta}{\alpha\alpha cc}Q+ii.i'i'.\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$

Scholion.

85. Hinc facile perſpicitur, quomodo determinatio ſpatii diffuſionis, ad plures lentes extendi debeat;

 vnde

vnde problema generale ſtatim ad numerum lentium quemcunque accommodabo, idque pro quacunque lentium craſſitie.

Problema 4.

86. Si radii ab obiecto $E\varepsilon$ emiſſi per lentes quotcunque PP, QQ, RR, SS etc. ſuper communi axe diſpoſitas refringantur, definire ſpatium diffuſionis a data apertura lentis primae oriundum vt et magnitudinem imaginis repraeſentatae.

Solutio.

Poſita obiecti magnitudine $E\varepsilon = z$ imagines eius principales contemplemur: cadat igitur eius imago principalis per lentem primam PP in $F\zeta$ deinde per ſecundam QQ in $G\eta$, tum per tertiam RR in $H\theta$, porro per quartam SS in $I\iota$, per quintam TT in $K\varkappa$ etc. Hinc pro ſingulis lentibus habebimus diſtantias determinatrices quas ita litteris indicemus.

craſſitiem

Pro lente PP obiecti $EA = a$, imaginis $aF = \alpha$; $Aa = v$
Pro lente QQ obiecti $FB = b$; imaginis $bG = \beta$; $Bb = v'$
Pro lente RR obiecti $GC = c$; imaginis $cH = \gamma$; $Cc = v''$
Pro lente SS obiecti $HD = d$; imaginis $dI = \delta$; $Dd = v'''$
Pro lente TT obiecti $IE = e$; imaginis $eK = \varepsilon$; $Ee = v''''$

etc.

Deinde

Deinde cum determinatio cuiusuis lentis non solum has distantias determinatrices cum crassitie cuiusque sed etiam quantitatem quandam arbitrariam inuoluat, a qua spatium diffusionis cuiusque pendet, ponamus si quaelibet lens esset solitaria eiusque aperturae semidiameter $= x$:

Pro lente	quant: arbitr:	Spatium Diffusionis
Prima PP	k	$\mathrm{P}\alpha\alpha xx$
Secunda QQ	k'	$\mathrm{Q}\beta\beta xx$
Tertia RR	k''	$\mathrm{R}\gamma\gamma xx$
Quarta SS	k'''	$\mathrm{S}\delta\delta xx$
Quinta TT	k''''	$\mathrm{T}\varepsilon\varepsilon xx$
	etc.	

Hinc ergo constructio cuiusque lentis ita se habebit posito $n = \frac{31}{20}$: vel si refractio differat, cuique lenti suus tribuatur valor primae n, secundae n', tertiae n'' etc.

Erit nempe pro	Radius faciei anterioris	Radius faciei posterioris
Lente prima PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente secunda QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Lente tertia RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$
Lente quarta SS	$\frac{(n-1)d(k'''+v''')}{k'''+v'''+2nd}$	$\frac{(n-1)\delta(k'''-v''')}{k'''-v'''-2n\delta}$
Lente quinta TT	$\frac{(n-1)e(k''''+v'''')}{k''''+v''''+2ne}$	$\frac{(n-1)\varepsilon(k''''-v'''')}{k''''-v''''-2n\varepsilon}$
	etc.	

at

at ſi ponamus breuitatis gratia:

$$\frac{k-v}{k+v}=i;\ \frac{k'-v'}{k'+v'}=i';\ \frac{k''-v''}{k''+v''}=i'';\ \frac{k'''-v'''}{k'''+v'''}=i''';\ \frac{k''''-v''''}{k''''+v''''}=i''''\ \text{etc.}$$

pro ſpatiis diffuſionis habebimus hos valores:

$$P=\frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2+ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

$$Q=\frac{n}{2(n-1)^2}\left(\frac{1}{i'i'}\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2+i'i'\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\right)$$

$$R=\frac{n}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{c}+\frac{2}{k''+v''}\right)^2+i''i''\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{\gamma}-\frac{2}{k''-v''}\right)^2\right)$$

$$S=\frac{n}{2(n-1)^2}\left(\frac{1}{i'''i'''}\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{d}+\frac{2}{k'''+v'''}\right)^2+i'''i'''\left(\frac{n}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{1}{\delta}-\frac{2}{k'''-v'''}\right)^2\right)$$

$$T=\frac{n}{2(n-1)^2}\left(\frac{1}{i''''i''''}\left(\frac{n}{e}+\frac{2}{k''''+v''''}\right)\left(\frac{1}{e}+\frac{2}{k''''+v''''}\right)^2+i''''i''''\left(\frac{n}{\varepsilon}-\frac{2}{k''''-v''''}\right)\left(\frac{1}{\varepsilon}-\frac{2}{k''''-v''''}\right)^2\right)$$

etc.

His conſtitutis pro magnitudine ſingularum imaginum habebimus:

		ad ſitum
pro vna lente imaginem	$F\zeta=\frac{1}{i}\cdot\frac{\alpha}{a}z$	inuerſum
pro duabus lentibus	$G\eta=\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z$	erectum
pro tribus lentibus	$H\theta=\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}z$	inuerſum
pro quatuor lentibus	$I\iota=\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta}{abcd}z$	erectum
pro quinque lentibus	$K\varkappa=\frac{1}{ii'i''i'''i''''}\cdot\frac{\alpha\beta\gamma\delta\varepsilon}{abcde}z$	inuerſum

etc.

Deni-

Denique dum aperturae lentium non ſint minores, quam ſequentes formulae exhibent:

Semid. aperturae Lentis	faciei anterioris	faciei poſterioris
primae PP	x	ix
ſecundae QQ	$i.\frac{bx}{\alpha}$	$ii'.\frac{bx}{\alpha}$
tertiae RR	$ii'.\frac{bcx}{\alpha\beta}$	$ii'i''.\frac{bcx}{\alpha\beta}$
quartae SS	$ii'i''.\frac{bcdx}{\alpha\beta\gamma}$	$ii'i''i'''.\frac{bcdx}{\alpha\beta\gamma}$
quintae TT	$ii'i''i'''.\frac{bcdex}{\alpha\beta\gamma\delta}$	$ii'i''i'''i''''.\frac{bcdex}{\alpha\beta\gamma\delta}$
	etc.	

erit vt ſequitur pro quolibet lentium numero:

I. Pro vna lente

Spatium diffuſionis $Ff = \alpha\alpha xx.\ P$

inclinatio radiorum in f concurrentium ad axem $= i.\frac{x}{\alpha}$

II. Pro duabus lentibus

Spatium diffuſionis

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii.\frac{bb}{\alpha\alpha}Q\right)$$

et radiorum in g concurrentium

inclinatio ad axem $= ii'.\frac{bx}{\alpha\beta}$

III. Pro tribus lentibus

Spatium diffuſionis:

$$Hh = \gamma\gamma xx\left(\frac{1}{i'i'.i''i''}\cdot\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}\cdot\frac{bb\beta\beta}{\alpha\alpha cc}\cdot Q + ii.i'i'.\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$

et radiorum in h concurrentium

inclinatio ad axem $= ii'i''.\frac{bcx}{\alpha\beta\gamma}$.

IV. Pro quatuor lentibus

Spatium diffusionis:

$$Ii = \delta\delta xx \left\{ \begin{array}{l} \frac{1}{i'i'.i''i''.i'''i'''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd} P + \frac{ii}{i''i''.i'''i'''} \cdot \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd} Q \\ + \frac{ii.i'i'}{i'''i'''} \cdot \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta ad} R + ii.i'i'.i''i'' \cdot \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma} S \end{array} \right\}$$

et radiorum in i concurrentium

inclinatio ad axem $= ii'i''i''' \cdot \frac{bcdx}{\alpha\beta\gamma\delta}$.

V. Pro quinque lentibus

Spatium diffusionis:

$$Kk = \varepsilon\varepsilon xx \left\{ \begin{array}{l} + \frac{1}{i'i'.i''i''.i'''i'''.i''''i''''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee} P \\ + \frac{ii}{i''i''.i'''i'''.i''''i''''} \cdot \frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee} Q \\ + \frac{ii.i'i'}{i'''i'''.i''''i''''} \cdot \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee} R \\ + \frac{ii.i'i'.i''i''}{i''''i''''} \cdot \frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee} S \\ + ii.i'i'.i''i''.i'''i''' \cdot \frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta} T \end{array} \right\}$$

et radiorum in k concurrentium

inclinatio ad axem $= ii'i''i'''i'''' \cdot \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$.

Vnde progressio harum formularum ad plures adhuc lentes satis est manifesta. Si in lentibus ratio refractionis sit diuersa atque ad singulas lentes ordine his litteris indicetur n, n', n'', n''', etc.; haec diuersitas facile ad formulas hic inuentas accommodabitur. Primum enim haec correctio occurrit in formulis pro radiis lentium,

lentium, ita, vt quemadmodum formulae f et g pro prima lente numerum n inuoluunt, ita pro secunda lente numerus n', pro tertia n'' et ita porro introducatur. Similem correctionem etiam requirunt valores litterarum P, Q, R, S etc. et loco litterae n, quae in valore P occurrit, in valoribus Q, R, S etc. scribi oportet n', n'', n''' etc.

Coroll. 1.

87. Si obiectum sit tantum punctum in axe positum, sufficit vt lentium aperturae sint tantae, quantas assignauimus sin autem obiectum habeat quandam magnitudinem, tum aperturae praeter primam eo magis mensuras assignatas superare debent, quo maior fuerit obiecti magnitudo z.

Coroll. 2.

88. In expressione spatii diffusionis quadratum semidiametri aperturae primae faciei xx primo multiplicatur per quadratum distantiae postremae imaginis ab vltima lentium: quae ergo si fuerit infinita, etiam spatium diffusionis fit infinitum.

Coroll. 3.

89. Ceteris ergo paribus, quotcunque fuerint lentes, spatium diffusionis semper est proportionale quadrato diametri aperturae primae faciei, hoc est ipsi huic aperturae. Vnde diametro aperturae primae

faciei ad femiſſem redacto ſpatium diffuſionis quadruplo fiet minus.

Scholion.

90. Confiderauimus hic ſtatim loca ſingularum imaginum principalium tanquam data ex iisque ſtructuram cuiusque lentis quantitatem arbitrariam introducendo determinauimus. Quod ſi vero ipſae lentes fuerint datae ita vt tam radii ambarum facierum cuiusque quam craſſities, vna cum earum interuallis cognoſcantur tum ope formularum exhibitarum, viciſſim diſtantiae determinatrices innoteſcunt. Sint ſcilicet radii facierum anterioris et poſterioris primae lentis PP, f, g, ſecundae lentis QQ, f', g'; tertiae lentis RR, f'', g'' etc. craſſitie earum exiſtente v, v' v'' etc. tum vero dentur diſtantiae $aB = F$; $bC = G$; $cD = H$ etc. Praeterea autem diſtantia obiecti ante lentem primam ſit $AE = a$, ac ſequenti modo omnia elementa ad problema ſuperius neceſſaria elicientur;

$$\left\{\begin{array}{l} 1.\ \frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+v}, \text{ hinc reperitur } k \\ 2.\ \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-v}, \text{ hinc vero } \alpha \\ 3.\ F = \alpha + b \text{ vnde } b = F - \alpha \end{array}\right.$$

$$\left\{\begin{array}{l} 4.\ \frac{n-1}{f'} = \frac{1}{b} + \frac{2n}{k'+v'}, \text{ hinc reperitur } k' \\ 5.\ \frac{n-1}{g'} = \frac{1}{\mathfrak{b}} - \frac{2n}{k'-v'}, \text{ hinc vero porro } \mathfrak{b} \\ 6.\ G = \mathfrak{b} + c \text{ vnde } c = G - \mathfrak{b} \end{array}\right.$$

$$\begin{cases} 7.\ \frac{n-1}{f''} = \frac{1}{c} + \frac{2n}{k''+v''}, & \text{hinc reperitur } k'' \\ 8.\ \frac{n-1}{g''} = \frac{1}{\gamma} - \frac{2n}{k''-v''}, & \text{hinc vero } \gamma \\ 9.\ H = \gamma + d & \text{vnde } d = H - \gamma \end{cases}$$

etc.

Vtcunque ergo lentes datae fuerint diſpoſitae ſuper axe communi, ſi ante eas conſtituatur obiectum in data diſtantia $AE = a$, inde ſingulae diſtantiae determinatrices $\alpha, b, \mathfrak{E}, c, \gamma$ etc. cum arbitrariis k, k', k'', k''' etc. facile definiuntur ex iisque porro ſpatium diffuſionis cum reliquis phaenomenis in ſolutione problematis commemoratis aſſignabitur. Operae pretium autem erit caſum, quo craſſities lentium vt euaneſcens ſpectatur, accuratius euoluiſſe.

Problema 5.

91. Si craſſities lentium euaneſcat, et quotcunque huiusmodi lentes ſuper communi axe fuerint diſpoſitae, ante quas exiſtat obiectum $E\varepsilon$, definire ſpatium diffuſionis, per quod imago erit diſſipata, vt et magnitudinem imaginis.

Solutio.

Si obiecti magnitudo $E\varepsilon = z$, cuius imagines principales ſucceſſiue cadant in $F\zeta$, $G\eta$, $H\theta$, $I\iota$, $K\kappa$ etc. hincque pro ſingulis lentibus ſequentes habebimus diſtantias determinatrices, cum imago per quamuis lentem repraeſentata reſpectu lentis ſequentis vicem obiecti gerat.

 diſtan-

	distantia	distantia
Pro lente PP	obiecti $EA = a$;	imaginis $aF = \alpha$
Pro lente QQ	obiecti $FB = b$;	imaginis $bG = \beta$
Pro lente RR	obiecti $GC = c$;	imaginis $cH = \gamma$
Pro lente SS	obiecti $HD = d$;	imaginis $dI = \delta$
Pro lente TT	obiecti $IE = e$;	imaginis $eK = \varepsilon$

etc.

Porro autem sint numeri arbitrarii vnitate maiores cuiusque lentis figuram determinantes, λ pro lente PP, λ' pro QQ, λ'' pro RR, λ''' pro SS, λ'''' pro TT, etc. ita vt ponendo breuitatis gratia

$$\rho = 0,190781,\ \sigma = 1,627401,\ \tau = 0,905133$$

Pro lente PP radius faciei $\begin{cases} \text{anter.} = \dfrac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda - 1)}} \\ \text{poster.} = \dfrac{a\alpha}{\rho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda - 1)}} \end{cases}$

Pro lente QQ radius faciei $\begin{cases} \text{anter.} = \dfrac{b\beta}{\rho\beta + \sigma b \pm \tau(b+\beta)\sqrt{(\lambda' - 1)}} \\ \text{poster.} = \dfrac{b\beta}{\rho b + \sigma\beta \mp \tau(b+\beta)\sqrt{(\lambda' - 1)}} \end{cases}$

Pro lente RR radius faciei $\begin{cases} \text{anter.} = \dfrac{c\gamma}{\rho\gamma + \sigma c \pm \tau(c+\gamma)\sqrt{(\lambda'' - 1)}} \\ \text{poster.} = \dfrac{c\gamma}{\rho c + \sigma\gamma \mp \tau(c+\gamma)\sqrt{(\lambda'' - 1)}} \end{cases}$

Pro lente SS radius faciei $\begin{cases} \text{anter.} = \dfrac{d\delta}{\rho\delta + \sigma d \pm \tau(d+\delta)\sqrt{(\lambda''' - 1)}} \\ \text{poster.} = \dfrac{d\delta}{\rho d + \sigma\delta \mp \tau(d+\delta)\sqrt{(\lambda''' - 1)}} \end{cases}$

Pro lente TT radius faciei $\begin{cases} \text{anter.} = \dfrac{e\varepsilon}{\rho\varepsilon + \sigma e \pm \tau(e+\varepsilon)\sqrt{(\lambda'''' - 1)}} \\ \text{poster.} = \dfrac{e\varepsilon}{\rho e + \sigma\varepsilon \mp \tau(e+\varepsilon)\sqrt{(\lambda'''' - 1)}} \end{cases}$

etc.

Deinde

Deinde si quaelibet lens cum binis suis distantiis determinatricibus seorsim consideretur, eiusque aperturae semidiameter foret $= x$, posito $\mu = 0,938191$ et $\nu = 0,232692$ esset spatium diffusionis

$$\text{Lentis } PP = \mu\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

$$\text{Lentis } QQ = \mu\beta\beta xx\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu}{b\beta}\right)$$

$$\text{Lentis } RR = \mu\gamma\gamma xx\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{c\gamma}\right)$$

$$\text{Lentis } SS = \mu\delta\delta xx\left(\tfrac{1}{d}+\tfrac{1}{\delta}\right)\left(\lambda'''\left(\tfrac{1}{d}+\tfrac{1}{\delta}\right)^2+\tfrac{\nu}{d\delta}\right)$$

$$\text{Lentis } TT = \mu\varepsilon\varepsilon xx\left(\tfrac{1}{e}+\tfrac{1}{\varepsilon}\right)\left(\lambda''''\left(\tfrac{1}{e}+\tfrac{1}{\varepsilon}\right)^2+\tfrac{\nu}{e\varepsilon}\right)$$

etc.

His constitutis pro magnitudine singularum imaginum habebimus

Pro vna lente $F\zeta = \frac{\alpha}{a}z$ situ inuerso

Pro duabus lentibus $G\eta = \frac{\alpha\beta}{ab}z$ situ erecto

Pro tribus lentibus $H\theta = \frac{\alpha\beta\gamma}{abc}z$ situ inuerso

Pro quatuor lentibus $I\iota = \frac{\alpha\beta\gamma\delta}{abcd}z$ situ erecto

Pro quinque lentibus $K\kappa = \frac{\alpha\beta\gamma\delta\varepsilon}{abcde}z$ situ inuerso

etc.

At si semidiameter aperturae primae lentis PP ponatur $= x$, necesse est, vt reliquarum lentium aperturae superent sequentes valores

Semi-

Semidiameter aperturae
Lentis ſecundae $QQ > \frac{b}{\alpha}x$
Lentis tertiae $RR > \frac{bc}{\alpha\beta}x$
Lentis quartae $SS > \frac{bcd}{\alpha\beta\gamma}x$
Lentis quintae $TT > \frac{bcde}{\alpha\beta\gamma\delta}x$
etc.

Hinc ſpatium diffuſionis pro quolibet lentium numero ita ſe habebit.

I. Pro vna lente

ſpatium diffuſionis

$$Ff = \mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

radiorum in f concurrentium inclinatio ad axem $= \frac{x}{\alpha}$

II. Pro duabus lentibns

ſpatium diffuſionis

$$Gg = \mu\beta\beta xx\begin{cases}+\frac{\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\+\frac{bb}{\alpha\alpha}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right)\end{cases}$$

et radiorum in g concurrentium inclinatio ad axem $= \frac{bx}{\alpha\beta}$

III. Pro tribus lentibus

ſpatium diffuſionis

$$Hh = \mu\gamma\gamma xx\begin{cases}+\frac{\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{\alpha a}\right)\\+\frac{bb\beta\beta}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right)\\+\frac{bbcc}{\alpha\alpha\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right)\end{cases}$$

et

et radiorum in b concurrentium
inclinatio ad axem $\frac{bcx}{\alpha\beta\gamma}$

IV. Pro quatuor lentibus
ſpatium diffuſionis

$$Ii = \mu\delta\delta xx \begin{cases} + \frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ + \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ + \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ + \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \end{cases}$$

et radiorum in i concurrentium
inclinatio ad axem $= \frac{bcdx}{\alpha\beta\gamma\delta}$

V. Pro quinque lentibus
ſpatium diffuſionis

$$Kk = \mu\varepsilon\varepsilon xx \begin{cases} + \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ + \frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ + \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ + \frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \\ + \frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)\left(\lambda''''\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)^2+\frac{\nu}{e\varepsilon}\right) \end{cases}$$

et radiorum in k concurrentium
inclinatio ad axem $= \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$
neque ergo caſus, quibus plures occurrunt lentes, vlla amplius laborant difficultate.

Si lentes ratione refractionis diſcrepent, ad easque referendae ſint litterae n, n', n'', n''', etc. formu-

 lae

lae in hoc problemate inuentae fequenti modo facile ad hunc cafum latius patentem adcommodabuntur. Primo fcilicet in formulis pro radiis facierum inuentis litterae ϱ, σ, et τ tantum ad primam lentem pertinent, earumque loco pro fecunda lente fcribi oportet ϱ', σ', et τ'; pro tertia autem ϱ'', σ'' et τ'' et ita porro. Praeterea vero fpatia diffufionis hinc aliquam mutationem requirunt, eritque fpatium diffufionis.

I. Pro vna lente

$$\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\mu\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right).$$

II. Pro duabus lentibus

$$\beta\beta xx\begin{cases}\frac{\mu\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb}{\alpha\alpha}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right).\end{cases}$$

III. Pro tribus lentibus

$$\gamma\gamma xx\begin{cases}+\frac{\mu\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb\beta\beta}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right)\\ +\frac{\mu'' bbcc}{\alpha\alpha\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right).\end{cases}$$

IV. Pro quatuor lentibus

$$\delta\delta xx\begin{cases}+\frac{\mu\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right)\\ +\frac{\mu'' bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right)\\ +\frac{\mu''' bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu'''}{d\delta}\right).\end{cases}$$

valores autem harum litterarum μ', ν', μ'', ν'' etc. iam fupra definiuimus §. 77.

Coroll.

Coroll. 1.

92. Si lentis primae PP ponatur radius faciei anterioris $=f$ et posterioris $=g$ erit

$$\frac{1}{f}=\frac{\rho}{a}+\frac{\sigma}{\alpha}\pm\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

$$\frac{1}{g}=\frac{\rho}{\alpha}+\frac{\sigma}{a}\mp\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

vnde si detur distantia obiecti EA$=a$, primo inuenitur α ex hac aequatione

$$\frac{1}{f}+\frac{1}{g}=(\rho+\sigma)\left(\frac{1}{a}+\frac{1}{\alpha}\right)=1,818182\left(\frac{1}{a}+\frac{1}{\alpha}\right)=\frac{20}{11}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

Inuenta autem distantia α numerus λ reperitur ex hac aequatione.

$$\frac{1}{f}-\frac{1}{g}+(\sigma-\rho)\left(\frac{1}{a}-\frac{1}{\alpha}\right)=2\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}.$$

Coroll. 2.

93. Deinde si distantia secundae lentis a prima sit $=F$, ob $F=\alpha+b$ habetur $b=F-\alpha$; qua distantia b cognita, si pro lente secunda datus sit radius faciei anterioris $=f'$, et posterioris $=g'$ habebuntur iterum duae aequationes

$$\frac{1}{f'}=\frac{\rho}{b}+\frac{\sigma}{\mathfrak{b}}\pm\tau\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)\sqrt{(\lambda'-1)}$$

$$\frac{1}{g'}=\frac{\rho}{\mathfrak{b}}+\frac{\sigma}{b}\mp\tau\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)\sqrt{(\lambda'-1)}$$

ex quibus cum distantiam $\mathfrak{b}$ tum numerum λ' definire licet. Similique modo ex forma sequentium lentium earumque distantia reliqua elementa innotescent.

Coroll. 3.

94. Si singulae lentes ad minimum spatium diffusionis fuerint accommodatae, erit $\lambda=1, \lambda'=1, \lambda''=1, \lambda'''=1$ etc. sin autem hae lentes alia forma fuerint praeditae, isti numeri erunt vnitate maiores.

Scholion.

95. Quo plures fuerint lentes eo pluribus constabit membris spatium diffusionis ab iis productum; neque tamen propterea aucto lentium numero spatium diffusionis necessario augetur. Cum enim quantitates α, b, $\mathfrak{b}$, c, γ, d, δ, etc. valores quoque negatiuos recipere queant, dummodo binarum summae $\alpha+b$; $\mathfrak{b}+c$; $\gamma+d$; $\delta+e$ etc. vtpote lentium distantiae maneant positiuae; fieri potest, vt vnum vel aliquot membra fiant negatiua, hincque spatium diffusionis diminuatur, quin etiam interdum prorsus euanescat, quo casu repraesentatio sine dubio erit perfectissima. Verum in instrumentis dioptricis ad visionem instructis veluti Telescopiis ac Microscopiis non tam hoc, quod definiuimus, spatium diffusionis quam confusio in ipsa visione orta spectari debet; quae autem etsi a spatio diffusionis plurimum differt, tamen ex eo definiri potest vti mox explicabimus. Ante autem conueniet lentes compositas seu multiplicatas considerare, cuiusmodi oriuntur, si duae pluresue lentes, quarum crassities tam est parua vt negligi queat, immediate iungantur quo quidem pacto instar lentium simplicium spectari possunt; verum tali coniunctione effici potest, vt spatium diffusionis multo fiat minus quam si lens simplex adhiberetur, atque adeo euanescat valorque numeri λ istiusmodi lenti compositae conueniens vnitate minor sit prodirurus vnde maxima commoda ad confusionem diminuendam obtinebuntur.

CAPVT III.

CAPVT III.

DE LENTIBVS COMPOSITIS SEV MVLTIPLICATIS.

Definitio I.

96.

Lens duplicata oritur, si duae lentes super communi axe sibi immediate iungantur.

Craſſitiem hic vtriusque lentis tanquam nullam aſſumo et quia diſtantia inter lentes nulla ponitur, craſſities etiam lentis duplicatae pro nulla haberi poterit.

Coroll. I.

97. Binae ergo lentes PP et QQ ſibi ad contactum fere coniunctae lentem duplicatam conſtituunt; de qua tamen notandum eſt, eius craſſitiem minus tuto negligi poſſe quam vtriusque lentis ſimplicis ſeorſim ſumtae. Si enim lentes immediate ſe in puncto contingerent; phaenomena colorum a *Newtono* obſeruata eſſent metuenda tum vero etiam oſtendemus, quomodo ratio diſtantiae inter binas lentes haberi poſſit. Tab. II. Fig. 7.

Coroll. 2.

98. Si lentis anterioris PP diſtantiae determinatrices ſint a et α, lentis poſterioris vero QQ, b et β, neceſſe eſt vt ſit $\alpha + b = 0$ ſeu $\alpha = -b$. Tum vero obiecti ante lentem ad diſtantiam $AE = a$ poſiti imago principalis repraeſentabitur poſt lentem ad diſtantiam $G = \beta$.

Coroll. 3.

99. Erunt ergo a et β quaſi diſtantiae determinatrices lentis duplicatae; ac ſumendo α vel b ad libitum infinita paria lentium pro his diſtantiis exhiberi poſſunt. Cum deinde vtraque lens praeterea numerum indefinitum λ recipiat inſuper infinita varietas locum habet.

Coroll. 4.

100. Quia craſſities pro nihilo reputatur aperturae in ſingulis faciebus eadem eſt ratio; ſcilicet ſi in prima facie ſemidiameter aperturae ſit $= x$, in reliquis quoque faciebus apertura eadem vel ſaltem non minor eſſe debet.

Scholion.

101. Non opus eſt, vt lentes plane ad contactum coniungantur quoniam forte refractionis lex turbari poſſet: hoc autem vel minima interpoſita diſtantia euitabitur id quod ad inſtitutum noſtrum ſufficit, cum etiam craſſities vtriusque non omnino ſit nulla.

Proble-

Problema I.

102. Omnes lentes duplicatas deſcribere, quibus obiectum $E\varepsilon$ in data ante lentem diſtantia AE propoſitum poſt lentem in data diſtantia bG repraeſentetur, ſimulque diffuſionem imaginis Gg pro data lentis apertura definire.

Solutio.

Sit diſtantia obiecti $AE = a$, imaginis principalis $bG = \beta$, tum vero lentis primae PP diſtantiae determinatrices a, et α; lentis poſterioris vero QQ, b et β; iam quia diſtantia lentium eſt nulla, erit $\alpha + b = 0$, ſeu $\alpha = -b$; Hinc ſi obiecti magnitudo ſit $E\varepsilon = z$. erit imaginis principalis magnitudo $G\gamma = \frac{\beta}{a} z$ pro ſitu inuerſo. Cum porro vtraque lens infinitis modis formari poſſit, ſit λ numerus arbitrarius pro prima PP et λ' pro ſecunda QQ, quarum ergo conſtructio ita ſe habebit:

Pro Lente

$$PP \text{ radius faciei } \begin{cases} \text{anterioris} = \dfrac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda - 1)}} \\ \text{poſterioris} = \dfrac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda - 1)}} \end{cases}$$

$$QQ \text{ radius faciei } \begin{cases} \text{anterioris} = \dfrac{b\beta}{\varrho\beta + \sigma b \pm \tau(b+\beta)\sqrt{(\lambda' - 1)}} \\ \text{poſterioris} = \dfrac{b\beta}{\varrho b + \sigma\beta \mp \tau(b+\beta)\sqrt{(\lambda' - 1)}} \end{cases}$$

Si ratio refractionis ſit diuerſa; pro ſecunda lente ſcribi debet ϱ', σ' et τ', loco ϱ, σ et τ.

Pro

Pro ſpatio autem diffuſionis Gg inueniendo, ſit ſemidiameter aperturae lentis duplicatae $= x$, eritque

$$Gg = \mu \mathfrak{E}\mathfrak{E}xx. \begin{cases} +\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\left(\frac{1}{b}+\frac{1}{\mathfrak{E}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{E}}\right)^2+\frac{\nu}{b\mathfrak{E}}\right) \end{cases}$$

et radiorum in g concurrentium inclinatio ad axem $= \frac{x}{\mathfrak{E}}$. Si ratio refractionis diſcrepet; in parte ex lente ſecunda orta ſcribatur μ', ν', loco μ, ν.

Coroll. 1.

103. Huius ergo lentis duplicatae diſtantiae determinatrices ſunt a et $\mathfrak{E}$, praeterea vero duo numeri arbitrarii λ et λ', vna cum diſtantia α vel b eius perfectam determinationem conſtituunt, vnde in huiusmodi lentibus multo maior varietas locum habet, quam in lentibus ſimplicibus.

Coroll. 2.

104. Si ex eisdem diſtantiis determinatricibus a et $\mathfrak{E}$ adiungendo numero arbitrario λ^0 lens ſimplex conſtruatur, ea imaginem eadem magnitudine $G\gamma = \frac{\mathfrak{E}}{a} z$ referet; ſed pro eadem apertura, cuius ſemidiameter $= x$ habebitur ſpatium diffuſionis

$$Gg = \mu \mathfrak{E}\mathfrak{E}xx\left(\frac{1}{a}+\frac{1}{\mathfrak{E}}\right)\left(\lambda^0\left(\frac{1}{a}+\frac{1}{\mathfrak{E}}\right)^2+\frac{\nu}{a\mathfrak{E}}\right)$$

Coroll. 3.

105. Fieri ergo poterit vt lens duplicata modo maiorem modo minorem diffuſionem gignat. Lens autem

autem ſimplex minimam parit diffuſionem, ſi $\lambda^{\circ} = 1$; ergo tum demum lentes duplicatae ſimplicibus erunt praeferendae, cum adhuc minorem diffuſionem producent.

Scholion.

106. Concipi quidem ſemper poterit lens ſimplex eandem diffuſionem gignens ac lens duplicata, ſi pro numero λ° omnes valores admittamus; quomodocunque enim lens duplicata fuerit comparata, ſi ſpatium diffuſionis inde productum huic ex lente ſimplici nato aequale ſtatuatur, determinatus valor pro numero λ° elicitur: qui ſi fuerit poſitiuus et vnitate maior, realis lens ſimplex aequiualens exhiberi poterit, ſin autem prodeat vnitate minor, vel adeo negatiuus, lens ſimplex inter imaginaria erit referenda. Quando autem ſit $\lambda^{\circ} > 1$, euidens eſt lentem ſimplicem eundem plane effectum eſſe edituram ac duplicatam ideoque ſemper expediet lente ſimplici potius vti quam duplicata; quodſi vero prodierit $\lambda^{\circ} < 1$, quo caſu lens ſimplex ſit imaginaria, tum lentes duplicatae effectum praeſtabunt a ſimplicibus non expectandum, qui adeo cum inſigni hoc commodo, quod ſpatium diffuſionis futurum ſit minus, erit coniunctus. Talibus ergo lentibus duplicatis maximo cum ſucceſſu vti poterimus, eoque magis eae ſimplicibus erunt anteponendae, quo minor fuerit valor numeri λ° iis reſpondens.

Problema 2.

107. Data lente duplicata ad binas diſtantias determinatrices $AE = a$ et $bG = \beta$ relata pro iisdem diſtantiis definire lentem ſimplicem, quae pro eadem apertura, eandem diffuſionem imaginis producat.

Solutio.

Totum ergo negotium huc redit, vt ſpatium diffuſionis Gg a lente ſimplici productum (104) aequale ponatur ſpatio diffuſionis a lente duplicata orto, cuius expreſſio in problemate praecedente (101) eſt inuenta, indeque valor numeri λ^{0} pro conſtructione lentis ſimplicis eliciatur. Quae inueſtigatio quo commodius inſtitui poſſit, ponamus:

$$\frac{1}{\alpha} = \frac{f-1}{a} + \frac{f}{\beta} \text{ eritque } \frac{1}{b} = \frac{-f+1}{a} - \frac{f}{\beta}$$

ita vt loco quantitatis α vel b numerum f introducamus eritque

$$\frac{1}{a} + \frac{1}{\alpha} = f\left(\frac{1}{a} + \frac{1}{\beta}\right) \text{ et } \frac{1}{b} + \frac{1}{b} = (1-f)\left(\frac{1}{a} + \frac{1}{\beta}\right)$$

vnde ſpatium diffuſionis a lente duplicata ortum prodit:

$$Gg = \mu\beta\beta xx \begin{cases} +f\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\lambda ff\left(\frac{1}{a}+\frac{1}{\beta}\right)^2 + \nu\left(\frac{f-1}{aa}+\frac{f}{a\beta}\right)\right) \\ +(1-f)\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\lambda'(1-f)^2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2 + \nu\left(\frac{1-f}{\alpha\beta}-\frac{f}{\beta\beta}\right)\right) \end{cases}$$

quae expreſſio reducitur ad hanc formam:

$$Gg = \mu\beta\beta xx\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\left(\lambda f^3 + \lambda'(1-f)^3\right)\left(\frac{1}{a}+\frac{1}{\beta}\right)^2 + \nu\left(\frac{f(f-1)}{aa} + \frac{1-2f+2ff}{a\beta} + \frac{f(f-1)}{\beta\beta}\right)\right).$$

Verum poſtremum membrum $\frac{f(f-1)}{aa} + \frac{1-2f+2ff}{a\beta} + \frac{f(f-1)}{\beta\beta}$ mutatur

mutatur in $f(f-1)\left(\frac{1}{a}+\frac{1}{\beta}\right)^2+\frac{1}{a\beta}$, ſicque habebimus pro lente duplicata:

$$Gg=\mu\beta\beta xx\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\left(\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)\right)\left(\frac{1}{a}+\frac{1}{\beta}\right)^2+\frac{\nu}{a\beta}\right)$$

quae forma iam facillime cum ſpatio diffuſionis lentis ſimplicis comparatur indeque manifeſto colligitur:

$$\lambda^{\circ}=\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)$$

Cum autem loco quantitatum a et b numerum f introduxerimus conſtructio lentis duplicatae ita ſe habebit

Pro lente radius faciei

prima PP $\begin{cases} \text{anterioris} = \dfrac{a\beta}{(\rho-\sigma(1-f))\beta+\sigma fa\pm\tau f(a+\beta)\sqrt{(\lambda-1)}} \\ \text{poſterioris} = \dfrac{a\beta}{(\sigma-\rho(1-f))\beta+\rho fa\mp\tau f(a+\beta)\sqrt{(\lambda-1)}} \end{cases}$

ſecunda QQ $\begin{cases} \text{anterioris} = \dfrac{a\beta}{(\sigma-\rho f)a+\rho(1-f)\beta\pm\tau(1-f)(a+\beta)\sqrt{(\lambda'-1)}} \\ \text{poſterioris} = \dfrac{a\beta}{(\rho-\sigma f)a+\sigma(1-f)\beta\mp\tau(1-f)(a+\beta)\sqrt{(\lambda'-1)}} \end{cases}$

Tum vero inuento numero λ° conſtructio lentis ſimplicis aequiualentis erit

Radius faciei $\begin{cases} \text{anterioris} = \dfrac{a\beta}{\rho\beta+\sigma a\pm\tau(a+\beta)\sqrt{(\lambda^{\circ}-1)}} \\ \text{poſterioris} = \dfrac{a\beta}{\rho a+\sigma\beta\mp\tau(a+\beta)\sqrt{(\lambda^{\circ}-1)}} \end{cases}$

Coroll. I.

108. Quoties ergo fuerit

$$\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)>1$$

ſemper lens ſimplex parari poteſt duplicatae aequiualens

lens iisque ergo casibus praestabit lente simplici vti potius quam lente duplicata.

Coroll. 2.

109. Verum si fuerit

$$\lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f) < 1$$

ob $\lambda^0 < 1$, constructio lentis simplicis fit impossibilis, ac lens duplicata minorem pariet diffusionem quam per vllam lentem simplicem obtineri potest.

Coroll. 3.

110. Si esset $f=0$, prodiret $\lambda^0=\lambda'$, et in lente duplicata anteriori nullam refractionem produceret ob facies parallelas, resque eodem rediret, ac si posterior sola adesset. Sin autem sumatur $f=1$, fit $\lambda^0=\lambda$, et lens posterior superflua; vtroque ergo casu nullum lucrum impetratur.

Coroll. 4.

111. Cum autem numeri λ et λ' vnitate nequeant esse minores, sitque $\nu=0,232692$, patet pro f nullum valorem inter limites o et 1 assumi posse, vnde fiat $\lambda^0=0$. At si f extra hos limites capiatur, vtique pro λ et λ' eiusmodi numeri vnitate maiores assignari poterunt, vt fiat $\lambda^0=0$.

Scholion 1.

112. Ratione ergo numeri f tres casus lentium duplicatarum considerari conueniet, prout vel f intra limi-

limites o et 1 continetur, vel fuerit $f > 1$ vel $f < o$. Primo casu euenire non potest, vt fiat $\lambda^o = o$; sed plurimum intererit eam determinasse lentem duplicatam, pro qua λ^o minimum obtineat valorem qui quo magis infra vnitatem cadat, eo perfectior lens erit censenda, maioreque iure simplicibus anteferenda. Binis reliquis vero casibus $f > 1$ et $f < o$ eiusmodi adeo lentes duplicatae parari poterunt, quae praebeant $\lambda^o = o$, quae ergo pro perfectissimis essent habendae. Verum hic quoque ad praxin est respiciendum, quae cum semper a praeceptis Theoriae aberrare soleat, euenire potest, vt leui errore commisso numerus λ^o non solum non euanescat, sed adeo vnitatem excedat: quo casu vtique expediret lente vti simplici.

Scholion 2.

113. Cum tanti sit momenti rationem aberrationis a qua praxis vix liberari potest habere, eam etiam in lentibus simplicibus perpendi conueniet. Postulauimus autem pro datis distantiis determinatricibus lentem construi posse in qua numerus λ datum obtineat valorem dum ne sit vnitate minor; hic igitur obseruari oportet, quo maior fuerit λ, eo difficilius fore errorem euitare; si enim in constructione leuissimus error committatur alius eo magis diuersus valor pro λ orietur, quo maior fuerit λ. Verum e contrario cum vnitas sit minimus valor, quem λ recipere potest ex natura minimi liquet etiamsi in praxi a praescripta regula notabiliter recedatur tamen inde vix

ſenſibile diſcrimen in valorem λ eſſe redundaturum. Ex quo concludimus feliciſſimo cum ſucceſſu eiusmodi lentes ſimplices parari poſſe, pro quibus futurum ſit $\lambda = 1$, neque hic errores praxeos, niſi fuerint enormes, admodum eſſe pertimeſcendos. Deinde quo minus numerus λ vnitatem ſuperare debeat, eo certiores eſſe poterimus de ſucceſſu, ſed non eo gradu, quo caſu, $\lambda = 1$; at ſi opus ſit eiusmodi lente, pro qua valor ipſius λ debeat eſſe numerus ſatis magnus, difficillime per praxin ſatisfiet ac fortaſſe ingentem lentium numerum parare oportebit, antequam vna obtineatur ſcopo ſatisfaciens. Quamobrem ſi praxi conſulere velimus, vix alias lentes exigere debemus, niſi pro quibus numerus λ vel ſit vnitas ipſa, vel parumper maior. Sin autem ad inſigne aliquod commodum aliae lentes requirantur, labori non erit parcendum, cum fortaſſe non niſi poſt plurimos conatus irritos voti tandem compotes reddi queamus.

Problema 3.

114. Definire eam lentem duplicatam, pro qua, ſi numerus f intra limites 0 et 1 accipiatur numerus λ^0 minimum adipiscatur valorem.

Solutio.

Poſitis iisdem, quae in praecedentibus problematibus ſunt conſtituta, inuenimus eſſe

$$\lambda^0 = \lambda f^3 + \lambda' (1-f)^3 - \nu f(1-f)$$

vbi cum f intra limites 0 et 1 aſſumi debeat, ambo termini

termini λf^3 et $\lambda'(1-f)^3$ erunt positiui. Quare vt λ^0 omnium minimum valorem nanciscatur, necesse est vtrique numero λ et λ' minimum valorem cuius est capax tribui.

Sit ergo $\lambda = 1$ et $\lambda' = 1$, ac habebimus:

$$\lambda^0 = 1 - 3f + 3ff - \nu f + \nu ff = 1 - (3+\nu)f(1-f)$$

quae expressio vt minima reddatur, oportet fieri $f(1-f)$ maximum, id quod fit sumendo $f = \frac{1}{2}$; hincque oritur

$$\lambda^0 = 1 - \tfrac{1}{4}(3+\nu) = \frac{1-\nu}{4} = 0,191827$$

Quare constructio huius lentis duplicatae ita se habebit.

$$\text{Pro lente PP radius faciei} \begin{cases} \text{anterioris} = \dfrac{2\alpha\beta}{(2\varrho-\sigma)\beta+\sigma\alpha} \\ \text{posterioris} = \dfrac{2\alpha\beta}{(2\sigma-\varrho)\beta+\varrho\alpha} \end{cases}$$

$$\text{Pro lente QQ radius faciei} \begin{cases} \text{anterioris} = \dfrac{2\alpha\beta}{(2\sigma-\varrho)\alpha+\varrho\beta} \\ \text{posterioris} = \dfrac{2\alpha\beta}{(2\varrho-\sigma)\alpha+\sigma\beta} \end{cases}$$

et si aperturae semidiameter sit $= x$, erit spatium diffusionis

$$Gg = \mu\beta\beta xx\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)\left(0,191827\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)^2+\frac{\nu}{\alpha\beta}\right)$$

Si eiusmodi vitro vtamur, pro quo est $n = 1,60 = \frac{8}{5}$ tum ob $\nu = \frac{4}{15}$, prodiret $\lambda^0 = 0,183333$ ideoque haec vitri species adhuc minorem confusionem pareret.

Coroll.

Coroll. 1.

115. Si pro iisdem diſtantiis determinatricibus a et $\mathfrak{b}$ lens ſimplex minimam diffuſionem pariens conſtruatur quod fit ſumendo

$$\text{radium faciei} \begin{cases} \text{anterioris} = \frac{a\mathfrak{b}}{\rho\mathfrak{b}+\sigma a} \\ \text{poſterioris} = \frac{a\mathfrak{b}}{\rho a+\sigma\mathfrak{b}} \end{cases}$$

ſpatium diffuſionis foret

$$\mu\,\mathfrak{b}\mathfrak{b}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left(\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2+\tfrac{\nu}{a\mathfrak{b}}\right).$$

Coroll. 2.

116. Apparet ergo a lente duplicata deſcripta multo minorem oriri diffuſionem, quam a lente ſimplici, etiamſi haec iam ad minimam diffuſionem ſit inſtructa: Cum enim ceterae partes ſint pares coefficiens membri $\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^2$ plus quam quintuplo minor eſt in duplicata quam in ſimplici.

Coroll. 3.

117. Si ponamus $\lambda=1$ et $\lambda'=1$, vel ſaltem $\lambda'=\lambda$, minor valor pro λ° obtineri nequit, quam inuenimus, etiamſi pro f alios valores admittere velimus. Vnde ſi vtraque lens per ſe iam minimam diffuſionem pariat, pro lente duplicata valor ipſius λ° minor quam 0,191827 fieri nequit.

Scholion.

118. Huiusmodi ergo lentes duplicatae maxime ſunt notatu dignae cum loco ſimplicium adhibitae multo

multo minorem diffufionem pariant, ex quo in conftructione Telefcopiorum et Microfcopiorum earum ampliffimus erit vfus. Neque vero hac infigni proprietate funt praeditae, fed etiam earum conftructio in praxi minimis difficultatibus eft obnoxia: propterea quod etfi a praefcriptis regulis parumper aberretur, effectus tamen inde vix vllam mutationem patiatur. Siue enim in conftructione vtriusque feorfim leuis error committatur, valores numerorum λ et λ' vnitatem haud fenfibiliter excedent; fiue in quantitate f valor iuftus $f = \frac{1}{2}$ non exacte obferuetur, error vix fentietur, quoniam hi numeri ex natura minimi funt eruti. Quomodocunque autem a regulis praefcriptis aberretur, valor ipfius λ° inde paulifper maior prodibit. Veluti fi eiusmodi errores committantur, vt fit $\lambda = 1 + \frac{1}{10}$; $\lambda' = 1 + \frac{1}{10}$ et $f = \frac{1}{2} + \frac{1}{20}$, prodibit $\lambda^{\circ} = 0, 191827 + 0, 02558$ feu $\lambda^{\circ} = 0, 2174$, ita vt difcrimen partem tantum quadragefimam vnitatis conficiat. Facile autem intelligitur, dummodo λ° prodeat minus quam $\frac{1}{4}$, quod nunquam non, facile praeftari poffe videtur, ab his lentibus duplicatis infignem vtilitatem expectandam effe. Etfi ergo eiusmodi lentes duplicatae confici poffunt, pro quibus numerus λ° plane euanefcat, ob harum lubricam conftructionem illae iftis anteferendae videntur, vt mox clarius patebit.

Problema 4.

119. Pro datis diftantiis determinatricibus $AE = a$ et $bG = \mathfrak{b}$ eas lentes duplicatas inuenire, in quibus fit $\lambda^{\circ} = 0$.

Solutio.

Cum fieri nequeat $\lambda^{\circ}=0$, nisi numerus f extra limites 0 et 1 accipiatur, simulque numeri λ et λ' fuerint inaequales, ita vt alterutra saltem lens non debeat seorsim minimam diffusionem parere; ponamus esse vel $f>1$ vel $f<0$. Sit ergo primo $f=1+\xi$, et cum sit

$$\lambda^{\circ}=\lambda(1+\xi)^3-\lambda'\xi^3+\nu\xi(1+\xi)$$

vt fiat $\lambda^{\circ}=0$ oportet esse:

$$\lambda'=\lambda(1+\tfrac{1}{\xi})^3+\frac{\nu(1+\xi)}{\xi^2}$$

vnde λ' necessario vnitatem superabit, cuius valor ne prodeat nimis magnus, sumi conueniet $\lambda=1$, ita vt sit

$$\lambda'=(1+\tfrac{1}{\xi})^3+\frac{0,232602(1+\xi)}{\xi^2} \text{ et } \lambda=1.$$

Quicunque ergo valor ipsi ξ tribuatur, lens duplicata habetur, pro qua sit $\lambda^{\circ}=0$, ac propterea spatium diffusionis $=\mu\mathfrak{b}\mathfrak{b}xx(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}})\tfrac{\nu}{a\mathfrak{b}}$. Videamus nonnullos casus speciales.

$f=\tfrac{3}{2}$; $\xi=\tfrac{1}{2}$; $\lambda=1$ et $\lambda'=28,396152$

$f=2$; $\xi=1$; $\lambda=1$ et $\lambda'=8,465384$

$f=3$; $\xi=2$; $\lambda=1$ et $\lambda'=3,549519$

$f=4$; $\xi=3$; $\lambda=1$ et $\lambda'=2,473789$

$f=5$; $\xi=4$; $\lambda=1$ et $\lambda'=2,025841$

$f=6$; $\xi=5$; $\lambda=1$ et $\lambda'=1,783846$

etc.

Pro

Pro altero caſu ſit $f = -\xi$, ideoque

$$\lambda^{\circ} = -\lambda\xi^{3} + \lambda'(1+\xi)^{3} + \nu\xi(1+\xi)$$

vnde facto $\lambda^{\circ} = 0$ prodit

$$\lambda = \lambda'(1+\tfrac{1}{\xi})^{3} + \frac{\nu(1+\xi)}{\xi^{2}}.$$

Statui ergo conueniet $\lambda' = 1$, ac pro λ notentur caſus ſequentes.

$f = -\tfrac{1}{2}$; $\xi = \tfrac{1}{2}$; $\lambda = 28,396152$ et $\lambda' = 1$
$f = -1$; $\xi = 1$; $\lambda = 8,465384$ et $\lambda' = 1$
$f = -2$; $\xi = 2$; $\lambda = 3,549519$ et $\lambda' = 1$
$f = -3$; $\xi = 3$; $\lambda = 2,473789$ et $\lambda' = 1$
$f = -4$; $\xi = 4$; $\lambda = 2,025841$ et $\lambda' = 1$
$f = -5$; $\xi = 5$; $\lambda = 1,783846$ et $\lambda' = 1$

ſicque patet infinitis modis huiusmodi lentes duplicatas parari poſſe, pro quibus ſit $\lambda^{\circ} = 0$, et ſpatium diffuſionis

$$Gg = \mu\mathfrak{E}\mathfrak{E}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}}\right)\frac{\nu}{a\mathfrak{E}}.$$

Scholion 1.

120. Si huiusmodi lentes accuratiſſime parari poſſent, nullum eſt dubium, quin praecedentibus eſſent anteferendae propterea quod diffuſio iis adhuc magis diminuitur. Verum dolendum eſt, quod minimus error in earum conſtructione commiſſus omnem

fere

fere vſum deſtruat. Quo hoc facilius diiudicare quaeramus, examinemus eum caſum, quo eſt

$$f = 5,\ \lambda = 1 \text{ et } \lambda' = 2,025841$$

hincque

$$\lambda^{\circ} = \lambda f^3 - \lambda'(f-1)^3 + \nu f(f-1) = 0.$$

Ponamus autem in conſtructione errorem eſſe commiſſum vt reuera non ſit $f = 5$ ſed $f = 5\frac{1}{10}$, dum numeri λ et λ' ſuos iuſtos valores obtineant: ob hunc autem vix vitandum errorem non ſit $\lambda^{\circ} = 0$, ſed adeo $\lambda^{\circ} = -2,011$; ſicque haec lens duplicata ſimplicibus longe eſt poſtponenda, ſimili modo ſi f eſſet $= 5$, ſed vel λ vel λ' tantillum a praeſcripto valore aberraret, enorme ſtatim diſcrimen in valorem ipſius λ° redundaret. Minus quidem error metuendus videtur in ea ſpecie, qua $\lambda = 1$, $\lambda' = 28,396152$ et $f = \frac{3}{2}$; ſed praeterquam, quod lens ſimplex poſterior difficillime parari queat, pro qua λ' praeciſe valorem aſſignatum conſequatur, talis lens ad vſum dioptricum plane eſt inepta, ob ingentem alterius faciei curuaturam. Quae cum ita ſint, quia tam exiguus error in paratione huiusmodi lentium commiſſus facit, vt λ° adeo ſupra vnitatem excreſcat, vix ſperare poterimus, vt vnquam talis lens duplicata perficiatur, pro qua λ° vsque ad $\frac{1}{5}$ diminuatur. In lentibus autem praecedentis generis ſucceſſus vix fallere poterit, niſi in praxi enormiter a praeſcripta regula aberretur: ex quo his ſolis lentibus duplicatis cum fructu vti licebit, dum

contra

contra eae, quas in praeſente problemate deſcripſimus, penitus profligandae videntur.

Scholion 2.

121. Pro datis ergo binis diſtantiis determinatricibus a et $\mathfrak{b}$ ſemper eiusmodi lens duplicata parari poteſt, ex qua naſcatur ſpatium diffuſionis

$$Gg = \mu \mathfrak{b}\mathfrak{b}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left(\lambda^{0}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^{2}+\tfrac{\nu}{a\mathfrak{b}}\right)$$

ita vt λ^{0} numerum quemcunque denotare poſſit. Ad hoc enim ſatisfieri oportet huic aequationi:

$$\lambda^{0} = \lambda f^{3} + \lambda'(1-f)^{3} - \nu f(1-f)$$

id quod ſemper fieri poteſt, cum f plane ab arbitrio noſtro pendeat, et numeri λ et λ' tantum non vnitate minores accipi debeant. Definitis autem his tribus numeris λ, λ' et f ita, vt λ^{0} datum valorem obtineat, binae lentes ſimplices, ex quibus duplicata eſt componenda, ſecundum formulas §. 107 datas confici debent. Vbi quidem tenenda ſunt ea, quae modo obſeruauimus, in praxi eas lentes facillime obtineri, quando numeri λ et λ' vnitatem parum ſuperant, f vero propemodum $\frac{1}{2}$ denotat, cum contra quo magis hi numeri ab iſtis terminis recedant, eo maius ſit periculum ne effectus enormiter fallat. Ceterum cum diffuſionem a lente duplicata oriundam ad eandem formam reduxerimus, qua diffuſio lentis ſimplicis exprimitur, inde id commodi conſequimur, vt ſimili modo diffuſionem a lentibus magis multiplicatis ortum definire valeamus.

 Sup-

Supplementum I.

De lentibus duplicatis.

Si pro lente anteriori ratio refractionis ſit $n : 1$, pro lente poſteriori vero alia ratio $n' : 1$ locum habeat; problemata hic tractata ſequenti modo reſolui poterunt

Pro Problemate I.

Si pro numeris ϱ, σ, τ, qui ex n oriuntur, quaerantur ſimili modo ex n' valores ϱ', σ', et τ' erunt

Pro Lente

$$P\,P \text{ radius faciei } \begin{cases} \text{anter.} = \dfrac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda - 1)}} \\[2ex] \text{poſter.} = \dfrac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda - 1)}} \end{cases}$$

$$Q\,Q \text{ radius faciei } \begin{cases} \text{anter.} = \dfrac{b\beta}{\varrho'\beta + \sigma' b \pm \tau'(b+\beta)\sqrt{(\lambda' - 1)}} \\[2ex] \text{poſter.} = \dfrac{b\beta}{\varrho' b + \sigma\beta \mp \tau'(b+\beta)\sqrt{(\lambda' - 1)}} \end{cases}$$

et definitis ſimili modo valoribus μ', ν' ex ratione $n' : 1$ reperietur ſpatium diffuſionis

$$\beta\beta xx \begin{cases} +\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\mu'\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right) \end{cases}$$

reliqua manent vt in problemate.

Pro Problemate II.

Quoniam hic duae vitri ſpecies occurrunt, ponamus lentem ſimplicem quaeſitam ex alio quocunque vitri genere parari, cuius ratio refractionis ſit $n^{\circ} : 1$, vnde prodeant numeri μ° et ν°; ſuperfluum autem foret, numeros ϱ°, σ° et τ° computari, quoniam radii

dii facierum fiunt imaginarii, ita, vt eos exprimere non fit opus, et quoniam pro hac lente fimplici aequiualente numerus λ° eft introductus, erit huius lentis fpatium diffufionis

$$\mathfrak{b}\mathfrak{b}xx.\mu^\circ(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}})(\lambda^\circ(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}})^2+\tfrac{\nu^\circ}{a\mathfrak{b}})$$

quod vt cum fpatio diffufionis lentis duplicatae aequale fiat; ponatur, vti in problemate eft factum,

$$\tfrac{1}{\alpha}=\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{b}} \text{ et } \tfrac{1}{b}=\tfrac{1-f}{a}-\tfrac{f}{\mathfrak{b}}$$

vt prodeat

$$\tfrac{1}{a}+\tfrac{1}{\alpha}=f(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}) \text{ et } \tfrac{1}{b}+\tfrac{1}{\mathfrak{b}}=(1-f)(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}})$$

Vnde peruenietur ad hanc aequationem

$$\mu f(\lambda f^2(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}})^2+\nu(\tfrac{f-1}{a^2}+\tfrac{f}{a\mathfrak{b}}))$$
$$+\mu'(1-f)(\lambda'.(1-f)^2(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}})^2+\nu'(\tfrac{1-f}{a\mathfrak{b}}-\tfrac{f}{\mathfrak{b}^2}))$$
$$=\mu^\circ(\lambda^\circ(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}})^2+\tfrac{\nu^\circ}{a\mathfrak{b}})$$

quae ita repraefentari poteft

$$(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}})^2(\mu\lambda f^3+\mu'\lambda'(1-f)^3)$$
$$+(\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{b}})(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\mathfrak{b}})$$
$$=\mu^\circ\lambda^\circ(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}})^2+\tfrac{\nu^\circ\mu^\circ}{a\mathfrak{b}})$$

Vnde

$$\lambda^\circ=\frac{\mu\lambda f^3+\mu'\lambda'(1-f)^3}{\mu^\circ}+$$
$$\frac{aa\mathfrak{b}\mathfrak{b}}{\mu^\circ(a+\mathfrak{b})^2}((\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{b}})(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\mathfrak{b}})-\tfrac{\nu^\circ\mu^\circ}{a\mathfrak{b}})$$

Deni-

Denique si ex ratione refractionis $n' : 1$ computentur numeri ϱ', σ' et τ', radii facierum lentis duplicatae erunt

Pro Lente

PP radius faciei $\begin{cases} \text{anterioris} = \dfrac{a\beta}{(\varrho-\sigma(1-f))\beta+\sigma f a \pm \tau f(a+\beta)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \dfrac{a\beta}{(\sigma-\varrho(1-f))\beta+\varrho f a \mp \tau f(a+\beta)\sqrt{(\lambda-1)}} \end{cases}$

QQ radius faciei $\begin{cases} \text{anter.} = \dfrac{a\beta}{(\sigma'-\varrho' f)a+\varrho'(1-f)\beta \pm \tau'(1-f)(a+\beta)\sqrt{(\lambda'-1)}} \\ \text{poster.} = \dfrac{a\beta}{(\varrho'-\sigma' f)a+\sigma'(1-f)\beta \mp \tau'(1-f)(a+\beta)\sqrt{\lambda'-1}} \end{cases}$

Ad Problema III.

Hoc problema non solum pro diuersa refractione n et n' hic generalius pertractabo, sed etiam rationem distantiae inter binas lentes habebo. Primum igitur vtramque lentem ad distantias determinatrices lentis duplicatae, a et β, reuocabo, ponendo

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\beta}\right) \text{ et } \frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{a}+\frac{1}{\beta}\right)$$

vt sit

$$\frac{1}{\alpha}=\frac{f-1}{a}+\frac{f}{\beta} \text{ et } \frac{1}{b}=\frac{g}{a}+\frac{g-1}{\beta}$$

hincque

$$\alpha=\frac{a\beta}{fa+(f-1)\beta} \text{ et } b=\frac{a\beta}{(g-1)a+g\beta}$$

ideoque distantia lentium

$$\alpha+b=\frac{a\beta(f+g-1)(a+\beta)}{(fa+(f-1)\beta)((g-1)a+g\beta)}$$

quae si deberet esse $=0$, capi oporteret $g=1-f$ sed si distantiam aliquam inter lentes admittamus, statuamus

tuamus $f+g-1=\omega$, denotante ω fractionem quandam minimam, siue positiuam siue negatiuam vt distantia lentium prodeat positiua et valde parua. Cum hinc igitur sit $g=1+\omega-f$; erit lentium distantia

$$a+b=\frac{a\,\varepsilon\,(a+\varepsilon)\,\omega}{(fa+(f-1)\varepsilon)((\omega-f)a+(1+\omega-f)\varepsilon)}.$$

Spatium autem diffusionis nunc ita exprimetur:

$$\varepsilon\,\varepsilon\,x\,x.\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)\begin{cases}+\lambda\mu f^3\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)^2+\lambda'\mu' g^3\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)^2\\ +\tfrac{\mu\nu f}{a}\left(\tfrac{f-1}{a}+\tfrac{f}{\varepsilon}\right)\\ +\tfrac{\mu'\nu' g}{\varepsilon}\left(\tfrac{g}{a}+\tfrac{g-1}{\varepsilon}\right)\end{cases}$$

quae cum in hoc problemate ita tractari debeat, vt tam f, quam g positiue sumantur, et haec formula minima reddatur; euidens est; litteris λ et λ' minimos valores tribui debere, scilicet $\lambda=1$ et $\lambda'=1$; deinde pro hoc casu minimi f et g conuenienter definiantur, vbi quia $f+g-1=\omega$ ideoque constans in differentiatione, habebimus $dg=-df$; vndeobtinebimus hanc aequationem

$$(3\mu f^2-3\mu' g^2)\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)^2+\tfrac{\mu\nu}{a}\left(\tfrac{2f-1}{a}+\tfrac{2f}{\varepsilon}\right)$$
$$-\tfrac{\mu'\nu'}{\varepsilon}\left(\tfrac{2g}{a}+\tfrac{2g-1}{\varepsilon}\right)=0$$

cui proxime satisfit, ponendo $f=g=\frac{1+\omega}{2}$ quibus valoribus substitutis spatium diffusionis ipsum minimum, erit proxime

$$\tfrac{\varepsilon^2 x^2 (1+\omega)}{8}\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)\begin{cases}\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)^2(1+\omega)^2(\mu+\mu')\\ +\tfrac{2\mu\nu}{a}\left(\tfrac{\omega-1}{a}+\tfrac{1+\omega}{\varepsilon}\right)\\ +\tfrac{2\mu'\nu'}{\varepsilon}\left(\tfrac{1+\omega}{a}+\tfrac{\omega-1}{\varepsilon}\right)\end{cases}$$

 Tum

Tum vero diſtantia lentium erit

$$a+b=\frac{4\alpha\mathfrak{b}(\alpha+\mathfrak{b})\omega}{((1+\omega)\alpha+(\omega-1)\mathfrak{b})((\omega-1)\alpha+(1+\omega)\mathfrak{b})}$$

$$=\frac{4\alpha\mathfrak{b}(\alpha+\mathfrak{b})\omega}{-(1-\omega^2)(\alpha^2+\mathfrak{b}^2)+2(1+\omega)^2\alpha\mathfrak{b}}$$

cuius denominator cum ſit negatiuus ob ω minimum; neceſſe eſt, fractionem ω ſumi debere negatiuam; hincque adeo ſpatium diffuſionis minus reddetur.

Coroll.

Si ergo diſtantia obiecti α fuerit infinita ſeu $\alpha = \infty$, habebitur primo diſtantia lentium $= \frac{-4\mathfrak{b}\omega}{1-\omega^2}$; et ſecundo ſpatium diffuſionis

$$\frac{x^2(1+\omega)}{8\mathfrak{b}}\begin{cases}(1+\omega)^2(\mu+\mu') \\ -2\mu'\nu'(1-\omega)\end{cases}$$

quod cum ω debeat eſſe negatiuum, non mediocriter minus erit, quam ſi diſtantia lentium eſſet nulla.

Ad Problema IV.

In hoc problemate etiam diſtantiam lentium non negligamus; factaque reductione, vt ante, ſtatuamus ſpatium diffuſionis plane euaneſcens; id quod fieri nequit, niſi altera litterarum f et g ſit negatiua, quod cum etiam fiat, quando confuſio a diuerſa radiorum refrangibilitate oriunda ad nihilum redigi debet, vt infra videbimus; hic caſus multo magis euolutionem meretur. Ponatur igitur $g = -\zeta f$, vbi ζ ex

illa

illa conditione determinatur, vt primo pro diſtantia lentium ſit

$$\alpha+b=\frac{-\alpha\mathfrak{G}(\alpha+\mathfrak{G})(f-\zeta f-1)}{(f\alpha+(f-1)\mathfrak{G})((\zeta f+1)\alpha+\zeta f\mathfrak{G})}$$

et poſito $f+g-1=\omega$, prodeat $f-\zeta f=1+\omega$ ideoque $f=\frac{1+\omega}{1-\zeta}$ et $g=\frac{-\zeta(1+\omega)}{1-\zeta}$ ſicque

$$\alpha+b=\frac{-\alpha\mathfrak{G}(\alpha+\mathfrak{G})\omega(1-\zeta)^2}{((1+\omega)\alpha+(\omega+\zeta)\mathfrak{G})((1+\omega\zeta)\alpha+\zeta(1+\omega)\mathfrak{G})}$$

ac ſi in denominatore ω reiiciatur, erit

$$\alpha+b=\frac{-\alpha\mathfrak{G}(\alpha+\mathfrak{G})(1-\zeta)^2\omega}{(\alpha+\zeta\mathfrak{G})}$$

ſicque patet ω negatiue capi debere.

Poſito autem $g=-\zeta f$, fit ſpatium diffuſionis

$$\mathfrak{G}^2x^2\left(\tfrac{1}{\alpha}+\tfrac{1}{\mathfrak{G}}\right)\begin{cases}(\lambda\mu f^3-\lambda'\mu'\zeta^3f^3)\left(\tfrac{1}{\alpha}+\tfrac{1}{\mathfrak{G}}\right)^2\\ +\frac{\mu\nu f}{\alpha}\left(\frac{f-1}{\alpha}+\frac{f}{\mathfrak{G}}\right)\\ -\frac{\mu'\nu'\zeta f}{\mathfrak{G}}\left(\frac{-\zeta f}{\alpha}-\frac{(\zeta f+1)}{\mathfrak{G}}\right)\end{cases}$$

ad nihilum redigendum; vnde ſequitur fore

$$\lambda'=\frac{\lambda\mu}{\mu'\zeta^3}+\frac{\mu\nu.\alpha.\mathfrak{G}^2}{\mu'\zeta^3f^2(\alpha+\mathfrak{G})^2}\left(\frac{f-1}{\alpha}+\frac{f}{\mathfrak{G}}\right)$$
$$+\frac{\nu'.\alpha^2\mathfrak{G}}{\zeta^2f^2.(\alpha+\mathfrak{G})^2}\left(+\frac{\zeta f}{\alpha}+\frac{\zeta f+1}{\mathfrak{G}}\right)$$

qui valor cum debeat eſſe maior vnitate, ſi forte eueniat, vt minor prodeat, tunc non λ' ſed λ definiri conueniet, vbi notandum eſt eſſe $f=\frac{1+\omega}{1-\zeta}$. Hincque per formulas ante datas facile eruuntur radii facierum vtriusque lentis.

Etſi formula ſuperius data pro ſpatio diffuſionis iam ad caſum, quo diſtantia lentium eſt nulla, eſt

 ad-

adcommodata; tamen quia hic diſtantiam minimam aſſumimus, nullus inde error eſt metuendus.

Definitio 2.

122. *Lens triplicata eſt, quae conſiſtit ex tribus lentibus ſimplicibus ſibi immediate iunctis ad communem axem.*

Hic quidem etiam craſſitiem negligo, etiamſi neceſſario maior ſit quam in lentibus duplicatis. In ſuplemento autem oſtendetur, quomodo etiam diſtantiarum inter lentes ratio ſit habenda.

Corollarium.

123. Poteſt ergo lens triplicata conſiderari, quaſi compoſita ex lente duplicata et lente ſimplici, hocque duplici modo, prout vel binae anteriores, vel binae poſteriores lentem duplicatam conſtituere concipiuntur.

Scholion.

124. Lentibus triplicatis tum demum vſum in praxi concedi conueniet, cum eadem commoda per lentes ſimplices vel duplicatas conſequi non licet. Haec autem commoda in paruitate numeri λ conſiſtunt, qui quamdiu vnitate fuerit maior, ſemper lente ſimplici vti praeſtat, cum vero circumſtantiae in expreſſione diffuſionis minorem numerum λ requirunt, ad lentes multiplicatas erit confugiendum. Quoniam igitur eiusmodi lentes duplicatas conficere docuimus, in

in quibus valor ipſius λ non ſolum ad nihilum vsque, ſed adeo ad negatiua diminui queat, vſus lentium triplicatarum ſuperfluus videtur. Verum iam obſeruauimus in praxi aegre eiusmodi duplicatas lentes parari poſſe, pro quibus valor ipſius λ minor ſit quam 0, 191827, propterea quod ſi leuiſſimus error committatur, omnis labor irritus reddatur. His igitur caſibus imprimis, quando minori valore numeri λ opus eſt, lentes triplicatae in vſum erunt vocandae: et quia reſpectu ad praxin habito non omnes aequo ſucceſſu conſtruere licet, errores ineuitabiles hic quoque imprimis ſpectari oportet, vt pateat quousque numerus λ cum ſucceſſu diminui queat, ac ſi adhuc minore valore opus fuerit lentes adeo quadruplicate erunt inducendae.

Problema 5.

125. Datis binis diſtantiis determinatricibus omnes lentes triplicatas definire, ſimulque ſpatium diffuſionis ab iis ortum. Tab. II. Fig. 8.

Solutio.

Sit Ee obiectum, cuius magnitudo $= z$, et diſtantia a lente $AE = a$: tum primae lentis PP diſtantiae determinatrices ſint a et α, ſecundae lentis QQ b et β; ac tertiae RR, c et γ. His poſitis quia lentes immediate iunctae ſumuntur, erit $\alpha + b = 0$, $\beta + c = 0$, et a et γ diſtantiae determinatrices lentis triplicatae, ita vt α et β arbitrio noſtro relinquantur.

 Si

Si igitur lens prima sola PP adesset, imago repraesentaretur in $F\zeta$, vt esset $AF=\alpha$ et $F\zeta=\frac{\alpha}{a}z$: si binae priores PP et QQ solae adessent imago exhiberetur in $G\eta$, vt esset $AG=\mathfrak{b}$ et $G\eta=-\frac{\mathfrak{b}}{a}z$; at per lentem triplicatam referetur in $H\theta$ vt sit $cH=\gamma$ et $H\theta=\frac{\gamma}{a}z$, pro situ inuerso ob $\frac{\alpha\mathfrak{b}}{b\,c}=1$. Hactenus scilicet res perinde se habet, ac si in A haberetur lens simplex ad distantias determinatrices a et γ accommodata.

At si ad spatium diffusionis Hh respiciamus figuram singularum lentium in computum ducere debemus, quatenus praeter distantias determinatrices numeri arbitrarii $\lambda, \lambda', \lambda''$ inuoluuntur; ex quibus facies singularum lentium supra §. 91 sunt definitae. Indidem autem colligitur pro apertura cuius semidiameter $=x$, spatium diffusionis ob $\frac{\alpha}{b}=-1$ et $\frac{\mathfrak{b}}{c}=-1$ fore:

$$Hh=\mu\gamma\gamma xx\begin{cases}+\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{\alpha a}\right)\\+\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu}{b\mathfrak{b}}\right)\\+\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right)\end{cases}$$

quae expressio, vt ad formam vni lenti respondentem reducatur statuamus:

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{b}+\frac{1}{\mathfrak{b}}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

et

et quia $\frac{1}{\alpha}+\frac{1}{b}=0$, et $\frac{1}{\mathfrak{b}}+\frac{1}{c}=0$, his aequationibus addendis adipiſcimur: $f+g+h=1$. Porro vero erit

$$\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{b}=\frac{1}{a}-f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{\mathfrak{b}}=(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{c}=\frac{1}{a}-(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

ſiue ob $1=f+g+h$

$$\frac{1}{a}=\frac{f+g+h}{a};\quad \frac{1}{\alpha}=\frac{-g-h}{a}+\frac{f}{\gamma}$$

$$\frac{1}{b}=\frac{g+h}{a}-\frac{f}{\gamma};\quad \frac{1}{\mathfrak{b}}=-\frac{h}{a}+\frac{f+g}{\gamma}$$

$$\frac{1}{c}=\frac{h}{a}-\frac{f-g}{\gamma};\quad \frac{1}{\gamma}=\frac{f+g+h}{\gamma}$$

Vnde cum ſpatium diffuſionis fiat.

$$Hh=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left((\lambda f^3+\lambda' g^3+\lambda'' h^3)\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\nu\left(\frac{f}{a\alpha}+\frac{g}{b\mathfrak{b}}+\frac{h}{c\gamma}\right)\right)$$

reducetur id ad hanc formam

$$Hh=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left((\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h))\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Radiorum autem in h concurrentium inclinatio ad axem erit $=\frac{x}{\gamma}$.

Coroll. 1.

126. Haec igitur lens triplicata idem producit ſpatium diffuſionis quod produceret lens ſimplex ad easdem diſtantias determinatrices inſtructa, numero eius arbitrario (per litteram λ indicato) exiſtente

$$\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h)$$

vbi quidem eſt $f+g+h=1$.

Coroll.

Coroll. 2.

127. Quatenus ergo haec quantitas reddi poteſt minor non ſolum vnitate ; ſed etiam fractione 0,191827 ita ſcilicet, vt praxis non enormi aberrationi ſit expoſita, eatenus lentibus triplicatis vſus erit concedendus.

Coroll. 3.

128. Si ſit vel $f=0$, vel $g=0$, vel $b=0$, vna lentium habebit facies parallelas, et lens triplicata aequiualebit duplicatae ac ſi duae litterarum f, g, b ſimul euaneſcant, tertia in vnitatem abeunte, caſus habebitur lentis ſimplicis.

Coroll. 4.

129. Si ſit $f=1$, ideoque $b=-g$, valor numeri λ pro lente triplicata erit $\lambda+(\lambda'-\lambda'')g^3$, ideoque ſi $\lambda''=\lambda'$ lens triplicata ſimplici aequiualebit, quod idem euenit ſi fuerit vel $g=1$ vel $b=1$.

Coroll. 5.

130. Sumtis autem pro f, g, b numeris idoneis ob $f+g+b=1$, conſtructio lentis triplicatae ex formulis §. 91 exhibitis eſt petenda ſumendo.

$$\frac{1}{\alpha}=\frac{-1+f}{a}+\frac{f}{\gamma};\quad \frac{1}{b}=\frac{1-f}{a}-\frac{f}{\gamma}$$

$$\frac{1}{6}=\frac{-b}{a}+\frac{1-b}{\gamma};\quad \frac{1}{c}=\frac{b}{a}-\frac{1+b}{\gamma}$$

Scholion

Scholion 1.

131. Quemadmodum hinc expressio inuenta prodeat, notandum est fore $\frac{f}{a\alpha}+\frac{g}{b\mathfrak{G}}+\frac{h}{c\gamma}=$

$$+\frac{1}{a\alpha}(-f(1-f)-gh(1-f))$$
$$+\frac{1}{a\gamma}(+ff+g(1-f)(1-h)+fgh+hh)$$
$$+\frac{1}{\gamma\gamma}(-fg(1-h)-h(1-h))$$

sed $-f(1-f)-gh(1-f)=-(1-f)(f+gh)=-(1-f)(1-g)(1-h)$ ob $f=1-g-h$ ideoque $f+gh=(1-g)(1-h)$. Simili modo pro $\frac{1}{\gamma\gamma}$ est $-fg(1-h)-h(1-h)=-(1-h)(h+fg)=-(1-f)(1-g)(1-h)$ ob $h=1-f-g$. Denique pro $\frac{1}{a\gamma}$, quia est $ff+hh=(f+h)^2-2fh=(1-g)^2-2fh=1-2g+gg-2fh$, hoc valore substituto coefficiens ipsius $\frac{1}{a\gamma}$ erit

$$1-2g-2fh+gg+fgh+g(1-f)(1-h)=$$
$$1+(g+fh)(g-2)-g(1-f)(1-h)=1-2(1-f)(1-g)(1-h)$$

ob $g+fh=(1-f)(1-h)$. Consequenter colligitur

$$\frac{f}{a\alpha}+\frac{g}{b\mathfrak{G}}+\frac{h}{c\gamma}=-(1-f)(1-g)(1-h)(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\frac{1}{a\gamma}.$$

Scholion 2.

132. Hoc problema etiam ope praecedentium facilius sequenti modo resolui potest. Considerentur scilicet binae lentes PP et QQ iunctim sumtae tanquam lens duplicata ad distantias determinatrices a et $\mathfrak{G}$ per numeros arbitrarios λ, λ' et f instructa, ac posito $\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)=\lambda^{(2)}$, spatium diffusionis ex ea sola ortum erit

$$\mu\mathfrak{G}\mathfrak{G}xx(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}})(\lambda^{(2)}(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}})^2+\tfrac{\nu}{a\mathfrak{G}})$$

 quae

quae lens ſi iam in compoſitione cum tertia R R tanquam ſimplex tractetur exinde elicitur ſpatium diffuſionis perinde atque ex coniunctione duarum ſimplicium

$$Hb = \mu\gamma\gamma xx \begin{cases} +\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)\left(\lambda^{(2)}\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu}{a\mathfrak{b}}\right) \\ +\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \end{cases}$$

vbi notandum eſt eſſe $\mathfrak{b}+c=0$, et conſtructio lentis duplicatae ex §. 107 erit petenda lentis vero ſimplicis RR ex diſtantiis determinatricibus $c=-\mathfrak{b}$ et γ vna cum numero arbitrario λ''. Ponatur iam $\frac{1}{\mathfrak{b}}=\frac{g-1}{a}+\frac{g}{\gamma}$ vt ſit $\frac{1}{c}=\frac{1-g}{a}-\frac{g}{\gamma}$, eritque ſpatium diffuſionis huius lentis triplicatae,

$$Hb=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\left(\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g)\right)\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Quare ſi pro lente triplicata ponatur

$$\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g)=\lambda^{(3)}$$

ita vt iam numerus g inſuper arbitrio noſtro relinquatur, habebitur ſpatium diffuſionis more hactenus recepto expreſſum

$$Hb=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Hinc iam id intelligitur, quod ex praecedente ſolutione minus patet; ſi numerus $\lambda^{(3)}$ fuerit vnitate maior loco binarum priorum lentium PP et QQ commodius vnicam ſimplicem adhiberi; ex quo eatenus tantum lentes triplicatae reſultare cenſendae ſunt, quatenus nume-

numerus $\lambda^{(2)}$ vnitate eſt minor. Vidimus autem ſucceſſum tuto ſperari non poſſe, niſi $\lambda^{(2)}$ aequalis ſit fractioni 0,191827, vel ea non multo maior; vnde ſi praxi conſulere velimus, ipſi $\lambda^{(2)}$ minorem valorem tribui non conuenit atque ob eandem rationem numerus g intra terminos 0 et 1 accipi debebit; cuius valor imprimis ad praxin erit accommodatus ſi reddat numerum $\lambda^{(3)}$ minimum, quia tum leues errores negotium minime turbant. Quodſi vero pro $\lambda^{(2)}$ valorem aſſignatum ſubſtituamus, obtinebimus

$$\lambda^{(3)} = \lambda f^3 g^3 + \lambda' g^3 (1-f)^3 + \lambda''(1-g)^3 - \nu g^3 f(1-f) - \nu g(1-g)$$

Conducet autem vtramque expreſſionem pro numero $\lambda^{(3)}$, quo ſpatium diffuſionis a lente triplicata ortum definitur, hic expoſuiſſe cum aliae concluſiones ex altera facilius deducantur. Etſi autem hinc omnes valores pro $\lambda^{(3)}$ obtineri poſſunt tamen eos tantum, qui prope minimum ſubſiſtunt ad praxin adhiberi conueniet.

Problema. 6.

133. Datis diſtantiis determinatricibus $AE = a$ et $cH = \gamma$ definire eam lentem triplicatam, quae minimum ſpatium diffuſionis producat.

Solutio I.

Duplici modo hoc problema ſolui poteſt, prout ſpatium diffuſionis vel ita exprimitur, vti in ſolutione problematis praecedentis, vel in Scholio 2. Priori

cui modo numeros f, g, h ita determinari oportet, vt minima reddatur haec expressio:

$$\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h)$$

vbi notandum est esse $f+g+h=1$. Eius ergo differentiali nihilo aequali posito habebimus:

$$3\lambda ffdf+3\lambda' ggdg+3\lambda'' hhdh$$
$$+\nu df(1-g)(1-h)+\nu dg(1-f)(1-h)+\nu dh(1-f)(1-g)=0.$$

Cum autem sit $dh=-df-dg$ erit

$$\left.\begin{array}{l} +3\lambda ffdf-3\lambda'' hhdf+\nu df(1-g)(f-h) \\ +3\lambda' ggdg-3\lambda'' hhdg+\nu dg(1-f)(g-h) \end{array}\right\}=0$$

quia vero bina differentialia df et dg a se inuicem non pendent ambo membra huius aequationis seorsim euanescere debent, vnde ob $1-g=f+h$ et $1-f=g+h$ has duas nanciscimur aequationes:

$$3\lambda ff-3\lambda'' hh+\nu ff-\nu hh=0$$
$$3\lambda' gg-3\lambda'' hh+\nu gg-\nu hh=0$$

ex quibus elicimus:

$$f=h\sqrt{\frac{3\lambda''+\nu}{3\lambda+\nu}} \text{ et } g=h\sqrt{\frac{3\lambda''+\nu}{3\lambda'+\nu}}.$$

Cum autem sit

$$f+g+h=1 \text{ seu } \frac{1}{h}=1+\frac{f}{h}+\frac{g}{h} \text{ erit}$$

$$\frac{1}{h}=1+\sqrt{\frac{3\lambda''+\nu}{3\lambda+\nu}}+\sqrt{\frac{3\lambda''+\nu}{3\lambda'+\nu}}$$

$$\frac{1}{g}=1+\sqrt{\frac{3\lambda'+\nu}{3\lambda+\nu}}+\sqrt{\frac{3\lambda'+\nu}{3\lambda''+\nu}}$$

$$\frac{1}{f}=1+\sqrt{\frac{3\lambda+\nu}{3\lambda'+\nu}}+\sqrt{\frac{3\lambda+\nu}{3\lambda''+\nu}}$$

Quicun-

Quicunque ergo numeri λ, λ', λ'' in conſtructione ſingularum lentium fuerint vſurpati, hinc numeri f, g, et h determinantur, ex quibus ſpatium diffuſionis minimum reſultet. Pro lentium autem conſtructione hinc diſtantiae α, b, $\mathfrak{b}$, c ita definiuntur, vt ſit

$$\frac{1}{\alpha} = \frac{-1+f}{a} + \frac{f}{\gamma}; \quad \frac{1}{b} = \frac{1-f}{a} - \frac{f}{\gamma}$$

$$\frac{1}{\mathfrak{b}} = \frac{-h}{a} + \frac{1-h}{\gamma}; \quad \frac{1}{c} = \frac{h}{a} - \frac{1+h}{\gamma}$$

eſt vero:

$$f = \frac{\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)} + \sqrt{(3\lambda+\nu)(3\lambda''+\nu)} + \sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

$$g = \frac{\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)} + \sqrt{(3\lambda+\nu)(3\lambda''+\nu)} + \sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

$$h = \frac{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)} + \sqrt{(3\lambda+\nu)(3\lambda''+\nu)} + \sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

Ex diſtantiis autem a, α, b, $\mathfrak{b}$, c, γ cum numeris λ, λ', λ'' lentes ipſae per formulas §. 91 exhibitas conſtruuntur.

Coroll. 1.

134. Si pro hac lente triplicata ponatur

$$\lambda f^3 + \lambda' g^3 + \lambda'' h^3 - \nu(1-f)(1-g)(1-h) = \lambda^{(3)}$$

ſubſtituendis his valoribus pro f, g, et h reperietur

$$\lambda^{(3)} = \frac{1}{3\left(\frac{1}{\sqrt{(3\lambda+\nu)}} + \frac{1}{\sqrt{(3\lambda'+\nu)}} + \frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2} - \frac{1}{3}\nu$$

vnde ſpatium diffuſionis fit

$$Hh = \mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2 + \frac{\nu}{a\gamma}\right)$$

O 3 Coroll.

Coroll. 2.

135. Hoc autem ſpatium diffuſionis omnium fiet minimum ſi numeris λ, λ', λ'' minimi valores, quos accipere poſſunt, tribuantur. Sit ergo $\lambda = 1$; $\lambda' = 1$, $\lambda'' = 1$, eritque

$$f = \tfrac{1}{3};\ g = \tfrac{1}{3},\ h = \tfrac{1}{3}$$

$$\text{et } \lambda^{(3)} = \frac{\nu+3}{27} - \frac{\nu}{3} = \frac{3-8\nu}{27} = 0,042165$$

ob 0, 232692; qui ergo valor multo eſt minor, quam caſu lentium duplicatarum.

Coroll. 3.

136. Hoc porro caſu erit $\frac{1}{a} = -\frac{2}{3\alpha} + \frac{1}{3\gamma}$; $\frac{1}{b} = \frac{2}{3\alpha} - \frac{1}{3\gamma}$; $\frac{1}{\mathfrak{b}} = \frac{-1}{3\alpha} + \frac{2}{3\gamma}$ et $\frac{1}{c} = \frac{1}{3\alpha} - \frac{2}{3\gamma}$; vnde conſtructio lentium ternarum ſimplicium ita ſe habebit.

Pro Lente

Prima radius faciei $\begin{cases} \text{anterioris} = \dfrac{3\alpha\gamma}{(3\varrho-2\sigma)\gamma+\sigma\alpha} \\ \text{poſterioris} = \dfrac{3\alpha\gamma}{(3\sigma-2\varrho)\gamma+\varrho\alpha} \end{cases}$

Secunda radius faciei $\begin{cases} \text{anterioris} = \dfrac{3\alpha\gamma}{(2\varrho-\sigma)\gamma+(2\sigma-\varrho)\alpha} \\ \text{poſterioris} = \dfrac{3\alpha\gamma}{(2\sigma-\varrho)\gamma+(2\varrho-\sigma)\alpha} \end{cases}$

Tertia radius faciei $\begin{cases} \text{anterioris} = \dfrac{3\alpha\gamma}{\varrho\gamma+(3\sigma-2\varrho)\alpha} \\ \text{poſterioris} = \dfrac{3\alpha\gamma}{\sigma\gamma+(3\varrho-2\sigma)\alpha} \end{cases}$

Solutio altera Problematis.

137. Conſideremus binas lentes priores PP et QQ vt lentem duplicatam ad diſtantias determinatrices a et $\mathfrak{b}$ ita

ita instructam vt posito $\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)=\lambda^{(2)}$ spatium diffusionis inde oriundum sit

$$=\mu\,\mathfrak{G}\mathfrak{G}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)\left(\lambda^{(2)}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\tfrac{\nu}{a\mathfrak{G}}\right)$$

sed pro constructione binarum lentium simplicium, ex quibus haec lens est composita, recordandum est esse

$$\tfrac{1}{\alpha}=\tfrac{-1+f}{a}+\tfrac{f}{\mathfrak{G}} \text{ et } \tfrac{1}{b}=\tfrac{1-f}{a}-\tfrac{f}{\mathfrak{G}}$$

Adiuncta iam tertia lente RR ad distantias determinatrices $c=-\mathfrak{G}$ et γ per numerum arbitrarium λ'' instructa si ponamus $\tfrac{1}{\mathfrak{G}}=\tfrac{-1+g}{a}+\tfrac{g}{\gamma}=-\tfrac{1}{c}$, et

$$\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g)=\lambda^{(3)}$$

erit spatium diffusionis

$$Hh=\mu\gamma\gamma xx\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{a\gamma}\right)$$

quod vt fiat minimum, valor ipsius $\lambda^{(3)}$ minimus reddi debet, quaeratur ergo primo g dum $\lambda^{(2)}$ vt numerus datus spectatur et habebimus

$$3\lambda^{(2)}gg-3\lambda''(1-g)^2-\nu+2\nu g=0$$

vnde elicitur

$$g=\frac{-3\lambda''-\nu+\sqrt{(3\lambda^{(2)}+\nu)(3\lambda''+\nu)}}{3\lambda^{(2)}-3\lambda''} \quad \text{siue}$$

$$g=\frac{\sqrt{(3\lambda''+\nu)}}{\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{(3\lambda''+\nu)}}$$

et hinc valor ipsius $\lambda^{(3)}$ erit

$$\lambda^{(3)}=\frac{1}{3\left(\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2}-\frac{1}{3}\nu$$

vel

vel

$$\frac{1}{3\lambda^{(3)}+\nu}=\left(\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2$$

Sicque fit:

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Simili modo si f ita definiatur, vt $\lambda^{(2)}$ fiat minimum reperietur:

$$f=\frac{\sqrt{(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)}+\sqrt{(3\lambda'+\nu)}}$$

hocque valore substituto

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}$$

Quare si numeris $\lambda, \lambda', \lambda''$, arbitrio nostro relictis bini numeri f et g ita definiantur, vt $\lambda^{(3)}$ consequatur valorem minimum erit

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

vnde idem valor pro $\lambda^{(3)}$ reperitur, quem ante inuenimus.

Coroll. 1.

138. Ex hac ergo solutione numeri f et g ita definiuntur vt sit

$$\frac{1}{f\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}} \quad \text{et}$$

$$\frac{1}{g\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

siue

sive hoc modo

$$\left(\frac{1}{f}-1\right)\frac{1}{\surd(3\lambda+\nu)}=\frac{1}{\surd(3\lambda'+\nu)} \text{ et}$$

$$\left(\frac{1}{g}-1\right)\left(\frac{1}{\surd(3\lambda+\nu)}+\frac{1}{\surd(3\lambda'+\nu)}\right)=\frac{1}{\surd(3\lambda''+\nu)}$$

Coroll. 2.

139. Ex inuentis minimis valoribus numerorum $\lambda^{(2)}$ et $\lambda^{(3)}$ numeri f et g etiam ita definiuntur vt sit

$$\frac{1}{f\surd(3\lambda+\nu)}=\frac{1}{\surd(3\lambda^{(2)}+\nu)} \text{ seu } f=\surd\left(\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}\right) \text{ et}$$

$$\frac{1}{g\surd(3\lambda^{(2)}+\nu)}=\frac{1}{\surd(3\lambda^{(3)}+\nu)} \text{ seu } g=\surd\left(\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}\right)$$

Coroll. 3

140. Distantiae autem determinatrices singularum lentium ita per a et γ prodibunt expressae:

$$\frac{1}{\alpha}=-\frac{1}{b}=\frac{-1+fg}{a}+\frac{fg}{\gamma}$$

$$\frac{1}{\beta}=-\frac{1}{c}=\frac{-1+g}{a}+\frac{g}{\gamma}$$

vbi cum sit

$$g=\surd\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu} \text{ notandum est esse } fg=\surd\frac{3\lambda^{(3)}+\nu}{3\lambda+\nu}.$$

Coroll. 4.

141. Eliminando autem numero $\lambda^{(2)}$ erit

$$\frac{1}{\alpha}=-\frac{1}{b}=-\frac{1}{a}\surd\frac{3\lambda^{(3)}+\nu}{3\lambda'+\nu}-\frac{1}{a}\surd\frac{3\lambda^{(3)}+\nu}{3\lambda''+\nu}+\frac{1}{\gamma}\surd\frac{3\lambda^{(3)}+\nu}{3\lambda+\nu}$$

$$\frac{1}{\beta}=-\frac{1}{c}=-\frac{1}{a}\surd\frac{3\lambda^{(3)}+\nu}{3\lambda''+\nu}+\frac{1}{\gamma}\surd\frac{3\lambda^{(3)}+\nu}{3\lambda+\nu}+\frac{1}{\gamma}\surd\frac{3\lambda^{(3)}+\nu}{3\lambda'+\nu}$$

ex quibus formulis, inuento iam valore minimo $\lambda^{(3)}$ singulae lentes commodissime determinantur.

Coroll. 5.

142. Si praeterea singulae lentes ita fuerint comparatae vt per se minimam confusionem pariant quod fit si $\lambda = \lambda' = \lambda'' = 1$, erit

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} = \frac{2}{\sqrt{(3+\nu)}} \text{ hincque } 3\lambda^{(2)}+\nu = \frac{3+\nu}{4}$$

vnde fit $\lambda^{(2)} = \frac{1-\nu}{4}$. Deinde vero habebitur:

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}} = \frac{3}{\sqrt{(3+\nu)}} \text{ hincque } 3\lambda^{(3)}+\nu = \frac{3+\nu}{9}$$

ac propterea

$\lambda^{(3)} = \frac{1-8\nu}{27}$. Sin autem fuerit tantum $\lambda = \lambda' = \lambda''$ reperietur:

$$\lambda^{(2)} = \frac{3\lambda - 3\nu}{12} \text{ et } \lambda^{(3)} = \frac{3\lambda - 8\nu}{27}.$$

Coroll. 6.

143. Eodem autem casu, quo $\lambda = \lambda' = \lambda''$, ob $\sqrt{(3\lambda^{(3)}+\nu)} = \frac{1}{3}\sqrt{(3\lambda+\nu)}$, distantiae determinatrices pro lentibus simplicibus erunt:

$$\frac{1}{\alpha} = \frac{-2}{3a} + \frac{1}{3\gamma}; \quad \frac{1}{b} = \frac{2}{3a} - \frac{1}{3\gamma}$$

$$\frac{1}{\beta} = \frac{-1}{3a} + \frac{2}{3\gamma}; \quad \frac{1}{c} = \frac{1}{3a} - \frac{2}{3\gamma}$$

vnde eaedem formulae pro earum constructione nascuntur quae supra (136) sunt allatae nisi quod iam in denominatoribus membra $\pm\tau(a+\gamma)\sqrt{(\lambda-1)}$ adiungi debeant.

Scholi-

Scholion.

144. Ternarum ergo lentium ſimplicium idonea coniunctione effici poteſt, vt in expreſſione ſpatii diffuſionis $Hh = \mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(2)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$ numerus $\lambda^{(3)}$ fiat $= 0,042165$. In lentibus autem duplicatis vidimus minimum valorem numeri $\lambda^{(2)}$ eſſe $= 0,191827$: ſicque in triplicatis hic numerus fere quinquies minor reddi poteſt at is fere vicies quater eſt minor, quam per lentes ſimplices obtineri poteſt. Loquor hic autem de lentibus ex principio minimi petitis, quippe quae ad praxin maxime ſunt accommodatae, dum conſtructio leuibus erroribus non admodum turbatur. Quanquam enim lentes triplicatae perinde ac duplicatae parari poſſent, pro quibus numerus conueniens λ non ſolum nihilo aequalis, ſed etiam negatiuus reſultaret tamen earum conſtructio tam eſt lubrica, vt minimus error totum laborem irritum reddat. Interim tamen periculum in triplicatis non tantum eſt quam duplicatis, vnde ſequens problema ſoluiſſe operae erit pretium.

Problema 7.

145. Pro datis diſtantiis determinatricibus a et γ eas definire lentes triplicatas, pro quibus valor ipſius $\lambda^{(3)}$ prorſus in nihilum abeat.

Solutio.

Conſideretur numerus $\lambda^{(2)}$ ex duabus prioribus

 lenti-

lentibus natus vt datus, et cum ſit ſecundum ſolutionem poſteriorem praecedentis problematis

$$\lambda^{(3)} = \lambda^{(\prime)} g^3 + \lambda''(1-g)^3 - \nu g(1-g)$$

definiri debet g ita vt iſta quantitas euaneſcat. Verum cum $\lambda^{(\prime)}$ commode nequeat minor effici quam $\frac{1-\nu}{4}$, ſtatuamus $\lambda^{(\prime)} = \frac{1-\nu}{4}$, fierique oportet:

$$o = \tfrac{1}{4} g^3 + \lambda''(1-g)^3 - \nu g(1-\tfrac{1}{2}g)^2$$

ſed quia λ'' vnitate minor eſſe nequit, neceſſe eſt vt g capiatur vnitate maior euoluantur ergo quidam caſus.

I. $g = \frac{5}{4}$; $o = \frac{125}{4\cdot 64} - \frac{\lambda''}{64} - \frac{5\cdot 9}{4\cdot 64}\nu$ et $\lambda'' = \frac{125 - 45\nu}{4}$

II. $g = \frac{6}{4}$; $o = \frac{216}{4\cdot 64} - \frac{8\lambda''}{64} - \frac{6\cdot 4}{4\cdot 64}\nu$ et $\lambda'' = \frac{216 - 24\nu}{4\cdot 8} = \frac{27 - 3\nu}{4}$

III. $g = \frac{7}{4}$; $o = \frac{343}{4\cdot 64} - \frac{27\lambda''}{64} - \frac{7\cdot 1}{4\cdot 64}\nu$ et $\lambda'' = \frac{343 - 7\nu}{4\cdot 27}$

IV. $g = \frac{8}{4}$; $o = \frac{512}{4\cdot 64} - \frac{64\lambda''}{64}$ et $\lambda'' = 2$

V. $g = \frac{9}{4}$; $o = \frac{729}{4\cdot 64} - \frac{125\lambda''}{64} - \frac{9\cdot 1}{4\cdot 64}\nu$ et $\lambda'' = \frac{729 - 9\nu}{4\cdot 125}$

VI. $g = \frac{10}{4}$; $o = \frac{1000}{4\cdot 64} - \frac{216\lambda''}{64} - \frac{10\cdot 4}{4\cdot 64}\nu$ et $\lambda'' = \frac{1000 - 40\nu}{4\cdot 216} = \frac{125 - 5\nu}{4\cdot 27}$.

In primo et ſecundo caſu fit valor ipſius λ'' nimis magnus, quam vt iſta lens commode in praxin recipi queat; ac ſi ipſi g multo maior tribuatur valor, leuis error ingentem effectum producit. Ponamus enim loco g per errorem ſumi $g + \omega$, et cum λ'' ex g rite fuerit definitum, fiet

$$\lambda^{(3)} = \omega\left(\tfrac{3}{4}gg - 3\lambda''(1-g)^2 - \nu(1-\tfrac{1}{2}g)(1-\tfrac{3}{2}g)\right)$$

et

et pro λ'' ſubſtituto valore:

$$\lambda^{(3)} = \frac{\omega}{4(1-g)}(3gg - \nu(4-gg))$$

vnde patet, quo minus g vnitatem excedat, ac ſimul quo maius fuerit g, valorem ipſius $\lambda^{(3)}$ ob errorem ω eo fieri maiorem. Intelligitur autem hunc errorem fieri minimum ſi capiatur

$$g = 1 + \sqrt{\frac{3-2\nu}{\nu+3}},$$ ex hoc autem valore elicitur,

$$\lambda'' = \frac{(3+\nu)\sqrt{3(1-\nu)(3+\nu)} + 3(3+\nu\nu)}{8(1-\nu)}$$

Capi ergo debebit $g = 1,84384$ vnde colligitur:

$$\lambda'' = \frac{g^3 - \nu g(2-g)^2}{4(g-1)^3} = 2,60372$$

et ſi hae menſurae exacte obſeruentur, fiet $\lambda^{(3)} = 0$. At ſi in valore ipſius g particula ω aberretur, vt ſit $g = 1,84384 + \omega$, prodibit ob hunc errorem:

$$\lambda^{(3)} = -2,981\,\omega,$$

ita ſi eſſet error $\omega = +\frac{1}{10}$, loco $\lambda^{(3)} = 0$, prodiret:

$\lambda^{(3)} = \mp 0,2981$ ideoque lens triplicata poſtponenda duplicatae, longe tamen praeferenda foret ſimplici.

In genere igitur pro quouis valore alio ipſius $\lambda^{(2)}$ idem commodum inueſtigemus: ac primo cum ſit

$$\lambda'' = \frac{\lambda^{(2)}g^3 + \nu g(g-1)}{(g-1)^3}$$

ſi loco iuſti valoris g capiatur $g+\omega$ fiet

$$\lambda^{(3)} = \omega(3\lambda^{(2)}g^2 - 3\lambda''(g-1)^2 - \nu + 2\nu g)$$

vbi ſi pro λ'' valor ſubſtituatur, erit

$$\lambda^{(3)} = \frac{\omega(\nu - (3\lambda^{(2)}+\nu)gg)}{g-1}$$

 qui

qui error vt minimus reddatur capi debet

$$g = 1 + \sqrt{\frac{3\lambda^{(2)}}{3\lambda^{(2)}+\nu}}$$

qui eſt valor maxime idoneus pro g ſumendus, ex quo elicitur

$$\lambda'' = \frac{1}{3\lambda^{(2)}}\left(\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{3\lambda^{(2)}}\right)\left(\tfrac{1}{3}(6\lambda^{(2)}+\nu)\sqrt{3\lambda^{(2)}}+(2\lambda^{(2)}+\nu)\sqrt{(3\lambda^{(2)}+\nu)}\right)$$

et ſumto per errorem $g+\omega$ pro g erit

$$\lambda^{(3)} = -2\omega\left(3\lambda^{(2)}+\nu+\sqrt{3\lambda^{(2)}(3\lambda^{(2)}+\nu)}\right) \text{ ſeu}$$

$$\lambda^{(3)} = -2\omega\left(\sqrt{3\lambda^{(2)}}+\sqrt{(3\lambda^{(2)}+\nu)}\right)\sqrt{(3\lambda^{(2)}+\nu)}$$

Quo minor igitur iam fuerit valor ipſius $\lambda^{(2)}$ eo minus erit error metuendus, vnde ſolutio ante ex valore $\lambda^{(2)} = \frac{1-\nu}{4}$ eruta prae ceteris eſt commendanda. Tum autem erit $f = \frac{1}{2}$; $\lambda = 1$, $\lambda' = 1$, $\lambda'' = 2,60372$ et $g = 1,84384$, vnde pro lentium conſtructione habemus,

$$\frac{1}{\alpha} = -\frac{1}{b} = \frac{-2+g}{2a} + \frac{g}{2\gamma}$$

$$\frac{1}{\beta} = -\frac{1}{c} = \frac{-1+g}{a} + \frac{g}{\gamma}$$

vnde ſingulae lentes per formulas §. 91 conſtruentur.

Coroll. I.

146. Lens ergo prima PP conſtrui debet ex diſtantiis determinatricibus a et $\frac{2a\gamma}{ga-(2-g)\gamma}$ cum numero $\lambda = 1$

Lens vero ſecunda QQ ex diſtantiis determinatricibus

$\frac{-2a\gamma}{ga-(2-g)\gamma}$ cum numero $\lambda' = 1$

at

at lens tertia RR ex distantiis determinatricibus

$\frac{-\alpha\gamma}{ga-(1-g)\gamma}$ et γ cum numero $\lambda''=2,60372$

existente $g=1,84384$.

Coroll. 2.

147. Quoniam in formula spatium diffusionis exprimente quae ob $\lambda^{(3)}=0$ est $\mu\gamma\gamma xx(\frac{1}{a}+\frac{1}{\gamma})\frac{\nu}{a\gamma}$, reiecto factore primo $\mu\gamma\gamma xx$, cuius ratio in his inuestigationibus non est habita, distantiae a et γ inter se permutari possunt; hinc etiam alia lens triplicata quaesito aeque satisfaciens exhiberi poterit.

Coroll. 3.

148. Nempe pro hac altera lente triplicata lens prima PP construi debet ex distantiis determinatricibus

a et $\frac{+a\gamma}{(1-g)a-g\gamma}$ cum numero $\lambda=2,60372$

Lens secunda QQ ex distantiis determinatricibus

$\frac{-a\gamma}{(1-g)a-g\gamma}$ et $\frac{+2a\gamma}{(2-g)a-g\gamma}$ cum numero $\lambda'=1$

at lens tertia ex distantiis determinatricibus

$\frac{-2a\gamma}{(2-g)a-g\gamma}$ et γ cum numero $\lambda''=1$

existente vt ante $g=1,84384$.

Coroll. 4.

149. Hinc ergo duas nacti sumus lentes triplicatas pro distantiis a et γ, quae producunt spatium diffu-

diffusionis $Hb = \mu\gamma\gamma xx(\frac{1}{a}+\frac{1}{\gamma})\frac{v}{a\gamma}$. Atque hae inter infinitas alias eundem effectum praestantes hac gaudent praerogatiua, vt leuis error in constructione commissus scopum minime perturbet.

Coroll. 5.

150. Si in constructione harum lentium per errorem numerus g parumper maior accipiatur, quam 1, 84384, tum pro lente triplicata numerus $\lambda^{(3)}$ prodit nihilo minor seu negatiuus. Sin autem numerus g in praxi aliquantillum maior sumatur numerus $\lambda^{(3)}$ fit nihilo maior, sicque lens triplicata ad naturam duplicatarum accedet.

Coroll. 6.

151. Si ergo opus fuerit lente, pro qua numerus λ valorem habeat negatiuum, huic scopo satisfieri commode poterit per lentes descriptas triplicatas, dummodo pro g numerus aliquanto maior quam 1, 84384 assumatur. Scilicet si sumatur

$$g = 1,84384 + \omega \text{ fiet } \lambda^{(3)} = -2,981\,\omega.$$

Supplementum II.

De lentibus triplicatis

Si pro singulis lentibus refractio sit diuersa, pro prima $n : 1$ pro secunda $n' : 1$, pro tertia

n''

n'' : 1, radiique facierum lentium sequenti modo definiantur

	Dist. determinatrices	Refractio et litterae inde pendentes	
I.	a et α	$n : 1,\ \mu,\ \nu,\ \rho,\ \sigma,\ \tau.$	λ
II.	b et $\mathfrak{b}$	$n' : 1,\ \mu',\ \nu',\ \rho',\ \sigma',\ \tau'.$	λ'
III.	c et γ	$n'' : 1,\ \mu'',\ \nu'',\ \rho'',\ \sigma'',\ \tau''.$	λ''

scilicet si pro lente prima vocetur radius faciei anterioris $= F$; posterioris $= G$; erit

$$\frac{1}{F} = \frac{\rho}{a} + \frac{\sigma}{\alpha} \mp \tau\left(\frac{1}{a} + \frac{1}{\alpha}\right)\sqrt{\lambda - 1}$$

$$\text{et } \frac{1}{G} = \frac{\rho}{\alpha} + \frac{\sigma}{a} \pm \tau\left(\frac{1}{a} + \frac{1}{\alpha}\right)\sqrt{\lambda - 1}$$

similique modo pro reliquis lentibus, nempe pro secunda

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{\mathfrak{b}} \mp \tau'\left(\frac{1}{b} + \frac{1}{\mathfrak{b}}\right)\sqrt{\lambda' - 1}$$

$$\frac{1}{G'} = \frac{\rho'}{\mathfrak{b}} + \frac{\sigma'}{b} \pm \tau'\left(\frac{1}{b} + \frac{1}{\mathfrak{b}}\right)\sqrt{\lambda' - 1}$$

tum vero pro tertia

$$\frac{1}{F''} = \frac{\rho''}{c} + \frac{\sigma''}{\gamma} \mp \tau''\left(\frac{1}{c} + \frac{1}{\gamma}\right)\sqrt{\lambda'' - 1}$$

$$\frac{1}{G''} = \frac{\rho''}{\gamma} + \frac{\sigma''}{c} \pm \tau''\left(\frac{1}{c} + \frac{1}{\gamma}\right)\sqrt{\lambda'' - 1}$$

Deinde quia distantiae lentium pro nihilo habentur scilicet $a + b = 0$ et $\mathfrak{b} + c = 0$, erit spatium diffusionis

$$= \gamma\gamma xx \begin{cases} +\mu\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right) \\ +\mu'\left(\frac{1}{b} + \frac{1}{\mathfrak{b}}\right)\left(\lambda'\left(\frac{1}{b} + \frac{1}{\mathfrak{b}}\right)^2 + \frac{\nu'}{b\mathfrak{b}}\right) \\ +\mu''\left(\frac{1}{c} + \frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c} + \frac{1}{\gamma}\right)^2 + \frac{\nu''}{c\gamma}\right) \end{cases}$$

 statua-

ſtatuatur nunc

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

vt fiat

$$\frac{1}{\alpha}=\frac{f-1}{a}+\frac{f}{\gamma}-\frac{1}{b}; \quad \text{et}$$

$$\frac{1}{c}=\frac{h}{a}+\frac{h-1}{\gamma}-\frac{1}{\beta}$$

vnde fit

$$\frac{1}{b}+\frac{1}{\beta}=\frac{1-f-h}{a}+\frac{1-f-h}{\gamma}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

hincque $f+g+h=1$; quibus valoribus ſubſtitutis erit primo pro radiis facierum

$$\frac{1}{F}=\frac{\varrho+(f-1)\sigma}{a}+\frac{\sigma f}{\gamma}\mp\tau f\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$

$$\frac{1}{G}=\frac{\sigma+(f-1)\varrho}{a}+\frac{\varrho f}{\gamma}\pm\tau f\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$

$$\frac{1}{F'}=\frac{(1-f)\varrho'-h\sigma'}{a}+\frac{(1-h)\sigma'-f\varrho'}{\gamma}\mp\tau' g\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$

$$\frac{1}{G'}=\frac{(1-f)\sigma'-h\varrho'}{a}+\frac{(1-h)\varrho'-f\sigma'}{\gamma}\pm\tau' g\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$

et pro lente tertia

$$\frac{1}{F''}=\frac{h.\varrho''}{a}+\frac{(h-1)\varrho''+\sigma''}{\gamma}\mp\tau''.h\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

$$\frac{1}{G''}=\frac{h.\sigma''}{a}+\frac{(h-1)\sigma''+\varrho''}{\gamma}\pm\tau''.h\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

Spatium

Spatium vero diffusionis iam ita exprimetur

$$\gamma\gamma\, xx\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)\begin{cases}\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2(\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda''.h^3)\\ +\frac{\nu\mu f}{a}\left(\frac{f-1}{a}+\frac{f}{\gamma}\right)\\ +\nu'\mu' g\left(\frac{1-f}{a}-\frac{f}{\gamma}\right)\left(\frac{1-h}{\gamma}-\frac{h}{a}\right)\\ +\frac{\nu''\mu''.h}{\gamma}\left(\frac{h}{a}+\frac{h-1}{\gamma}\right)\end{cases}$$

Nunc igitur circa has lentes triplicatas sequentia sunt obseruanda

I. Diuersa media refringentia eum tantum in finem adhiberi solent, vt non solum spatium diffusionis hic determinatum ad nihilum redigatur, sed etiam confusio a diuersa radiorum refrangibilitate oriunda tollatur, quippe quod per lentes eiusdem refractionis obtineri nequit. Infra autem videbimus ad hanc conditionem implendam requiri, vt sit $\zeta f+\eta g+\vartheta h=0$, existente $\zeta=\frac{dn}{n-1}$; $\eta=\frac{dn'}{n'-1}$ et $\vartheta=\frac{dn''}{n''-1}$; ex quibus formis iam perspicitur, si haec differentialia dn, dn' et dn'' essent ipsis $n-1$; $n'-1$; $n''-1$ proportionalia, vti *Newtonus* statuerat; tum proditura esse $\zeta=\eta=\vartheta$ siue $f+g+h=0$; at iam vidimus, esse debere $f+g+h=1$; quare si *Newtoni* sententia esset vera; tum ne quidem diuersis refractionibus adhibendis diffusioni a diuersa refrangibilitate oriundae remedium adferri posset. Eatenus igitur tantum hoc incommodum vitari poterit, quatenus litterae ζ, η et ϑ sunt diuersae, ita, vt simul esse possit, et $f+g+h=1$ et $\zeta f+\eta g+\vartheta h=0$ ex quo perspicuum est, quantitatum f, g et h vnam

vel adeo duas eſſe debere negatiuas ſicque hac conditione ſuperaddita caſus ille principalis, quo omnes tres litterae f, g et h poſitiuae ſunt aſſumtae hic locum inuenire nequit.

II. Vt ergo noſtrum ſpatium diffuſionis euaneſcat, ſatisfaciendum eſt huic aequationi

$$\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2 (\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' h^3)$$
$$+\frac{\nu\mu f}{a}\left(\frac{f-1}{a}+\frac{f}{\gamma}\right)$$
$$+\nu'\mu' g\left(\frac{1-f}{a}-\frac{f}{\gamma}\right)\left(\frac{1-h}{\gamma}-\frac{h}{a}\right)$$
$$+\frac{\nu''\mu'' h}{\gamma}\left(\frac{h}{a}+\frac{h-1}{\gamma}\right)=0$$

Vnde vel λ vel λ' vel λ'' quaeri poteſt, dummodo caueatur, ne valor vel vnitate minor vel nimis magnus prodeat, quia prius naturae repugnat; alterum autem, quia conſtructio lentium fieret nimis lubrica. Hoc autem praeſtito circa binas reliquas litteras λ nihil amplius definitur neque etiam circa litteras f, g et h, praeterquam quod ſupra allatis conditionibus $f+g+h=1$ et $\zeta f+\eta g+\vartheta h=0$ continetur, vnde ex data vna harum litterarum duae reliquae ſponte definiuntur ſequenti ſcilicet modo:

$$g=\frac{(\vartheta-\zeta)f-\vartheta}{\eta-\vartheta}; \text{ et}$$
$$h=\frac{(\zeta-\eta)f+\eta}{\eta-\vartheta}$$

III. In praxi autem huiusmodi lentes triplicatae ideo potiſſimum quaeruntur, vt loco lentis obie-

ctiuae

ctiuae in telescopiis substitui queant, pro quibus est $a = \infty$. Statuamus ergo statim $a = \infty$ et pro radiis facierum singularum lentium habebimus

$$\frac{1}{F} = \frac{\sigma f}{\gamma} \mp \frac{\tau f}{\gamma} \sqrt{\lambda - 1}; \quad \frac{1}{G} = \frac{\rho f}{\gamma} \pm \frac{\tau f}{\gamma} \sqrt{\lambda - 1}$$

pro secunda lente

$$\frac{1}{F'} = \frac{(1-b)\sigma' - f\varrho'}{\gamma} \mp \frac{\tau' g}{\gamma} \sqrt{\lambda' - 1}; \quad \frac{1}{G'} = \frac{(1-b)\varrho' - f\sigma'}{\gamma} \pm \frac{\tau' g}{\gamma} \sqrt{\lambda' - 1}$$

pro tertia lente

$$\frac{1}{F''} = \frac{(b-1)\varrho'' + \sigma''}{\gamma} \mp \frac{\tau''.b}{\gamma} \sqrt{\lambda'' - 1}; \quad \frac{1}{G''} = \frac{(b-1)\sigma'' + \varrho''}{\gamma} \pm \frac{\tau''.b}{\gamma} \sqrt{\lambda'' - 1}$$

Spatium autem diffusionis tum ita exprimetur

$$\frac{x^2}{\gamma}(\mu\lambda f^3 + \mu'\lambda' g^3 + \mu''\lambda'' b^3 - \nu'\mu'.fg(1-b) - \nu''\mu''.b(1-b))$$

Ita vt satisfieri oporteat huic aequationi

$$\mu\lambda f^3 + \mu'\lambda' g^3 + \mu''\lambda'' b^3 - \nu'\mu'.fg(1-b) - \nu''\mu''.b(1-b) = 0.$$

Vnde vnus valorum λ, λ', λ'', qui ad vsum commodissimus videtur, determinari debet.

IV. Si tantum lentibus vitreis vti velimus, sufficiet duas tantum vitri species adhiberi; si igitur statuamus, lentem tertiam et primam ex eadem vitri specie parari, vt $\mu'' = \mu$; $\nu'' = \nu$; $\varrho'' = \varrho$; $\sigma'' = \sigma$; $\tau'' = \tau$ et $\zeta = \vartheta$ ob $n'' = n$; pro litteris autem f, g et b hae determinationes habebuntur, 1°. ex aequatione

$$\zeta f + \eta g + \zeta b = 0 \text{ fit } f + b = \frac{-\eta}{\zeta}.g,$$

quo valore substituto fiet

$$g\frac{(\zeta - \eta)}{\zeta} = 1; \quad g = \frac{\zeta}{\zeta - \eta} \text{ ideoque } f + b = \frac{-\eta}{\zeta - \eta};$$

vnde patet, prouti littera g fuerit vel positiua vel negatiua, fore vicissim summam $f+h$ vel negatiuam vel positiuam et aequatio resoluenda iam erit

$$\mu(\lambda f^3+\lambda'' h^3)+\mu'\lambda' g^3-\frac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-h)-\mu\nu.h(1-h)=0$$

Hinc igitur elicitur

$$\mu'\lambda' g^3=-\mu(\lambda f^3+\lambda'' h^3)+\frac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-h)+\mu\nu.h(1-h)$$

cui facile erit pro casu quouis proposito satisfacere.

IV. Dum lens prima et tertia ex eadem vitri specie parantur, media constat ex aqua vel alia materia fluida, vt lens triplicata intra duas lentes vitreas contineat fluidum, et quia fluidum plerumque minorem refractionem patitur, quam vitrum; erit $n' < n$ indeque porro $\eta < \zeta$. Quia igitur pro hoc casu fit g positiuum seu lens aquea conuexa, lentes vitreae vel ambae vel vna saltem debent esse concauae.

Inprimis autem praeter determinationes iam inventas necesse est, vt radius faciei anterioris pro lente media aequalis et contrarius sit radio faciei posterioris lentis primae, eodemque modo radius faciei posterioris aequalis et contrarius radio faciei anterioris lentis tertiae vnde hae aequalitates nascentur

$$\frac{1}{F'}=\frac{-1}{G}, \text{ seu } F'+G=0; \text{ et } \frac{1}{G'}=\frac{-1}{F''} \text{ seu } F''+G'=0.$$

Ideoque satisfieri oportet istis aequationibus:

$$(1-h)\sigma'-f\varrho'\mp\tau' g.\sqrt{\lambda'-1}=-\varrho f\mp\tau f\sqrt{\lambda-1} \text{ et}$$

$$(1-h)\varrho'-f\sigma'\pm\tau'.g.\sqrt{\lambda'-1}=(1-h)\varrho-\sigma\mp\tau'' h\sqrt{\lambda''-1}$$

En

En igitur duas conditiones, quibus satisfieri oportet, vnde vel numeri λ et λ'' vel alter eorum cum alterutra litterarum f et h definiri debent; quem in finem probe obseruandum est, formulas $\sqrt{\lambda - 1}$ et $\sqrt{\lambda'' - 1}$ pro lubitu siue positiuas siue negatiuas assumi posse, neque a se inuicem pendere. Pro formula autem $\sqrt{\lambda' - 1}$ notandum est, si ea in priore aequatione positiue ponatur, in posteriore necessario negatiue sumi debere et viciffim.

Problema 8.

152. Determinare eas lentes quadruplicatas ad datas distantias determinatrices $AE = a$ et $dI = \delta$ accommodatas, quae minimum spatium diffusionis si producant. Tab. II. Fig. 9.

Solutio.

Prima lens PP ad distantias determinatrices $AE = a$ et $AF = \alpha$ cum numero λ construatur, secunda QQ ad distantias $b = -\alpha$ et $AG = \mathfrak{b}$ cum numero λ', tertia RR ad distantias $c = -\mathfrak{b}$ et $AH = \gamma$ cum numero λ'': et quarta SS ad distantias $d = -\gamma$ et $dI = \delta$ cum numero λ''' construatur: vbi scilicet crassitiem lentium vt euanescentem spectamus. Posito iam semidiametro aperturae lentis x, sola prima lens PP produceret spatium diffusionis

$$Ff = \mu . aaxx \left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right)$$

Adiun-

Adiuncta autem secunda lente QQ, positoque

$$\frac{1}{\alpha}=\frac{-1+f}{a}+\frac{f}{\mathfrak{b}} \text{ et } \frac{1}{b}=\frac{1-f}{a}-\frac{f}{\mathfrak{b}}$$

si breuitatis gratia statuamus

$$\lambda^{(2)}=\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)$$

vidimus fore spatium diffusionis:

$$Gg=\mu\mathfrak{b}\mathfrak{b}xx\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)\left(\lambda^{(2)}\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu}{a\mathfrak{b}}\right)$$

Adiungatur insuper tertia lens RR, numerusque g ita sumatur vt sit

$$\frac{1}{\mathfrak{b}}=\frac{-1+g}{a}+\frac{g}{\gamma} \text{ et } \frac{1}{c}=\frac{1-g}{a}-\frac{g}{\gamma}$$

ac si breuitatis ergo ponamus:

$$\lambda^{(3)}=\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g)$$

erit spatium diffusionis:

$$Hh=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Nunc denique adiungatur lens quarta SS, et numero h ita in calculum introducto, vt sit

$$\frac{1}{\gamma}=\frac{-1+h}{a}+\frac{h}{\delta} \text{ et } \frac{1}{d}=\frac{1-h}{a}-\frac{h}{\delta}$$

si simili modo ponamus

$$\lambda^{(4)}=\lambda^{(3)}h^3+\lambda'''(1-h)^3-\nu h(1-h)$$

erit spatium diffusionis a lente quadruplicata productum:

$$Ii=\mu\delta\delta xx\left(\frac{1}{\alpha}+\frac{1}{\delta}\right)\left(\lambda^{(4)}\left(\frac{1}{a}+\frac{1}{\delta}\right)^2+\frac{\nu}{a\delta}\right)$$

quod igitur minimum reddi debet. Hunc in finem considerentur numeri λ, λ', λ'' et λ''' vt dati, et quaerantur

rantur idonei valores pro numeris f, g, et h: atque vt valor $\lambda^{(4)}$ minimus euadat, necesse est quoque valores $\lambda^{(3)}$ et $\lambda^{(2)}$ minimos fieri. Incipiamus ergo a valore $\lambda^{(2)}$ qui minimus redditur sumendo

$$f = \frac{\sqrt{(3\lambda' + \nu)}}{\sqrt{(3\lambda + \nu)} + \sqrt{(3\lambda' + \nu)}} \text{ seu } \frac{1}{f} = 1 + \sqrt{\frac{3\lambda + \nu}{3\lambda' + \nu}}$$

vnde fit

$$\frac{1}{\sqrt{(3\lambda^{(2)} + \nu)}} = \frac{1}{\sqrt{(3\lambda + \nu)}} + \frac{1}{\sqrt{(3\lambda' + \nu)}}.$$

Deinde numerus $\lambda^{(3)}$ minimum induet valorem capiendo

$$g = \frac{\sqrt{(3\lambda'' + \nu)}}{\sqrt{(3\lambda^{(2)} + \nu)} + \sqrt{(3\lambda'' + \nu)}} \text{ seu } \frac{1}{g} = 1 + \sqrt{\frac{3\lambda^{(2)} + \nu}{3\lambda'' + \nu}}$$

hincque colligitur

$$\frac{1}{\sqrt{(3\lambda^{(3)} + \nu)}} = \frac{1}{\sqrt{(3\lambda^{(2)} + \nu)}} + \frac{1}{\sqrt{(3\lambda'' + \nu)}}$$

Denique numerus $\lambda^{(4)}$ ideoque et spatium diffusionis Ii minimum efficietur sumendo

$$h = \frac{\sqrt{(3\lambda''' + \nu)}}{\sqrt{(3\lambda^{(3)} + \nu)} + \sqrt{(3\lambda''' + \nu)}} \text{ seu } \frac{1}{h} = 1 + \sqrt{\frac{3\lambda^{(3)} + \nu}{3\lambda''' + \nu}}$$

vnde obtinetur

$$\frac{1}{\sqrt{(3\lambda^{(4)} + \nu)}} = \frac{1}{\sqrt{(3\lambda^{(3)} + \nu)}} + \frac{1}{\sqrt{(3\lambda''' + \nu)}}.$$

Quod si hic valores ante inuentos substituamus, nanciscemur

$$\frac{1}{\sqrt{(3\lambda^{(4)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}.$$

Pro praecedentibus vero erit

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

et

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}.$$

Tum vero ex his porro consequimur:

$$f=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}},\ g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}},\ h=\sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(3)}+\nu}}$$

Superest ergo vt constructionem singularum lentium luculentius exponamus: et earum distantias determinatrices per solas propositas a et δ exprimamus: erit igitur

$$\frac{1}{\gamma}=\frac{-1+h}{a}+\frac{h}{\delta};\quad \frac{1}{d}=\frac{1-h}{a}-\frac{h}{\delta}$$

$$\frac{1}{\beta}=\frac{-1+gh}{a}+\frac{gh}{\delta};\quad \frac{1}{c}=\frac{1-gh}{a}-\frac{gh}{\delta}$$

$$\frac{1}{\alpha}=\frac{-1+fgh}{a}+\frac{fgh}{\delta};\quad \frac{1}{b}=\frac{1-fgh}{a}-\frac{fgh}{\delta}$$

Ex superioribus vero formulis colligitur:

$$h=\sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(3)}+\nu}};\ g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}};\ f=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}}$$

vnde

vnde fit

$$gh = \sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(2)}+\nu}} \text{ et } fgh = \sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda+\nu}}.$$

et ſuperiores valores ita exprimi poterunt

$$= -\frac{1}{b} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\left(\frac{1}{\sqrt{(3\lambda'+\nu)}} + \frac{1}{\sqrt{(3\lambda''+\nu)}} + \frac{1}{\sqrt{(3\lambda'''+\nu)}}\right) + \frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta} \cdot \frac{1}{\sqrt{(3\lambda+\nu)}}.$$

$$= -\frac{1}{c} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\left(\frac{1}{\sqrt{(3\lambda''+\nu)}} + \frac{1}{\sqrt{(3\lambda'''+\nu)}}\right) + \frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\left(\frac{1}{\sqrt{(3\lambda+\nu)}} + \frac{1}{\sqrt{(3\lambda'+\nu)}}\right)$$

$$= -\frac{1}{d} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a} \cdot \frac{1}{\sqrt{(3\lambda'''+\nu)}} + \frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\left(\frac{1}{\sqrt{(3\lambda+\nu)}} + \frac{1}{\sqrt{(3\lambda'+\nu)}} + \frac{1}{\sqrt{(3\lambda''+\nu)}}\right).$$

Coroll. 1.

153. Si pro lentibus ſimplicibus numeri λ, λ' λ'' et λ''' ſumantur inter ſe aequales, fiet pro minimo ſpatio diffuſionis:

$$\sqrt{(3\lambda^{(2)}+\nu)} = \tfrac{1}{2}\sqrt{(3\lambda+\nu)}; \quad \lambda^{(2)} = \frac{3\lambda - 1.3\nu}{3.4}$$

$$\sqrt{(3\lambda^{(3)}+\nu)} = \tfrac{1}{3}\sqrt{(3\lambda+\nu)}; \quad \lambda^{(3)} = \frac{3\lambda - 2.4\nu}{3.9}$$

$$\sqrt{(3\lambda^{(4)}+\nu)} = \tfrac{1}{4}\sqrt{(3\lambda+\nu)}; \quad \lambda^{(4)} = \frac{3\lambda - 3.5\nu}{3.16}$$

hinc $f = \frac{1}{2}$; $g = \frac{2}{3}$; $h = \frac{3}{4}$, et pro conſtructione lentium ſimplicium:

$$\frac{1}{\alpha} = -\frac{3}{4a} + \frac{1}{4\delta}; \quad \frac{1}{b} = +\frac{3}{4a} - \frac{1}{4\delta}$$

$$\frac{1}{\beta} = -\frac{2}{4a} + \frac{2}{4\delta}; \quad \frac{1}{c} = +\frac{2}{4a} - \frac{2}{4\delta}$$

$$\frac{1}{\gamma} = -\frac{1}{4a} + \frac{3}{4\delta}; \quad \frac{1}{d} = +\frac{1}{4a} - \frac{3}{4\delta}$$

Coroll. 2.

154. Hinc ex §. 91 ſequens quatuor lentium ſimplicium conſtructio obtinetur:

Pro lente radius faciei

Prima PP $\begin{cases} \text{anterioris} = \dfrac{4a\delta}{(4\varrho - 3\sigma)\delta + \sigma a \pm \tau(a+\delta)\surd(\lambda - 1)} \\ \text{poſterioris} = \dfrac{4a\delta}{(4\sigma - 3\varrho)\delta + \varrho a \mp \tau(a+\delta)\surd(\lambda - 1)} \end{cases}$

Secunda QQ $\begin{cases} \text{anterioris} = \dfrac{4a\delta}{(3\varrho - 2\sigma)\delta + (2\sigma - \varrho)a \pm \tau(a+\delta)\surd(\lambda - 1)} \\ \text{poſterioris} = \dfrac{4a\delta}{(3\sigma - 2\varrho)\delta + (2\varrho - \sigma)a \mp \tau(a+\delta)\surd(\lambda - 1)} \end{cases}$

Tertia RR $\begin{cases} \text{anterioris} = \dfrac{4a\delta}{(2\varrho - \sigma)\delta + (3\sigma - 2\varrho)a \pm \tau(a+\delta)\surd(\lambda - 1)} \\ \text{poſterioris} = \dfrac{4a\delta}{(2\sigma - \varrho)\delta + (3\varrho - 2\sigma)a \pm \tau(a+\delta)\surd(\lambda - 1)} \end{cases}$

Quarta SS $\begin{cases} \text{anterioris} = \dfrac{4a\delta}{\varrho\delta + (4\sigma - 3\varrho)a \pm \tau(a+\delta)\surd(\lambda - 1)} \\ \text{poſterioris} = \dfrac{4a\delta}{\sigma\delta + (4\varrho - 3\sigma)a \mp \tau(a+\delta)\surd(\lambda - 1)} \end{cases}$

Coroll. 3.

155. Si praeterea numeri λ, λ', λ'', λ''' vnitati aequales ſtatuantur, qui eſt valor minimus, quem recipere poſſunt erit: ob $\nu = 0,232692$

$$\lambda^{(2)} = \frac{3 - 3\nu}{3 \cdot 4} = 0,191827$$

$$\lambda^{(3)} = \frac{3 - 8\nu}{3 \cdot 9} = 0,042165$$

$$\lambda^{(4)} = \frac{3 - 15\nu}{3 \cdot 16} = -0,010216$$

ſicque pro lente quadruplicata valor numeri $\lambda^{(4)}$ adeo infra nihilum deprimitur.

Coroll.

Coroll. 4.

156. Maiorem ergo valorem numeris $\lambda, \lambda', \lambda'', \lambda'''$ tribuendo effici poterit, vt valor ipſius $\lambda^{(+)}$ praeciſe nihilo aequalis prodeat; quippe hoc fiet ſumendo $\lambda = 5\nu = 1,163460$. Hinc habebitur $\lambda - 1 = 0,163460$ et $\tau\sqrt{(\lambda - 1)} = 0,365947$, vnde ſingulae lentes ſimplices duplici modo per formulas exhibitas conſtrui poterunt.

Scholion 1.

157. Si inter ſe comparemus hos duos caſus, quibus eſt vel $\lambda^{(+)} = -0,010216$ vel $\lambda^{(+)} = 0$, videmus, etiamſi diſcrimen vix partem centeſimam vnitatis ſuperet, in conſtructione tamen lentium ſimplicium ſatis magnum diſcrimen deprehendi, cum denominatores formularum, (154) ſiue augeri ſiue diminui debeant quantitate: $0,365947\,(a+\delta)$; quae differentia maior eſt, quam errores, qui forte ab artifice non nimis rudi committi queant. Ex quo viciſſim colligimus, etiamſi in conſtructione harum lentium quadruplicatarum ab artifice leues errores committantur, inde vix perceptibilem effectum in ſpatio diffuſionis vel valore ipſius $\lambda^{(+)}$ eſſe metuendum, quam ob cauſam hae lentes imprimis ad praxin accommodatae videntur. Si ſcilicet opus fuerit lente, pro qua valor ipſius λ in nihilum abeat, multo magis his lentibus quadruplicatis Coroll. 4 deſcriptis erit vtendum, quam triplicatis, quas ſupra definiui. Quin etiam eas lentes quadruplicatas adhibere licebit,

in quibus est $\lambda^{(4)} = -0,010216$, quia hic numerus vix nihilo est minor; ac si in constructione a praescriptis mensuris aberretur, ille valor adhuc propius ad nihilum perducatur, ita vt hoc casu adeo errores commissi scopo magis attingendo inseruiant. Infra autem videbimus, plurimos dari casus, quibus eiusmodi lentibus vti conueniat, pro quibus valor ipsius λ non solum sit nihilo aequalis, sed etiam tantillum infra nihilum deprimatur; tunc optimo cum successu huiusmodi lentes quadruplicatas in vsum vocabimus. Sin autem eiusmodi lentes sufficiant, pro quibus valor ipsius λ sit $0,042165$ vel aliquantillum excedat, lentes triplicatae erunt commendandae, quarum constructionem supra (§. 136) dedimus; quemadmodum duplicatae negotium conficient si non opus fuerit minore valore ipsius λ quam $0,191827$. Tota autem instrumentorum dioptricorum perfectio in hoc maxime consistit, vt lentes habeantur, pro quibus valor ipsius λ sit quam minimus, cum eae sint aptissimae ad confusionem penitus tollendam; ex quo lentium quadruplicatarum hic descriptarum vsus erit amplissimus.

Scholion 2.

158. Si haec, quae de lentibus quadruplicatis hic tradidimus, attente considerentur, facile patebit, quomodo lentes quintuplicatae magisque multiplicatae ad vsum sint accommodandae. Eiusmodi scilicet semper constructione erit opus, quae a natura minimi parumper

rumper recedat, quoniam hoc modo errores in praxi commiſſi effectum propoſitum minime perturbant. Cum autem vix vnquam vſu veniat, vt lentibus opus ſit, pro quibus valor numeri λ magis vltra nihilum imminuatur, ſuperfluum foret, conſtructionem lentium quintuplicatarum magisue multiplicatarum vlterius proſequi. Interim tamen iuuabit, ſi pro huiusmodi lentibus numeri λ lentium numero conuenienter ſignis $\lambda^{(5)}$ $^{(6)}$ etc. indicentur, eorum valores, quos ex natura minimi recipiunt, expoſuiſſe. Sumamus omnes numeros λ, λ', λ'', λ''', λ'''', λ''''' etc. lentibus ſimplicibus reſpondentes vnitati aequales, et eorum qui lentibus multiplicatis conueniunt ex natura minimi ita ſe habebunt.

Pro Lente

Solitaria . . .	$\lambda^{(1)} = \frac{3 - 0\nu}{3.\,1}$	$= 1,000000$
Duplicata . . .	$\lambda^{(2)} = \frac{3 - 3\nu}{3.\,4}$	$= 0,191827$
Triplicata . . .	$\lambda^{(3)} = \frac{3 - 8\nu}{3.\,9}$	$= 0,042165$
Quadruplicata	$\lambda^{(4)} = \frac{3 - 15\nu}{3.\,16}$	$= -0,010216$
Quintuplicata	$\lambda^{(5)} = \frac{3 - 24\nu}{3.\,25}$	$= -0,034461$
Sextuplicata	$\lambda^{(6)} = \frac{3 - 35\nu}{3.\,36}$	$= -0,047632$
Septuplicata	$\lambda^{(7)} = \frac{3 - 48\nu}{3.\,49}$	$= -0,055573$
Octuplicata	$\lambda^{(8)} = \frac{3 - 63\nu}{3.\,64}$	$= -0,060727$
Noncuplicata	$\lambda^{(9)} = \frac{3 - 80\nu}{3.\,81}$	$= -0,064261$
Decuplicata	$\lambda^{(10)} = \frac{3 - 99\nu}{3.\,100}$	$= -0,066788$

ac

ac ſi huiusmodi lentes in infinitum multiplicentur, valor numeri reſpondentis $\lambda^{(\infty)}$ erit $= -\frac{\nu}{3} = -0,077564$ ita vt nunquam infra hunc numerum deprimi poſſit: ex quo patet vix vnquam caſum exiſtere poſſe, quo lente ſaltem quintuplicata opus eſſet. Ceterum etiam in genere, quicunque alii valores praeter vnitatem, numeris λ, λ', λ'', λ''' etc. tribuantur, ex formulis ſuperioribus numeri $\lambda^{(5)}$, $\lambda^{(6)}$ etc. facile deriuabuntur; quin etiam diſtantiae determinatrices ſingularum lentium ſimplicium indidem ſine difficultate definientur, cum lex progreſſionis ſatis ſit manifeſta. Verum per totum hoc caput probe tenendum eſt, vbique craſſitiem lentium tanquam euaneſcentem eſſe conſideratam, in ſequente autem capite iterum lentes in genere non neglecta craſſitie ſumus contemplaturi.

CAPVT

CAPVT IV.

DE CONFVSIONE VISIONIS NEC NON DE MAGNITVDINE APPARENTE ET CLARITATE.

Definitio I.

159.

Visio est diſtincta, ſi omnes radii, qui ex quolibet obiecti puncto in oculum ingrediuntur in fundo oculi ſuper retina iterum in vnum punctum congregantur.

Scholion.

160. Ad viſionem diſtinctam requiritur, vt obiecta in certa quadam ab oculo diſtantia reperiantur, quae diſtantia pro varia oculorum indole maxime ſolet eſſe diuerſa, dum Myopes exiguam, ii qui oculis valent ingentem, ac presbytes non ſolum infinitam ſed quandoque etiam negatiuam exigunt; cuiusmodi diſtantia cum in veris obiectis locum habere nequeat, ope perſpicillorum ſibi ſatisfacere ſolent. Quilibet ergo oculus ad certam quandam diſtantiam obiectorum eſt inſtructus quam eius diſtantiam iuſtam appellabo; vbi quidem inſignis latitudo locum habet,

propterea quod ſtructura oculi ita eſt artificioſa, vt contractione ac elongatione quadam ſe ad diſtantias aliquanto maiores et minores accommodare poſſit. Quando ergo obiecta in diſtantia ab oculo iuſta reperiuntur, viſio eſt diſtincta, dum ſingula obiectorum puncta ſuper retina ſingulis punctis exprimuntur.

Definitio 2.

161. Viſio eſt confuſa, ſi radii ex quolibet obiecti puncto in oculum immiſſi non in vno retinae puncto congregantur, ſed per aliquod ſpatium retinam afficiunt.

Corollarium 1.

162. Eo maior ergo erit confuſio, quo maius fuerit hoc ſpatium in retina, per quod radii ex eodem obiecti puncto emiſſi diſſipantur. Ex quo huius ſpatii magnitudo veram confuſionis menſuram ſuppeditabit.

Coroll. 2.

163. Viſio ergo erit confuſa, cum obiecti viſi diſtantia multum fuerit diuerſa ab oculi diſtantia iuſta. Paruum enim diſcrimen vel per ſe nullam confuſionem parit vel oculus ſe ad obiecti diſtantiam, ſua qua pollet, volubilitate accommodare valet.

Scholion.

164. Si obiecta per lentes diſtincte repraeſentarentur, viſio imaginum eadem lege teneretur atque ipſo-

ipſorum obiectorum; iuſta ſcilicet earum ab oculo diſtantia viſionem diſtinctam, admodum autem diuerſa confuſam produceret. Verum ſi imago per aliquod ſpatium fuerit diffuſa etiamſi ab oculo ad diſtantiam iuſtam ſit remota, inde tamen in viſione confuſio oriatur neceſſe eſt. Quod ſi nempe non obiecta ipſa, ſed earum imagines per vnam pluresue lentes repraeſentatas intueamur ob duplicem cauſam viſio poterit eſſe confuſa; altera, ſi diſtantia imaginis ab oculo multum fuerit diuerſa a diſtantia iuſta, altera vero ſi ipſa imago per aliquod ſpatium diffundatur. Priorem quidem cauſam tollere in noſtra eſt poteſtate ſiquidem lentes ita diſponere licet, vt imaginem in iuſta ab oculo diſtantia exhibeant; quam lentium diſpoſitionem propterea hic perpetuo aſſumamus. Quamobrem in hoc capite inueſtigare conſtitui quanta confuſio in viſione imaginum a ſpatio diffuſionis oriri debeat, eiusque quantitate determinata deinceps in hoc erit elaborandum quemadmodum lentes formatas ac diſpoſitas eſſe oporteat, vt confuſio inde in viſione nata datum limitem, quo adhuc eſt tolerabilis, non excedat. In genere quidem certum eſt, quo maius fuerit ſpatium diffuſionis, eo maiorem inde in viſionem induci debere confuſionem, verum tamen mox videbimus, confuſionem viſionis non eſſe ſpatio diffuſionis proportionalem, ſed aliam omnino legem ſequi, quam accurate determinaſſe maximi erit momenti, cum ex hoc fonte conſtructio omnium inſtrumentorum Dioptricorum ad viſionem accommodatorum ſit repetenda.

Problema I.

165. Si oculus aſpiciat imaginem cuiuspiam obiecti ab vna pluribusue lentibus per ſpatium Ll diffuſam, atque imago principalis L in iuſta ab oculo diſtantia reperiatur, definire confuſionem, qua viſio huius imaginis afficietur.

Tab. II. Fig. 10.

Solutio.

In ſpatio diffuſionis Ll, in quo punctum L a radiis per lentium medium, punctum l vero a radiis per lentium extremitates transmiſſis exhibeatur, conſiderari oportet primo eius magnitudinem Ll deinde radiorum in l concurrentium inclinationem ad axem. Supra autem vidimus has res a primae lentis apertura, cuius ſemidiameter ſit $= x$, ita pendere vt ſit ipſum ſpatium $Ll = \mathfrak{V}xx$, et angulus $OlT = \mathfrak{B}x$ pro quouis ſcilicet lentium numero valores horum coefficientium $\mathfrak{V}$ et $\mathfrak{B}$ determinauimus. Repraeſentet iam circulus OV oculum, quem hic vt exiguam cameram obſcuram ſpectare licet, ſed ita per ectam, vt radios ex vno puncto emiſſos iterum in vno puncto colligat, etſi enim in oculo plures fiunt refractiones, tamen conditione illa ſeruata vnica lens earum loco conſiderari poteſt, quae ſit in TOT, a qua retina remota ſit interuallo $OV = u$. Cum nunc imago principalis L in debita ab oculo diſtantia, quae vocetur $OL = l$ exiſtat, puncti L imago in oculo in ipſam retinam incidet, et in V diſtin-

cte

cte depingetur: puncti vero l imago non in V, sed ante retinam in v referetur, quod interuallum Vv, si lentis in TOT conceptae crassities euanescat, secundum §. 62 ita exprimitur, vt sit $Vv = \frac{OV^2}{OL^2}$. $Ll = \frac{uu}{ll}$. Vxx. Cum autem radii punctum l formantes ad axem inclinati sint angulo $OlT = \mathfrak{B}x$, ii per lentis oculi puncta TT ita intrabunt, vt neglecto interuallo Ll prae distantia $OL = l$, sit $OT = l\mathfrak{B}x$, ex quo ii in puncto v concurrentes cum axe angulum facient $OvT = \frac{OT}{Ov} = \frac{OT}{OV} = \frac{l}{u}\mathfrak{B}x$. Hinc ergo vltra ad retinam pergentes super ea circulum UU effingent cuius radius erit $VU = \frac{l}{u}\mathfrak{B}x . Vv = \frac{l}{u} . \mathfrak{B}x . \frac{uu}{ll} . Vxx$ et a punctis inter L et l mediis hoc spatium circulare super retina replebitur. Quare imago per spatium Ll diffusa super retina circello repraesentabitur, cuius radius $VU = \frac{u}{l}\mathfrak{B}\, Vx^3$ qui circellus veram confusionis mensuram suppeditat. Quodlibet scilicet obiecti punctum, quod lentibus per spatium Ll diffusum exhibetur, in oculo super retina non puncto sed circello exprimetur cuius radius erit $= \frac{u}{l} . \mathfrak{B} . Vx^3$.

Coroll. I.

166. Videmus ergo confusionem, qua visio afficitur non solum a quantitate spatii diffusionis $Ll = Vxx$, sed insuper ab angulo, quo radii in l concurrentes ad axem inclinantur, qui est $= \mathfrak{B}x$ pendere; radiumque circelli confusionem metientis producto illius spatii per hunc angulum esse proportionalem.

Coroll. 2.

167. Cum igitur poſito ſemidiametro aperturae primae lentis $=x$, ſpatium diffuſionis ſit vt eius quadratum xx; radius circelli confuſionem metientis eſt vt eius cubus x^3. Ac ſi confuſio ipſa areae huius circelli proportionalis aeſtimetur erit ea vt x^6, ſeu vt cubus aperturae primae lentis.

Coroll. 3.

168 Hinc ergo intelligitur quanti interſit per idoneam lentium diſpoſitionem ſpatium diffuſionis Vxx diminuiſſe non ſolum enim in eadem ratione qua ſpatium diffuſionis Vxx ſeu quantitas V diminuitur ſed adeo in ratione duplicata, ipſa confuſio viſa minor redditur.

Scholion.

169. Aſſumſi hic pupillam tam late patere, vt radios ab l diuergentes recipiat; at ſi apertura pupillae minor eſſet, radios a puncto l venientes nequidem caperet: hoc ergo caſu res eodem rediret ac ſi contracta primae lentis apertura ſpatium diffuſionis Ll eo vsque diminueretur quoad pupilla omnes radios ab imagine emiſſos recipere poſſet hocque caſu manifeſtum eſt, confuſionem minorem eſſe prodituram. Verum in ſequentibus oſtendetur, tam in Teleſcopiis quam Microſcopiis hunc caſum vix vnquam locum inuenire cum plerumque conus radioſus ex puncto l emiſſus circa ingreſſum in oculum mul-

to

to tenuior ſit quam pupillae apertura; quamobrem ne opus quidem eſt hunc caſum etſi per ſe facile expediretur, hic expendere. Ceterum tametſi oculus non commode cum lente ſimplici cuius craſſities evaneſcat, conferri poſſit hincque expreſſio ſpatii V v ſecundum §. 84. aliquantum diuerſa prodire potuiſſet hic ad iſtam circumſtantiam non attendimus propterea quod ſpatium V v ſecundum certam quandam rationem auctum vel diminutum prodiiſſet; Hic autem non adeo neceſſe eſt ipſam quantitatem abſolutam confuſionis cognouiſſe, dummodo rationem quam ſequitur accurate definiuerimus. Cum enim noſtrum calculum cum experientia contulerimus, terminum cognoſcemus, quem ſi confuſio noſtro more expreſſa ſuperauerit, intolerabilis euadat; hincque perpetuo ſufficiet confuſionem ſimili modo expreſſam infra hunc terminum reduxiſſe. Interim tamen notaſſe iuvabit, per oculi conformationem confuſionem adhuc multo minorem effici poſſe.

Problema 2.

170. Poſitis iisdem, quae in praecedente problemate ſunt aſſumta, eam definire oculi conformationem, qua minima confuſio percipiatur. Tab. II. Fig. 11.

Solutio.

Oculus omnis facultate praeditus eſt ſeſe aliquantillum aliter conformandi, vt etiam obiecta, quorum diſtantia a iuſta non nimis differt, diſtincte videat:

videat: quod quomodocunque perficiatur, id ita fieri concipere licet, ac si retina ad pupillam seu potius eam lentem, quam oculi loco consideramus, propius admoueretur seu longius ab ea detorqueretur. Cum ergo ante retinam in V sitam simus contemplati, hic primo sumamus retinam per ipsum punctum v transire, ac manifestum est super ea punctum l distincte expressum iri. Verum quia punctum L alios radios nisi axi proximos non emittit etiam punctum L sine vlla confusione in v repraesentabitur; ita vt tantum puncta media inter L et l confusionem sint paritura. Consideretur ergo quoduis punctum intermedium λ, quod esset extremum in spatio confusionis, si semidiameter aperturae primae lentis minor foret quam x, qui ergo ponatur $=z$: eritque $L\lambda = Vzz$, et inclinatio radiorum in λ concurrentium ad axem $=\mathfrak{B}z$: hinc istius puncti imago intra oculum referetur in ϑ, vt sit $V\vartheta = \frac{uu}{ll}Vzz$, ideoque ob $Vv = \frac{uu}{ll}Vxx$ erit interuallum $v\vartheta = \frac{uu}{ll}V(xx-zz)$: et radiorum in ϑ conuergentium inclinatio ad axem $=\frac{l}{u}\mathfrak{B}z$; ex quo circelli super retina in v existentis radiis puncti λ expressi semidiameter erit $=\frac{u}{l}\mathfrak{B}Vz(xx-zz)$ qui euanescit, vti iam monuimus, siue sit $z=0$, siue $z=x$. Quaeratur iam ille valor ipsius z, quo ille circellus fiat maximus; id quod eueniet, si $xx-3zz=0$ seu $z=\frac{x}{\sqrt{3}}$; sicque circelli confusionem hoc casu metientis semidiameter erit $=\frac{2u}{3l\sqrt{3}}.\mathfrak{B}Vx^3$, quod est multo minus quam casu praecedente, quo retina erat in V.

At

At ſi retina aliquantillum a v verſus V remoueatur, hic circellus adhuc minor effici poterit. Ponatur enim ſpatium $VS = s$, vt ſit $vS = \frac{uu}{ll} Vxx - s$, et $S\upsilon = s - \frac{uu}{ll} Vzz$. Punctum ergo l, cuius effigies in u exhibetur, ſuper retina S circello referetur, cuius radius eſt $= \frac{l}{u} \mathfrak{B} x (\frac{uu}{ll} Vxx - s)$, punctum vero λ, cuius effigies eſt in υ, ſuper retina S circello, cuius radius erit $= \frac{l}{u} \mathfrak{B} z (s - \frac{uu}{ll} Vzz)$; nunc igitur id punctum λ inueſtigemus, vnde iſte circellus minimus euadat: quod fit ſi ſit $s = \frac{3uu}{ll} Vzz$; eritque huius circuli radius $= \frac{2u}{l} \mathfrak{B} Vz^3$, qui ſimul confuſionem exhiberet, ſi modo a puncto l, non maior circulus oriretur: ſed ſubſtituto pro s valore inuento huius circuli radius erit, $\frac{u}{l} \mathfrak{B} Vx (xx - 3zz)$, qui ergo illi $\frac{2u}{l} \mathfrak{B} Vz^3$ aequalis ſtatuatur, vnde naſcitur haec aequatio:

$$x^3 - 3xzz = 2z^3, \text{ hincque } z = \tfrac{1}{2} x.$$

Quare circelli, quo minima confuſio menſuratur, radius erit $= \frac{u}{4l} \mathfrak{B} Vx^3$, quadruplo minor, quam ſi retina eſſet in V; indeque ipſa confuſio ſedecies minor. Etſi autem illa aequatio cubica tres habet radices, praeter $z = \frac{1}{2}x$ binae reliquae ſunt aequales et $z = -x$, ſicque $zz = xx$, vnde non minima ſed quaſi maxima confuſio naſceretur.

Coroll. 1.

171. Cum igitur, vt oculus minimam confuſionem ſentiat, debeat eſſe $z = \frac{1}{2} x$, ideoque $s = \frac{3uu}{4ll} Vxx$, patet

patet, retinam in eum locum S cogi debere, vt fit $VS = \frac{3}{4} Vv$ et $vS = \frac{1}{4} Vv$.

Coroll. 2.

172. Cum ergo oculus non folum praeditus fit facultate fefe parumper immutandi fed etiam hac facultate vti foleat ad confufionem euitandam, nullum eft dubium quin circellorum confufionem producentium radius fit $= \frac{u}{4l} \mathfrak{B} V x^3$, ideoque quadruplo minor, quam problemate praecedente inueneramus dummodo imago fpectanda Ll propemodum in diftantia iufta reperiatur.

Coroll. 3.

173. Haec igitur expreffio $\frac{u}{4l} \mathfrak{B} V x^3$ iuftam nobis exhibet menfuram confufionis, cum exprimat radium circellorum, quibus fingula obiecti puncta, quatenus id per lentes fpectatur, fuper retina repraefentantur. Si modo imago in diftantia iufta ab oculo reperiatur.

Scholion.

174. Si hi circelli euanefcerent, vifio plane effet diftincta hoc autem fieri nequit, nifi ipfum interuallum diffufionis Vxx euanefcat: vnde patet fi apertura primae lentis ad nihilum reduceretur, nullam confufionem fentiri debere. Verum vifio non ita eft delicata, vt prorfus nullam confufionem pati poffit fed dummodo confufio certum quendam termi-

num

num non ſuperet, quaſi eſſet nulla conſiderari poteſt. Iſte terminus, ſeu valor quem expreſſio $\frac{u}{4l}\mathfrak{B}Vx^3$ excedere non debet, ex experientia potius peti debet, quam ex Theoria isque idcirco inſigni adhuc latitudine continetur: vnde fit vt pro diuerſo ſcopo modo maiore gradu confuſionis contenti eſſe ſoleamus, modo autem minorem gradum exigamus: quas circumſtantias cum ad praxin propius accedemus, accuratius ſumus examinaturi. Ceterum notandum eſt pro interuallo u, quo quaſi profunditas oculi exhibetur, vnum circiter pollicem aſſumi poſſe, quae quidem menſura fere erit arbitraria, cum deinceps limites confuſionis per experientiam conſtituemus.

Problema 3.

175. Si oculus per vnicam lentem obiectum aſpiciat ita vt imago viſa ab oculo in diſtantia iuſta reperiatur, definire confuſionem, qua viſio afficietur. Tab. I. Fig. 5.

Solutio.

Sit diſtantia obiecti $E\varepsilon$ ante lentem $AE = a$, imaginis vero principalis $F\zeta$ poſt lentem $BF = \alpha$, lentisque craſſities $AB = v$, quantitas vero arbitraria qua cum binis diſtantiis determinatricibus a et α lentis facies determinatur, ſit $= k$, ponatur autem breuitatis gratia $\frac{k-v}{k+v} = i$. Tum autem poſita ratione refractionis $= n$, debet eſſe lentis PP

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei poſterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$

ac ſi ſemidiameter aperturae faciei anterioris ſit $=x$, poſterioris vero $> ix$ ſpatium diffuſionis Ff erit $= P\alpha\alpha xx$ exiſtente

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2 + ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

radiorum vero in f concurrentium inclinatio ad axem $= i. \frac{x}{\alpha}$. His, quae §. 86 ſunt ſtabilita, praemiſſis, ſit diſtantia oculi poſt lentem $BO = O$, et quia imago $F\zeta$ ante oculum in diſtantia iuſta exiſtere aſſumitur, erit $O = \alpha + l$, ideoque $\alpha = O - l$, ſi locus oculi vt datus conſideretur. Hinc ergo habebimus $V = P\alpha\alpha$ et $\mathfrak{B} = \frac{i}{\alpha}$ ideoque $\mathfrak{B}V = i\alpha P$. Conſequenter radius circellorum in oculo confuſionem metientium, ſeu vt in poſterum loquemur menſura confuſionis erit

$$= \frac{u}{4l}. i\alpha x^3. P = \frac{iu}{4l}(O-l)x^3 P.$$

Si lentis craſſities v euaneſcat, et pro determinatione lentis numerus arbitrarius λ loco k introducatur, erit ex §. 91 $P = \mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right)$, et $i = 1$ ex quo loco citato etiam ipſa lentis conſtructio eſt petenda.

Coroll. 1.

176. Si igitur lens fuerit data, locus oculi poſt lentem ita definitur, vt debeat eſſe diſtantia $BO = O = \alpha + l$, exiſtente l diſtantia oculi iuſta: Sin autem locus oculi detur, pro lentis conſtructione diſtantia determinatrix α ita capi debet, vt ſit $\alpha = O - l$.

Coroll.

Coroll. 2.

177. Quare si oculus ita fuerit comparatus, vt exigat distantiam iustam $l = \infty$, fiet $\alpha = -\infty$, et mensura confusionis erit $= -\frac{1}{4} i u x^3 . P$ seu $= \frac{1}{4} i u x^3 . P$, quia signum $-$ nihil mutat in magnitudine circellorum confusionem producentium.

Coroll. 3.

178. Etsi ergo hoc casu quo $\alpha = \infty$ spatium diffusionis Ff est infinitum, tamen inde confusio in visione orta est finita, quia hoc non obstante valor ipsius P manet finitus: erit enim

$$P = \frac{n}{2(n-1)^2} \left(\frac{1}{ii} \left(\frac{n}{a} + \frac{2}{k+v} \right) \left(\frac{1}{a} + \frac{2}{k+v} \right)^2 - \frac{8ii}{(k-v)^3} \right)$$

Radius autem faciei posterioris fit $= -\frac{(n-1)}{2n}(k-v)$.

Coroll. 4.

179. Quia est $i = \frac{k-v}{k+v}$, valor ipsius P etiam in genere ita exprimi potest, vt sit:

$$P = \frac{n}{2(n-1)^2} \left(\left(\frac{n}{a} + \frac{2}{k+v} \right) \left(\frac{1}{ia} + \frac{2}{k-v} \right)^2 + \left(\frac{n}{\alpha} - \frac{2}{k-v} \right) \left(\frac{i}{\alpha} - \frac{2}{k+v} \right)^2 \right)$$

vbi est $\frac{n}{2(n-1)^2} = \frac{310}{121} = 2,561983$ ob $n = \frac{31}{20}$. Sicque etiam valores reliquarum litterarum Q, R, S etc. in §. 86 transformari poterunt.

Problema 4.

180. Si oculus per duas lentes obiectum $E\varepsilon$ aspiciat, ita vt imago per eas repraesentata $G\gamma$ in distantia iusta $OG = l$ ob oculo sit remota, definire confusionem, qua visio afficietur. Tab. I. Fig. 5.

Solutio.

Sint vt hactenus pro lente PP distantiae determinatrices $AE = a$, $aF = \alpha$, crassities $Aa = v$, et distantia arbitraria $= k$, ita vt sit

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei posterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$

tum vero posito $\frac{k-v}{k+v} = i$ ponatur

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$

Deinde pro lente posteriori QQ sint distantiae determinatrices $BF = b$, $bG = \beta$, crassities lentis $Bb = v'$ et distantia arbitraria $= k'$ vt sit

radius faciei anterioris $= \frac{(n-1)b(k'+v')}{k'+v'+2nb}$

radius faciei posterioris $= \frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$

tum vero posito $\frac{k'-v'}{k'+v'} = i'$ ponatur

$$Q = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

Atque iam erit spatium diffusionis

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii.\frac{bb}{\alpha\alpha}Q\right) = Vxx$$

et radiorum in g concurrentium inclinatio ad axem $ii'.\frac{bx}{\alpha\beta} = \mathfrak{V}x$.

Sit iam oculus in O existente $OG = l$, ac ponatur eius post lentem QQ distantia $bO = O$ erit $O = \beta + l$, ideoque

ideoque $\mathfrak{G} = O - l$; quibus positis cum sit mensura confusionis $= \frac{u}{4l} \mathfrak{B} x . V x x$ erit ea pro nostro casu:

$$\frac{i i' u}{4 l} \cdot \frac{b \mathfrak{G}}{\alpha} x^3 \left(\frac{1}{i' i'} \cdot \frac{\alpha\alpha}{bb} P + ii \cdot \frac{bb}{\alpha\alpha} Q \right)$$

vel pro $\mathfrak{G}$ posito valore $O - l$ et signo mutato

$$\tfrac{1}{4} i i' u \left(1 - \tfrac{O}{l} \right) \cdot \frac{b}{\alpha} x^3 \left(\frac{1}{i' i'} \cdot \frac{\alpha\alpha}{bb} P + ii \cdot \frac{bb}{\alpha\alpha} Q \right).$$

Coroll. 1.

181. Si ergo pro oculo fuerit $l = \infty$, ideoque et $\mathfrak{G} = \infty$, etsi spatium diffusionis Gg sit infinitum tamen ob $\frac{O}{l} = 0$ confusio visionem afficiens nihilominus erit finita: neque enim P neque Q ob $\mathfrak{G} = \infty$ fit infinita.

Coroll. 2.

182. Si crassities lentium euanescat, vt fit $v = 0$ et $v' = 0$ erit $i = 1$ et $i' = 1$. ideoque hoc casu mensura confusionis

$$\tfrac{1}{4} u \left(1 - \tfrac{O}{l} \right) \cdot \frac{b}{\alpha} x^3 \left(\frac{\alpha\alpha}{bb} P + \frac{bb}{\alpha\alpha} Q \right)$$

At si loco k et k' introducantur numeri λ et λ' erit

$$P = \mu \left(\tfrac{1}{a} + \tfrac{1}{\alpha} \right) \left(\lambda \left(\tfrac{1}{a} + \tfrac{1}{\alpha} \right)^2 + \tfrac{\nu}{a\alpha} \right) \text{ et}$$

$$Q = \mu \left(\tfrac{1}{b} + \tfrac{1}{\mathfrak{G}} \right) \left(\lambda' \left(\tfrac{1}{b} + \tfrac{1}{\mathfrak{G}} \right)^2 + \tfrac{\nu}{b\mathfrak{G}} \right)$$

existente

$$\mu = 0,938191 \text{ et } \nu = 0,232692$$

Coroll.

Coroll. 3.

183. Eodem autem hoc casu constructio binarum lentium ita se habebit:

radius faciei

Pro lente PP $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\surd(\lambda-1)} \\ \text{posterioris} = \dfrac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\surd(\lambda-1)} \end{cases}$

Pro lente QQ $\begin{cases} \text{anterioris} = \dfrac{b\beta}{\varrho\beta + \sigma b \pm \tau(b+\beta)\surd(\lambda'-1)} \\ \text{posterioris} = \dfrac{b\beta}{\varrho b + \sigma\beta \mp \tau(b+\beta)\surd(\lambda'-1)} \end{cases}$

existente $\varrho = 0,190781$; $\sigma = 1,627401$ et $\tau = 0,905133$.

Problema 5.

Tab. II. Fig. 6. 184. Si oculus per tres lentes PP, QQ et RR obiectum $E\varepsilon$ aspiciat, ita vt imago per eas repraesentata $H\theta$ in distantia iusta $OH = l$ ab oculo O sit remota, definire confusionem, qua visio afficietur.

Solutio.

Positis duabus prioribus lentibus PP et QQ vt in problemate praecedente, indeque determinatis valoribus P et Q cadat imago per has duas lentes repraesentata principalis in $G\eta$, post quam tertia lens RR ita collocata sit, vt sint eius distantiae determinatrices $CG = c$, $cH = \gamma$, crassities $Cc = v''$ et quantitas arbitraria $= k''$ vt sit:

radius faciei anterioris $= \dfrac{(n-1)c(k''+v'')}{k''+v''+2nc}$

radius faciei posterioris $= \dfrac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$

Tum

Tum vero posito $\frac{k''-v''}{k''+v''}=i''$ ponatur

$$R=\frac{n}{2(n-1)}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\gamma}-\frac{2}{k''+v''}\right)^2\right)$$

atque iam spatium diffusionis erit

$$Hb=\gamma\gamma xx\left(\frac{1}{i'i'.i''i''}\cdot\frac{\alpha\alpha\mathfrak{G}\mathfrak{G}}{bbcc}P+\frac{ii}{i''i''}\cdot\frac{bb\mathfrak{G}\mathfrak{G}}{\alpha\alpha cc}Q+ii\cdot i'i'.\frac{bbcc}{\alpha\alpha\mathfrak{G}\mathfrak{G}}R\right)$$

quod est valor ipsius Vxx. Radiorum vero in b concurrentium inclinatio ad axem est

$$ii'i''.\frac{bcx}{\alpha\mathfrak{G}\gamma}=\mathfrak{B}x$$

sit iam oculus in O, ac ponatur eius distantia post lentem RR $=O$ erit $O=\gamma+l$, ideoque $\gamma=O-l$. Hinc mensura confusionis in oculo ortae colligitur

$$\frac{1}{4}ii'i''u\left(1-\frac{O}{l}\right)\frac{bc}{\alpha\mathfrak{G}}x^3\left(\frac{1}{i'i'.i''i''}\cdot\frac{\alpha\alpha\mathfrak{G}\mathfrak{G}}{bbcc}P+\frac{ii}{i''i''}\cdot\frac{bb\mathfrak{G}\mathfrak{G}}{\alpha\alpha cc}Q+ii.i'i'.\frac{bbcc}{\alpha\alpha\mathfrak{G}\mathfrak{G}}R\right)$$

Coroll. 1.

185. Hic iterum vt ante patet, si fuerit $l=\infty$ ideoque et $\gamma=\infty$, quo casu diffusionis spatium in infinitum extenditur, mensuram confusionis terminis finitis contineri, quod etiam locum habet pro quouis lentium numero.

Coroll. 2.

186. Si lentium crassities pro euanescente habeatur, ob $i=1$ $i'=1$, $i''=1$, mensura confusionis ita simplicius exprimetur, vt sit

$$=\frac{1}{4}u\left(1-\frac{O}{l}\right)\frac{bc}{\alpha\mathfrak{G}}x^3\left(\frac{\alpha\alpha\mathfrak{G}\mathfrak{G}}{bbcc}P+\frac{bb\mathfrak{G}\mathfrak{G}}{\alpha\alpha cc}Q+\frac{bbcc}{\alpha\alpha\mathfrak{G}\mathfrak{G}}R\right)$$

At hoc casu loco k, k', k'' introductis numeris $\lambda, \lambda', \lambda''$ erit

$$P = \mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

$$Q = \mu\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right)$$

$$R = \mu\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right)$$

Coroll. 3.

187. Eodem vero casu constructio lentium per istos numeros $\lambda, \lambda', \lambda''$ ita erit dirigenda vt sit

Radius faciei

Pro lente PP $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{\varrho\alpha+\sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \dfrac{a\alpha}{\varrho a+\sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}} \end{cases}$

Pro lente QQ $\begin{cases} \text{anterioris} = \dfrac{b\beta}{\varrho\beta+\sigma b \pm \tau(b+\beta)\sqrt{(\lambda'-1)}} \\ \text{posterioris} = \dfrac{b\beta}{\varrho b+\sigma\beta \mp \tau(b+\beta)\sqrt{(\lambda'-1)}} \end{cases}$

Pro lente RR $\begin{cases} \text{anterioris} = \dfrac{c\gamma}{\varrho\gamma+\sigma c \pm \tau(c+\gamma)\sqrt{(\lambda''-1)}} \\ \text{posterioris} = \dfrac{c\gamma}{\varrho c+\sigma\gamma \mp \tau(c+\gamma)\sqrt{(\lambda''-1)}} \end{cases}$

Scholion.

188. Hinc satis manifestum est, quemadmodum hae formulae pro pluribus lentibus progrediantur: verum antequam eas exponam, conueniet alias quoque circumstantias, quae hinc facillime deducuntur, perpendi, scilicet magnitudinem obiecti visam, et copiam radiorum a singulis eius punctis in oculum transmissorum vt hae simul cum confusione visionis dein-

deinceps coniunctim pro quouis lentium numero exhiberi queant; quo pacto plures taediosas repetitiones euitabimus. Duae autem res ex hactenus allatis facile definiri possunt, quarum altera est quantitas, qua imago obiecti per lentes repraesentata ab oculo cernitur, quae quantitas aestimanda est ex angulo, sub quo imago videtur, vt is deinceps comparari possit, cum eo angulo sub quo ipsum obiectum in data distantia a nudo oculo spectaretur, vnde qua ratione magnitudo per lentes visa augeatur, intelligetur. Altera res in copia radiorum, a singulis obiecti punctis in oculum transmissorum versatur; qua claritas visione percepta continetur, a quolibet scilicet puncto conus seu cylindrus radiosus in oculum ingreditur, qui si pupillam penitus expleat, claritas ad summum gradum erit euecta, nisi forte maiori illustratione ipsi obiecto maius lumen concilietur. At si sectio illius coni aut cylindri, qua in oculum intrat, minor fuerit pupilla, in eadem ratione claritas decrescet; quod cum in omnibus instrumentis dioptricis, quibus magnitudinem visam vehementer augere propositum est, vsu venire soleat, plurimum intererit amplitudinem illius coni seu cylindri, qua in oculum penetrat, accurate determinasse.

Problema 6.

189. Definire quantitatem, sub qua quaeuis obiecti portio per lentes quotcunque ab oculo in distantia iusta ab imagine vltima remoto cernetur.

 Solutio.

Solutio.

Sit z linea in obiecto concepta, quae quanta per lentes oculo fit apparitura, definiri oporteat. Ostensum autem est in praecedentibus, quotcunque fuerint lentes, imaginem principalem huius lineae iterum esse lineam, cuius longitudo ad z certam teneat rationem a distantiis determinatricibus lentium et numeris i, i', i'', i''' etc. pendentem (86). Sit ergo haec longitudo imaginis $= Mz$, quae cum ab oculo in distantia l remoto aspiciatur, apparebit sub angulo $= \frac{Mz}{l}$; vel cuius tangens potius sit $= \frac{Mz}{l}$; sed quia hic angulus rarissime vltra aliquot gradus assurgere solet, tangens tuto pro ipso arcu assumitur. Effigies autem quae ab hac linea Mz in oculo exprimitur, erit $= \frac{Muz}{l}$; hic enim cogitationes a confusione abstraho, qua vtique fit, vt effigies maior imprimatur, propterea quod singula puncta circellis exhibentur. Iam positis iisdem lentium determinationibus, quibus supra §. 86 sum vsus, pro vario lentium numero angulus, sub quo linea z in obiecto sumta cernetur ita se habebit:

	Angulus visionis	situ
Pro vnica lente	$1 \cdot \frac{\alpha}{a} \cdot \frac{z}{l}$	inuerso
Pro duabus lentibus . . .	$\frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} \cdot \frac{z}{l}$	erecto
Pro tribus lentibus . . .	$\frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma}{abc} \cdot \frac{z}{l}$ . . .	inuerso
Pro quatuor lentibus . .	$\frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta}{abcd} \cdot \frac{z}{l}$ . . .	erecto

etc.

Scilicet

Scilicet si hae formulae sint positiuae, oculus lineam z situ siue erecto siue inuerso videbit, prout est notatum sin autem fuerint negatiuae, situm indicatum in contrarium verti oportet.

Coroll. 1.

190. Si eadem obiecti linea z in distantia $=h$ ab oculo nudo cerneretur, ea appaeritura esset sub angulo $=\frac{z}{h}$, vnde perspicitur, quanto ea vel maior vel minor per lentes videatur.

Coroll. 2.

191. Si distantia oculi a postrema lente ponatur $=O$, erit casu vnius lentis $\alpha=O-l$, ideoque $\frac{\alpha}{l}=-(1-\frac{O}{l})$ vnde angulus opticus lineae obiecti z respondens erit $=\frac{1}{l}(1-\frac{O}{l})\frac{z}{a}$ pro situ erecto, quia signum mutauimus.

Coroll. 3.

192. Simili modo casu duarum lentium ob $\mathfrak{b}=O-l$ erit iste angulus $=\frac{1}{l l'}(1-\frac{O}{l})\frac{\alpha z}{a b}$ pro situ inuerso.

Casu vero trium lentium ob $\gamma=O-l$ erit

iste angulus $=\frac{1}{l l' l''}(1-\frac{O}{l})\frac{\alpha \mathfrak{b} z}{a b c}$ pro situ erecto

Casu quatuor lentium ob $\delta=O-l$ erit

iste angulus $\frac{1}{l l' l''}(1-\frac{O}{l})\frac{\alpha \mathfrak{b} \gamma z}{a b c d}$ pro situ inuerso, et ita porro pro pluribus lentibus.

Scholion.

193. Hinc etiam modus ſe offert confuſionem ob lentium aperturam in oculo natam diſtinctius aeſtimandi. Scilicet cum ſingula obiecti puncta in oculo exprimantur circulis, quorum radius eſt $= \frac{u}{4l} \mathfrak{B} V x^3$, verus autem circulus cuius radius $= z$, oculo in diſtantia l expoſitus in oculo referatur circulo cuius radius $= \frac{u}{l} z$ ſingula puncta illius obiecti per lentes ſpectati, aeque magna apparebunt, ac orbes circulares radii $z = \frac{1}{4} \mathfrak{B} V x^3$, ſi in diſtantia ab oculo $= l$ ſpectarentur. Vel cum horum orbium ſemidiameter apparens ſit $= \frac{z}{l}$, ob confuſionem ſingula obiecti puncta inſtar circulorum videbuntur, quorum ſemidiameter apparens eſſet $= \frac{1}{4l} \mathfrak{B} V x^3$. In expreſſionibus igitur ante pro confuſione inuentis deleatur quantitas u, et habebitur ſemidiameter apparens circulorum confuſionem exprimentium. Hinc iudicari poterit quam parua eſſe debeat confuſio, vt non amplius ſentiatur, ſcilicet ſi oculus non amplius percipere valeat ſpatium circulare, cuius ſemidiameter eſſet 1″, ſeu $\frac{1}{\tau\sigma 3}$ pars radii circiter, euidens eſt, ſi fuerit noſtra expreſſio $\frac{1}{4l} \mathfrak{B} V x^3 = \frac{1}{\tau\sigma 3}$, confuſionem fore imperceptibilem. Ac experientiam conſulentes deprehendimus multo maiores angulos non amplius percipi poſſe, ita vt confuſio non ſit metuenda etiamſi expreſſio $\frac{1}{4l} \mathfrak{B} V x^3$ notabiliter maior fuerit, quam $\frac{1}{\tau\sigma 3}$; ne autem hic temere quicquam ſtatuamus, ponamus limitem, quem formula $\frac{1}{4l} \mathfrak{B} V x^3$ excedere non

non debeat, eſſe $= \frac{1}{4\varkappa^3}$, ita vt eſſe oporteat $\frac{1}{l}\mathfrak{B}\,V x^3 < \frac{1}{\varkappa^3}$. Poſtea igitur ad praxin deſcendentes poterimus pro $\varkappa$ numerum vel 40 vel minorem aſſumere prout experientia quouis caſu poſtulauerit. Quod ſi ergo hoc modo confuſionis rationem habeamus, profunditas oculi u non amplius in computum ingredietur.

Definitio 3.

194. Semidiameter confuſionis, eſt ſemidiameter apparens circuli, qui ab oculo aeque magnus videtur, ac ſingula obiecti puncta ipſi ob confuſionem apparent.

Corollarium.

195. Inueniemus igitur facile ſemidiametrum confuſionis ſi formulas ſupra pro confuſione repertas per profunditatem oculi u diuidamus quo pacto eae formulae ad numeros abſolutos reducentur.

Definitio 4.

196. Multiplicatio per lentes producta ex ratione quantitatis, qua obiecta per lentes ſpectantur, ad quantitatem, qua eadem obiecta in data diſtantia ab oculo nudo cernerentur aeſtimatur. Exponens autem multiplicationis inuenitur, ſi magnitudo, qua linea quaecunque in obiecto concepta per lentes videtur, diuidatur per magnitudinem, qua eadem linea in data diſtantia ab oculo nudo ſpectata eſſet apparitura.

Coroll.

Coroll. 1.

197. Inuoluit ergo diiudicatio multiplicationis distantiam quandam fixam, in qua eadem obiecta a nudo oculo aspici assumimus quae prout diuersa assumatur, multiplicatio alio atque alio modo exprimetur.

Coroll. 2.

198. Si haec distantia fixa, ex qua multiplicatio diiudicatur ponatur $= h$ et exponens multiplicationis $= m$, sit linea quaepiam in obiecto concepta $= z$, quae ergo nudo oculo in distantia h appareret sub angulo $= \frac{z}{h}$; eadem autem linea per lentes spectetur sub angulo $= \frac{Mz}{l}$ (88) ex quo erit exponens multiplicationis $m = \frac{Mh}{l}$.

Coroll. 3.

199. In ratione ergo $m : 1$ dimensiones lineares per lentes augeri sunt censendae; vnde superficies auctae apparebunt in ratione $mm : 1$, et ipsa corpora in ratione $m^3 : 1$. Cum exponente autem multiplicationis coniungi debet situs, quo obiecta apparent siue is sit erectus siue inuersus.

Scholion.

200. Distantia haec fixa h ad quam multiplicatio refertur non eodem modo perpetuo assumi solet, quippe quod etiam pro diuersitate obiectorum omnino fieri non posset. Nam si per lentes obiecta valde

valde remota veluti coeleſtia contuemur, quoniam ea nunquam in diſtantia modica ſpectare ſolemus, conuenit vtique magnitudinem per lentes viſam cum ea comparare, qua in ea ipſa a nobis diſtantia nudis oculis cernerentur: ideoque his caſibus diſtantia fixa *b* ipſi diſtantiae *a*, qua a lentibus obiecta ſunt remota, aequalis conſtitui ſolet. Scilicet ſi diſtantia *a* fuerit valde magna, qui eſt caſus Teleſcopiorum, ſtatuitur $b = a$, et magnitudo per haec inſtrumenta viſa, cum magnitudine per nudos oculos viſa in eadem diſtantia commodiſſime comparatur. Sic de Teleſcopiis dicitur, quoties diametri corporum coeleſtium multiplicentur, eoque caſu littera *m* exponentem huius rationis indicabit. Sin autem obiecta propiora contemplamur, qui eſt vſus Microſcopiorum ea plerumque ita prope ad inſtrumentum admouentur, vt in tam exigua diſtantia nudis oculis nunquam diſtincte cerni poſſent: neque ergo his caſibus ſtatui $b = a$ conueniret. Aliam ergo rationem ineundo pro *b* ſumi ſolet eiusmodi diſtantia modica, in qua obiecta commode ac diſtincte cernere liceat, quae etſi vtique pro diuerſa oculorum indole diuerſa ſumi deberet; tamen vt aliquid fixi ſtatuatur, pro *b* diſtantia 8 pollicum vtpote maximae oculorum parti conveniens accipi ſolet, ita vt his caſibus definiamus, quoties huiusmodi obiecta maiora *appareant*, quam ſi eadem nudis oculis in diſtantia 8 digitorum aſpicerentur. Interim tamen ſi multiplicatio ad hanc diſtantiam fuerit relata, non difficile erit eam ad quam-

 libet

libet aliam referre, ita vt haec hypotheſis naturam horum inſtrumentorum afficere non ſit cenſenda.

Problema 7.

Tab. II. Fig. 10. 201. Si obiectum per lentes quotcunque aſpiciatur, definire amplitudinem coni ſeu cylindri luminoſi, qui a ſingulis obiecti punctis in oculum transmittitur.

Solutio.

Reperiatur vt ſupra oculus in diſtantia iuſta l poſt imaginem poſtremam per lentes repraeſentatam quae etſi eſt per ſpatium aliquod Ll diffuſa, hic tamen a confuſione inde nata mentem abſtrahimus, quoniam confuſionem iam ſeorſim determinauimus. Ex puncto igitur l oculum verſus diffunditur conus luminoſus, cuius radii extremi ad axem inclinati ſunt angulo OlT quem poſuimus ſupra $= \mathfrak{B}x$. Huius igitur coni ſectio circa ingreſſum in oculum conſideretur, cuius ſemidiameter erit $= \mathfrak{B}lx$, vnde pro vario lentium numero hic ſemidiameter ſequenti modo definietur.

Pro vnica lente . . . . . . $il\frac{x}{\alpha}$

Pro duabus lentibus . . . . . $ii'l\frac{bx}{\alpha\beta}$

Pro tribus lentibus . . . . $ii'i''l\frac{bcx}{\alpha\beta\gamma}$

Pro quatuor lentibus . . . $ii'i''i'''l\frac{bcdx}{\alpha\beta\gamma\delta}$

etc.

hicque perinde eſt ſiue hae formulae ſint poſitiuae ſiue negatiuae; quia circulus ſiue radio poſitiuo ſiue negatiuo deſcribatur eiusdem prodit magnitudinis.

Coroll.

Coroll. 1.

202. Si iſte ſemidiameter $\mathfrak{B}lx$ maior fuerit ſemidiametro pupillae, tota pupillae apertura radiis impletur neque propterea viſio clarior procurari poterit, niſi forte ipſum obiectum fortiori illuſtratione ſplendidius reddatur.

Coroll. 2.

203. Sin autem haec quantitas $\mathfrak{B}lx$ minor fuerit ſemidiametro pupillae claritatis menſura exiſtet, quae eo maior erit quo maior fuerit iſta quantitas: dum contra minuta hac quantitate claritas tam exigua euadere poteſt, vt non amplius ſenſui viſus excitando ſufficiat.

Definitio 5.

204. Gradus claritatis per lentes perceptae commodiſſime definietur ſemidiametro coni luminoſi, qui a quouis obiecti puncto in oculum transmittitur.

Coroll. 1.

205. Gradus ergo claritatis definitur quantitate ſupra inuenta $\mathfrak{B}lx$, ita vt ſi gradum claritatis ponamus $= y$ habeamus $y = \mathfrak{B}lx$, quo cognito facillime iudicabimus, quonam gradu viſio ſit clara habenda.

Coroll. 2.

206. Scilicet ſi ſemidiametrum pupillae ponamus $= \omega$, quamdiu fuerit $y > \omega$, claritate plena

fruemur, quae nullius augmenti eſt capax niſi forte ipſam pupillam magis dilatare valeamus.

Coroll. 3.

207. At ſi fuerit $y < \omega$, claritatem vtique minorem percipiemus: ac ſi claritatem plenam vnitate deſignemus, claritas ex caſu $y < \omega$ reſultans erit $= \frac{yy}{\omega\omega}$; propterea quod copia radiorum in oculum immiſſorum eſt vt quadratum ſemidiametri y.

Coroll. 4.

208. Quod ſi gradus claritatis y eousque decreſcat, vt copia radiorum nimis ſit parua, quam vt ſenſum viſus excitare poſſit, nihil ob ſummam caliginem percipi poterit, vnde manifeſtum eſt ad viſionem requiri, vt gradus claritatis certum quempiam limitem ſuperet.

Scholion.

209. Tam in Teleſcopiis quam Microſcopiis maxime neceſſe eſt; vt obiecta certo claritatis gradu exhibeantur ne repraeſentatio nimis fiat obſcura. Hic autem gradus plurimum a lumine proprio obiectorum pendet, quae quo fuerint illuſtriora minor claritatis gradus iis ſatis clare videndis ſufficit; ideoque ſtellas illuſtriores contemplantes minori gradu claritatis contenti eſſe poſſumus: terreſtria vero obiecta multo maiorem claritatis gradum poſtulant. Quo igitur haec ad

ad omnes caſus accommodare valeamus gradum claritatis hic littera indefinita y contentum in computum ſum ducturus. Quamobrem his de multiplicatione et claritate praemiſſis, haec duo elementa ſimul cum confuſione pro quouis lentium numero exhibebo: ac primo quidem non neglecta lentium craſſitie, tum vero eadem ſeorſim lentium craſſitie neglecta exponi conueniet.

Problema 8.

210. Si oculus per quotcunque lentes PP, QQ, RR, SS etc. obiectum $E\varepsilon$ aſpiciat; ita vt imago poſtrema per eas repraeſentata ante oculum in iuſta diſtantia $= l$ reperiatur, determinare tam multiplicationem et claritatem, quam confuſionem, qua viſio perturbabitur.

Solutio

Quotcunque fuerint lentes, ſint pro ſingulis diſtantiae determinatrices, vt et craſſities cum quantitate arbitraria, vt ſequitur.

Pro Lente	Diſt: determinatr:	Craſſities	quant: arb:
Prima PP	$EA = a$; $aF = \alpha$;	$Aa = v$;	k
Secunda QQ	$FB = b$; $bG = \beta$;	$Bb = v'$;	k'
Tertia RR	$GC = c$; $cH = \gamma$;	$Cc = v''$;	k''
Quarta SS	$HD = d$; $dI = \delta$;	$Dd = v'''$;	k'''
	etc.		

atque hinc posita ratione refractionis $\frac{31}{20} = n$ constructio lentium ita se habebit;

Pro Lente	Radius Faciei anterioris	Radius Faciei posterioris
Prima PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Secunda QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Tertia RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$
Quarta SS	$\frac{(n-1)d(k'''+v''')}{k'''+v'''+2nd}$	$\frac{(n-1)\delta(k'''-v''')}{k'''-v'''-2n\delta}$

etc.

Tam posito breuitatis gratia:

$$\frac{k-v}{k+v} = i;\ \frac{k'-v'}{k'+v'} = i';\ \frac{k''-v''}{k''+v''} = i'';\ \frac{k'''-v'''}{k'''+v'''} = i''' \text{ etc.}$$

si aperturae primae lentis in facie anteriori semidiameter fuerit $= x$, tam pro facie posteriori quam pro vtraque facie singularum lentium sequentium aperturae maiores esse debent vel saltem non minores, quam sequens tabula ostendit:

Pro Lente	Semidiameter aperturae in facie anteriori	Semidiameter aperturae in facie posteriori
Prima PP	x	ix
Secunda QQ	$i.\frac{bx}{\alpha}$	$ii'.\frac{bx}{\alpha}$
Tertia RR	$ii'.\frac{bcx}{\alpha\beta}$	$ii'i''.\frac{bcx}{\alpha\beta}$
Quarta SS	$ii'i''.\frac{bcdx}{\alpha\beta\gamma}$	$ii'i''i'''.\frac{bcdx}{\alpha\beta\gamma}$

etc.

Denique

Denique ad abbreuiandum ponatur:

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$

$$Q = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

$$R = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\gamma}-\frac{2}{k''+v''}\right)^2\right)$$

$$S = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{i'''d}+\frac{2}{k'''-v'''}\right)^2+\left(\frac{n}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{i'''}{\delta}-\frac{2}{k'''+v'''}\right)^2\right)$$

etc.

His poſitis ponamus oculum in diſtantia $= O$ ab vltima lente locari ita vt poſt imaginem vltimam reperiatur in diſtantia $= l$: magnitudinem autem viſam comparari cum magnitudine qua idem obiectum nudo oculo in diſtantia fixa $= h$ cerneretur; ac ponatur exponens multiplicationis $= m$.

Deinde pro claritate ſit gradus claritatis $= y$, ita vt y indicet ſemidiametrum coni luminoſi in oculum intrantis.

Confuſio autem aeſtimetur per ſemidiametrum confuſionis ſupra (194) definitum.

Iam pro quouis lentium numero hae tres res ita ſe habebunt.

I. Pro vnica lente $O = \alpha + l$

1. Exponens multiplicationis $m = \frac{1}{i} \cdot \frac{\alpha b}{a i}$ ſitu inuerſo

2. Gradus claritatis $y = il. \frac{x}{\alpha}$

3. Semid: Confuſionis $= \frac{1}{4} i \frac{\alpha}{l} x^3 . P$

II.

II. Pro duabus Lentibus $O = \beta + l$

1. Exp. mult: $m = \frac{1}{ii'} \cdot \frac{\alpha\beta b}{abl}$ situ erecto

2. Gradus claritatis $y = ii'l. \frac{bx}{\alpha\beta}$

3. Semid. Confusionis $= \frac{1}{4} ii'. \frac{\beta}{l} \cdot \frac{b}{\alpha} x^3 \left(\frac{1}{i'i'} \cdot \frac{\alpha\alpha}{bb} P + ii. \frac{bb}{\alpha\alpha} Q \right)$.

III. Pro tribus Lentibus $O = \gamma + l$

1. Exp. mult: $m = \frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma b}{abcl}$ situ inuerso:

2. Gradus claritatis $y = ii'i''l. \frac{bcx}{\alpha\beta\gamma}$.

3. Semidiameter confusionis;

$$\frac{1}{4} ii'i''. \frac{\gamma}{l} \cdot \frac{bc}{\alpha\beta} x^3 \left(\frac{1}{i'i'.i''i''} \cdot \frac{\alpha\alpha\beta\beta}{bbcc} P + \frac{ii}{i''i''} \cdot \frac{bb\beta\beta}{\alpha\alpha cc} Q + ii.i'i'. \frac{bbcc}{\alpha\alpha\beta\beta} R \right).$$

IV. Pro quatuor Lentibus $O = \delta + l$

1. Exp. multipl: $m = \frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta b}{abcdl}$ situ erecto

2. Gradus Claritatis $y = ii'i''i'''l. \frac{bcdx}{\alpha\beta\gamma\delta}$

3. Semidiameter Confusionis;

$$\frac{1}{4} ii'i''i'''. \frac{\delta}{l} \cdot \frac{bcd}{\alpha\beta\gamma} x^3 \left\{ \begin{array}{l} \frac{1}{i'i'.i''i''.i'''i'''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd} P + \frac{ii}{i''i''i'''i'''} \cdot \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd} Q \\ + \frac{ii.i'i'}{i'''i'''} \cdot \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd} R + ii.i'i'.i''i''. \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma} S \end{array} \right\}$$

V.

V. Pro quinque Lentibus $O = \varepsilon + l$

1. Exp. multipl: $m = \frac{1}{i\,i'\,i''\,i'''\,i''''} \cdot \frac{\alpha\beta\gamma\delta\varepsilon b}{abcdel}$ ſitu inuerſo

2. Gradus claritatis $y = i\,i'\,i''\,i'''\,i''''\,l.\ \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$

3. Semidiameter confuſionis

$$\frac{1}{4}\, i\,i'\,i''\,i'''\,i''''.\ \frac{\varepsilon}{l} \cdot \frac{bcde}{\alpha\beta\gamma\delta}\, x^3 \left\{ \begin{array}{ll} + \dfrac{1}{i'\,i'.\ i''\,i''.\ i'''\,i'''.\ i''''\,i''''} & \cdot\ \dfrac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee}\, P \\ + \dfrac{i\,i}{i''\,i''.\ i'''\,i'''.\ i''''\,i''''} & \cdot\ \dfrac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee}\, Q \\ + \dfrac{i\,i.\ i'\,i'}{i'''\,i'''.\ i''''\,i''''} & \cdot\ \dfrac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}\, R \\ + \dfrac{i\,i.\ i'\,i'.\ i''\,i''}{i''''\,i''''} & \cdot\ \dfrac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}\, S \\ + i\,i.\ i'\,i'.\ i''\,i''.\ i'''\,i''' & \cdot\ \dfrac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}\, T \end{array} \right.$$

atque hinc etiam progreſſus ad plures lentes eſt manifeſtus.

Coroll. 1.

211. His omnibus caſibus euidens eſt fore generatim $my = \frac{bx}{a}$.

Datis ſcilicet multiplicatione m cum claritate y ſtatim definitur apertura primae lentis nempe $x = my.\ \frac{a}{b}$. Quo maior ſcilicet tam multiplicatio quam claritas deſideratur, eo maiorem eſſe oportet aperturam lentis primae.

Coroll. 2.

212. Quum autem x maius accipere non liceat, quam vt confuſio infra certum limitem contineatur, dato x cum exponente multiplicationis m

 defi-

definitur claritatis gradus $y = \frac{hx}{ma}$, vnde patet reliquis paribus, quo maior multiplicatio exigatur, eo minori claritate contentos nos esse oportere.

Coroll. 3.

213. Imprimis autem hic obseruandum est, has formulas aeque negotium conficere, quamcunque crassitiem lentes habuerint. *Euadent autem* tractabiliores, si lentium crassities negligatur, qui casus seorsim tractari meretur.

Problema 9.

214. Iisdem positis, quae in problemate praecedente, si lentium crassities vt euanescens consideretur, determinare tam multiplicationem et claritatem, quam confusionem, qua visio perturbabitur.

Solutio.

Haec tractatio a praecedenti in isto differt, quod lentium crassities v, v', v'' etc. euanescant, et loco quantitatum arbitrariarum k, k', k'' etc. numeri arbitrarii λ, λ', λ'' etc. in calculum introducantur. Ponatur ergo

Pro Lente		Dist. determinatr.	num. arb.
Prima	P P	$EA = a$; $aF = \alpha$	λ
Secunda	Q Q	$FB = b$; $bG = \beta$	λ'
Tertia	R R	$GC = c$; $cH = \gamma$	λ''
Quarta	S S	$HD = d$; $dI = \delta$	λ'''

etc.

Hinc

Hinc si sit breuitatis gratia!

$\varrho = 0,190781$; $\sigma = 1,627401$; et $\tau = 0,905133$

lentium constructio ita est instituenda

Pro Lente	Radius Faciei anterioris	Radius Faciei posterioris
Prima PP	$\frac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$	$\frac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$
Secunda QQ	$\frac{b\beta}{\varrho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}}$	$\frac{b\beta}{\varrho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}}$
Tertia RR	$\frac{c\gamma}{\varrho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda''-1)}}$	$\frac{c\gamma}{\varrho c+\sigma\gamma\mp\tau(c+\gamma)\sqrt{(\lambda''-1)}}$
Quarta SS	$\frac{d\delta}{\varrho\delta+\sigma d\pm\tau(d+\delta)\sqrt{(\lambda'''-1)}}$	$\frac{d\delta}{\varrho d+\sigma\delta\mp\tau(d+\delta)\sqrt{(\lambda'''-1)}}$

Cum iam lentis primae PP semidiameter aperturae sit $= x$, et in qualibet lente vtriusque faciei eadem sit ratio, vt omnes radii per primam ingressi simul per reliquas transmittantur, apertura reliquarum sequentes limites superare debet:

Semid. aperturae

Lentis secundae QQ $> \frac{b}{\alpha}x$

Lentis tertiae RR $> \frac{bc}{\alpha\beta}x$

Lentis quartae SS $> \frac{bcd}{\alpha\beta\gamma}x$

etc.

Tum vero posito breuitatis ergo $\mu = 0,938191$ et $\nu = 0,232692$ statuatur:

$$P = \mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

$$Q = \mu\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu}{b\beta}\right.$$

$$R = \mu\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{c\gamma}\right)$$

$$S = \mu\left(\tfrac{1}{d}+\tfrac{1}{\delta}\right)\left(\lambda'''\left(\tfrac{1}{d}+\tfrac{1}{\delta}\right)^2+\tfrac{\nu}{d\delta}\right)$$

etc.

 Sit

Sit iam O diſtantia oculi poſt vltimam lentem, ita vt ab imagine poſtrema diſtet interuallo $= l$, comparetur magnitudo viſa cum ea, qua idem obiectum in diſtantia fixa b nudo oculo cerneretur, ſitque exponens multiplicationis $= m$; et gradus claritatis $= y$, quibus poſitis erit pro quouis lentium numero vt ſequitur.

I. Pro vnica lente $O = \alpha + l$

1. Exponens multiplicationis $m = \frac{\alpha b}{a l}$ ſitu inuerſo
2. Gradus claritatis $y = l.\ \frac{x}{\alpha}$ hinc $my = \frac{bx}{a}$
3. Semid: confuſionis $= \frac{\alpha}{4l}.\ x^3\ P$

II. Pro duabus lentibus $O = \beta + l$

1. Exponens multiplicationis $m = \frac{\alpha\beta b}{a b l}$ ſitu erecto
2. Gradus claritatis $y = l.\ \frac{bx}{\alpha\beta}$, hinc $my = \frac{bx}{a}$
3. Semid: confuſionis $= \frac{\beta}{4l}.\ \frac{b}{\alpha} x^3 (\frac{\alpha\alpha}{bb} P + \frac{bb}{\alpha\alpha} Q)$

III. Pro tribus lentibus $O = \gamma + l$

1. Exponens multiplicationis $m = \frac{\alpha\beta\gamma b}{abcl}$ ſitu inuerſo
2. Gradus claritatis $y = l.\frac{bcx}{\alpha\beta\gamma}$ hinc $my = \frac{bx}{a}$
3. Semidiameter confuſionis

$$\frac{\gamma}{4l}.\ \frac{bc}{\alpha\beta} x^3 (\frac{\alpha\alpha\beta\beta}{bbcc} P + \frac{bb\beta\beta}{\alpha\alpha cc} Q + \frac{bbcc}{\alpha\alpha\beta\beta} R).$$

IV. Pro quatuor lentibus $O = \delta + l$

1. Exponens multiplicationis $m = \frac{\alpha\beta\gamma\delta b}{abcdl}$ ſitu erecto
2. Gradus claritatis $y = l.\frac{bcdx}{\alpha\beta\gamma\delta}$ hinc $my = \frac{bx}{a}$
3. Semidiameter confuſionis:

$$\frac{\delta}{4l}.\ \frac{bcd}{\alpha\beta\gamma} x^3 (\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd} P + \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd} Q + \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd} R + \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma} S)$$

V.

V. Pro quinque lentibus $O = \varepsilon + l$

1. Exponens multipl. $m = \frac{\alpha\beta\gamma\delta\varepsilon b}{abcdel}$ situ inuerso.

2. Gradus claritatis $y = l.\frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$ hinc $my = \frac{bx}{a}$

3. Semidiameter confusionis

$$\frac{\varepsilon}{4l} \cdot \frac{bcde}{\alpha\beta\gamma\delta} x^3 \begin{cases} + \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee} P + \frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee} Q \\ + \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee} R \\ + \frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee} S + \frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta} T \end{cases}$$

vnde non difficile erit has formulas ad lentes etiam plures continuare.

Coroll. 1.

215. Lentes simplices adhibendo numeros $\lambda, \lambda', \lambda''$, etc. vnitate minores accipi non possunt. Verum hic nihil obstat quo minus loco lentium simplicium lentes duplicatae, triplicatae vel etiam quadruplicatae in vsum vocentur, quo pacto in his formulis numeri $\lambda, \lambda', \lambda''$, etc. non solum infra vnitatem diminui sed ad nihilum vsque perduci poterunt. Tum autem constructio lentium harum multiplicatarum ex superiori capite peti et ad distantias determinatrices hic positas accommodari debet.

Coroll. 2.

216. Veluti si pro lente prima PP debeat esse

 $\lambda =$

$\lambda = 0,191827$, hanc lentem ita ex duabus componi oportet, vt sit

Pro lente	Radius faciei anterioris	Radius faciei posterioris
Priori	$\frac{2a\alpha}{(2\varrho-\sigma)\alpha+\sigma a}$;	$\frac{2a\alpha}{(2\sigma-\varrho)\alpha+\varrho a}$
Posteriori	$\frac{2a\alpha}{\varrho\alpha+(2\sigma-\varrho)a}$;	$\frac{2a\alpha}{\sigma\alpha+(2\varrho-\sigma)a}$.

Coroll. 3.

217. Sin autem velimus, vt pro lente prima sit $\lambda = 0,042165$, ea erit triplicanda, hoc modo

Pro lente	Radius faciei anterioris	Radius faciei posterioris
Priori	$\frac{3a\alpha}{(3\varrho-2\sigma)\alpha+\sigma a}$;	$\frac{3a\alpha}{(3\sigma-2\varrho)\alpha+\varrho a}$
Media	$\frac{3a\alpha}{(2\varrho-\sigma)\alpha+(2\sigma-\varrho)a}$;	$\frac{3a\alpha}{(2\sigma-\varrho)\alpha+(2\varrho-\sigma)a}$
Posteriori	$\frac{3a\alpha}{\varrho\alpha+(3\sigma-2\varrho)a}$;	$\frac{3a\alpha}{\sigma\alpha+(3\varrho-2\sigma)a}$

Coroll. 4.

218. At si pro lente prima requiratur $\lambda = -0,010216$, quadruplicata erit vtendum ita construenda:

Pro lente	Radius faciei anterioris	Radius faciei posterioris
Prima	$\frac{4a\alpha}{(4\varrho-3\sigma)\alpha+\sigma a}$;	$\frac{4a\alpha}{(4\sigma-3\varrho)\alpha+\varrho a}$
Secunda	$\frac{4a\alpha}{(3\varrho-2\sigma)\alpha+(2\sigma-\varrho)a}$;	$\frac{4a\alpha}{(3\sigma-2\varrho)\alpha+(2\varrho-\sigma)a}$
Tertia	$\frac{4a\alpha}{(2\varrho-\sigma)\alpha+(3\sigma-2\varrho)a}$;	$\frac{4a\alpha}{(2\sigma-\varrho)\alpha+(3\varrho-2\sigma)a}$
Quarta	$\frac{4a\alpha}{\varrho\alpha+(4\sigma-3\varrho)a}$;	$\frac{4a\alpha}{\sigma\alpha+(4\varrho-3\sigma)a}$

Coroll.

Coroll. 5.

219. Simili autem modo lens ſecunda QQ ex ſuis diſtantiis determinatricibus b et $\mathfrak{b}$ per multiplicationem erit conſtruenda, numerus que ei reſpondens λ' debet eſſe vel $0,191827$, vel $0,042165$ vel $-0,010216$; quod idem de reliquis lentibus eſt intelligendum.

Scholion 1.

220. Alias lentium ſpecies hic nollem in praxi adhiberi, cum hae ſolae ſine metu enormis erroris confici queant; tum vero etſi aliae ab his non admodum diſcrepantes fere aequo ſucceſſu in praxin introduci poſſent, tamen quia diſcrimen non admodum eſt notabile, iis facile carere poterimus. Cum igitur ſit $\varrho = 0,190781$ et $\sigma = 1,627401$, ideoque

$$\varrho = 0,190781; \qquad \sigma = 1,627401$$
$$2\varrho - \sigma = -1,245839; \quad 2\sigma - \varrho = 3,064021$$
$$3\varrho - 2\sigma = -2,682459; \quad 3\sigma - 2\varrho = 4,500641$$
$$4\varrho - 3\sigma = -4,119079; \quad 4\sigma - 3\varrho = 5,937261$$

easdem conſtructiones in numeris euolutis exhiberi conueniet.

I. Si igitur pro lente PP debeat eſſe $\lambda = 1$, ea erit ſimplex hoc modo conſtruenda.

Radius faciei anterioris poſterioris

$$\frac{a\alpha}{+0,190781\alpha + 1,627401\,a}; \quad \frac{a\alpha}{+1,627401\alpha + 0,190781\,a}$$

II.

II. Si pro lente PP debeat esse $\lambda = 0,191827$ ea erit duplicata hoc modo construenda

Radius Faciei

Pro lente	anterioris	posterioris
Priori	$\frac{a\alpha}{-0,622519\alpha + 0,013700a}$;	$\frac{a\alpha}{+1,532010\alpha + 0,095390a}$
Posteriori	$\frac{a\alpha}{+0,095390\alpha + 1,532010a}$;	$\frac{a\alpha}{+0,013700\alpha - 0,622519a}$

III. Si pro lente PP debeat esse $\lambda = 0,042165$ ea erit triplicata hoc modo construenda

Radius Faciei

Pro lente	anterioris	posterioris
Priori	$\frac{a\alpha}{-0,894153\alpha + 0,542467a}$;	$\frac{a\alpha}{+1,500214\alpha + 0,063594a}$
Media	$\frac{a\alpha}{-0,415200\alpha + 1,021340a}$;	$\frac{a\alpha}{+1,021340\alpha - 0,415280a}$
Posteriori	$\frac{a\alpha}{+0,063594\alpha + 1,500214a}$;	$\frac{a\alpha}{+0,542467\alpha - 0,894153a}$

IV. Si pro lente PP debeat esse $\lambda = -0,010216$ ea erit quadruplicata hoc modo construenda

Radius Faciei

Pro lente	anterioris	posterioris
Prima	$\frac{a\alpha}{-1,029770\alpha + 0,406850a}$;	$\frac{a\alpha}{+1,484315\alpha + 0,047695a}$
Secunda	$\frac{a\alpha}{-0,670615\alpha + 0,766005a}$;	$\frac{a\alpha}{+1,125160\alpha - 0,311460a}$
Tertia	$\frac{a\alpha}{-0,311460\alpha + 1,125160a}$;	$\frac{a\alpha}{+0,766005\alpha - 0,670615a}$
Quarta	$\frac{a\alpha}{+0,047695\alpha + 1,484315a}$;	$\frac{a\alpha}{+0,406850\alpha - 1,029770a}$

Scholion

Scholion 2.

221. Interim tamen ſi lentes deſideremus, in quibus valor ipſius λ maior ſit, quam hic eſt aſſumtus, conſtructio earum leui additione ex his ipſis formulis concinnari poterit. In binis ſcilicet fractionibus, quibus radii facierum cuiusque lentis deſignantur, alter denominator augeri alter vero diminui debet eadem quantitate, quae quantitas ſemper $= \tau(a+\alpha)\sqrt{v}$ denotante v exceſſum valoris ipſius λ ſupra ante aſſumtum. Ita ſi eſſe debeat

I. $\lambda = 1+v$; II. $\lambda = 0,191827+v$; III. $\lambda = 0,042165+v$
vel IV. $\lambda = -0,010216+v$

denominatores fractionum in Scholio praecedente traditarum pro quauis lente ſimplici alternatim ſunt augendi et minuendi quantitate $0,905133\,(a+\alpha)\sqrt{v}$. Vnde ſi lens quadruplicata deſideretur, pro qua ſit praeciſe $\lambda = 0$, erit $v = 0,010216$ et $\tau\sqrt{v} = 0,091487$; hincque pro quauis lente ſimplici denominatorum alter augeri alter vero diminui debet hac quantitate

$$0,091487\alpha + 0,091487a$$

ex quo talis naſcitur conſtructio huiusmodi lentis quadruplicatae pro qua eſt $\lambda = 0$

Pro lente	Radius faciei anterioris		Radius faciei poſterioris
Prima	$\dfrac{a\alpha}{-1,121257\alpha+0,315363a}$	;	$\dfrac{a\alpha}{+1,575802\alpha+0,139182a}$
Secunda	$\dfrac{a\alpha}{-0,762102\alpha+0,674518a}$	;	$\dfrac{a\alpha}{+1,216647\alpha-0,219973a}$
Tertia	$\dfrac{a\alpha}{-0,402547\alpha+1,033673a}$	;	$\dfrac{a\alpha}{+0,857452\alpha-0,579128a}$
Quarta	$\dfrac{a\alpha}{-0,043792\alpha+1,352828a}$	;	$\dfrac{a\alpha}{+0,458337\alpha-0,938283a}$

Sed hoc caſu quaelibet lens adhuc alio modo conſtrui poteſt: ſic quarta ordine retrogrado expoſita permutatis a et α dabit alteram formam lentis primae.

Supplementum III.

Ad Problem. 8.

Si lentes ratione refractionis diſcrepent, vt ſit ratio refractionis pro prima lente $= n$, pro ſecunda $= n'$, pro tertia $= n''$; inde neque in multiplicatione m neque in gradu claritatis quicquam mutatur; at vero in ſemidiametro confuſionis valores litterarum P, Q, R ſequenti modo immutari debent.

$$P = \frac{n}{2(n-1)^2}\left\{\begin{array}{l}\left(\frac{n}{\alpha}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2 \\ +\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\end{array}\right.$$

$$Q = \frac{n'}{2(n'-1)^2}\left\{\begin{array}{l}\left(\frac{n'}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2 \\ +\left(\frac{n''}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i''}{\beta}-\frac{2}{k'+v'}\right)^2\end{array}\right.$$

$$R = \frac{n''}{2(n''-1)^2}\left\{\begin{array}{l}\left(\frac{n''}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2 \\ +\left(\frac{n''}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\gamma}-\frac{2}{k''+v''}\right)^2\end{array}\right.$$

$$S = \frac{n'''}{2(n'''-1)^2}\left\{\begin{array}{l}\left(\frac{n'''}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{i'''d}+\frac{2}{k'''-v'''}\right)^2 \\ +\left(\frac{n'''}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{i'''}{\delta}-\frac{2}{k'''+v'''}\right)^2\end{array}\right.$$

etc.

Ad Problem. 9.

Si lentes ratione refractionis diſcrepent, loco litterarum ϱ, σ, τ in radiis facierum ſcribi oportet

pro

pro secunda lente ϱ', σ', τ'; pro tertia ϱ'', σ'', τ'', etc. tum vero expressiones pro multiplicatione et gradu claritatis nulla mutatione egent; pro confusione autem notari oportet, fore

$$P = \mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

$$Q = \mu'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu'}{b\beta}\right)$$

$$R = \mu''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu''}{c\gamma}\right)$$

etc.

CAPVT V.

DE CAMPO APPARENTE OCVLIQVE LOCO MAXIME IDONEO.

Problema I.

222.

Tab. II. Fig. 12. Si ex obiecti puncto extra axem sumto ε radius quicunque εM per lentem PP transmittatur, definire eius concursum cum axe O.

Solutio.

Sit $F\zeta$ imago principalis per lentem proiecta, a diffusione enim hic mentem abstrahimus, ac ponamus pro lente eius distantias determinatrices $AE = a$, $aF = \alpha$, eius crassitiem $Aa = v$, et quantitatem arbitrariam $= k$, sitque breuitatis ergo $\frac{k-v}{k+v} = i$. His positis si puncti ε imago cadat in ζ, voceturque $E\varepsilon = z$, erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha z}{a}$ at pro puncto lentis M statuatur eius distantia ab axe $AM = x$; ac supra ostendimus si radius a puncto E per punctum M transmitteretur, eum ita per m ad punctum F progressurum esse, vt foret $am = ix$. Radius igitur εM per punctum N ad ζ feretur, ita vt, cum obliquitas radiorum

rum in facies lentis incidentium perpetuo valde parva statuatur, sit proxime angulus $EM\varepsilon$ ad NMm vt n ad 1 posito $n = \frac{31}{20}$. Hinc erit $E\varepsilon$ ad mN in ratione composita istorum angulorum, et distantiarum AE ad Aa, seu $E\varepsilon : mN = na : v$, vnde fit $mN = \frac{vz}{na}$, ideoque $aN = ix - \frac{vz}{na}$. Iam vero radius a puncto N recta ad ζ pergit, et propterea axem ita in O secabit, vt sit $aN + F\zeta : aF = aN : aO$ sicque

$$aO = \left(iax - \frac{\alpha vz}{na}\right) : \left(ix - \frac{vz}{na} + \frac{\alpha z}{ia}\right) = \frac{niia\alpha x - i\alpha vz}{niiax - ivz + n\alpha z}$$

et $FO = \frac{n\alpha\alpha z}{niiax - ivz + n\alpha z}$

Si x sit semidiameter aperturae lentis in facie anteriori, haec intersectio O respondet casui, quo radius ab ε per lentis terminum summum transeat: sin autem is per terminum imum transmittatur, sumto x negatiuo fiet

$$FO = \frac{n\alpha\alpha z}{-niiax - ivz + n\alpha z}$$

At si radius ex ε per centrum lentis A transeat, punctum intersectionis ita cadet in O vt sit

$$FO = \frac{n\alpha\alpha}{n\alpha - iv}$$

Coroll. I.

223. Si igitur x denotet semidiametrum aperturae lentis in facie anteriori PMAP, vt omnes radii a puncto ε in hanc faciem incidentes per lentem transmittantur, necesse est, vt faciei posterioris

PNaP ſemidiameter ſit maior quam $\pm ix - \frac{vz}{na}$, ſumto x tam negatiuo quam poſitiuo. Vnde hic ſemidiater minor eſſe nequit, quam $ix + \frac{vz}{na}$.

Coroll. 2.

224. Si apertura in facie anteriori euaneſcat, vt ſit $x = 0$ radii a puncto ε, cuius ab axe diſtantia $E\varepsilon = z$, per lentem non tranſmittentur, niſi in facie poſteriori ſemidiameter aperturae ſit $= \frac{vz}{na}$ vel maior. Vnde patet quo maior fuerit lentis craſſities v eo maiori apertura in facie poſteriori eſſe opus.

Coroll. 3.

225. Viciſſim ergo ſi detur lentis apertura in facie poſteriori, cuius ſemidiameter ſit $= \frac{vz}{na}$; inde ſimul in obiecto extremum punctum ε determinatur, a quo radius in centrum lentis A incidens per eam tranſmittatur.

Coroll. 4.

226. Hic autem radius per A immiſſus poſt tranſitum cum axe in O occurret, vt ob $x = 0$ ſit interuallum $FO = \frac{n\alpha\alpha}{n\alpha - iv}$. ſeu $aO = \frac{-i\alpha v}{n\alpha - iv}$. Niſi ergo oculus in hoc axis loco teneatur, radius transmiſſus non in oculum ingredietur; ſi quidem apertura pupillae vt infinite parua ſpectetur.

Coroll.

Coroll. 5.

227. At ſi ſemidiameter pupillae ponatur $= \omega$, oculus etiam in o poſitus illum radium excipiet, ſi fuerit $o\,\omega = \omega$. Cum autem caſu $x = o$ ſit

$$a\mathrm{N}\left(\frac{vz}{na}\right) : a\mathrm{O}\left(\frac{i\alpha v}{n\alpha - iv}\right) = o\,\omega\,(\omega) : \mathrm{O}o \text{ erit}$$

$$\mathrm{O}o = \frac{nia\alpha\omega}{z(n\alpha - iv)}.$$

Quod interuallum cum aeque poſitiuum ac negatiuum accipi poſſit, pro loco oculi o habebimus

$$ao = -\frac{i\alpha vz \pm nia\alpha\omega}{z(n\alpha - iv)} = \frac{-i\alpha(vz \pm na\omega)}{z(n\alpha - iv)}.$$

Scholion.

228. In hac tractatione, vbi campum apparentem et locum oculi idoneum inueſtigamus, tam aperturam lentis obiectiuae in facie anteriori quam pupillae amplitudinem pro nihilo habebimus, vt quaeſtiones obtineamus determinatas. Quare horum elementorum, quae in inſtrumentis dioptricis ad viſionem accommodatis maximi ſunt momenti, ſequentes definitiones conſtituemus.

Definitio 1.

229. Campus apparens eſt ſpatium in obiecto, ex cuius ſingulis punctis radii in centrum lentis obiectiuae incidentes per reliquas lentes omnes transmittuntur. Quod ſpatium cum ſit circulare, eius radius vocatur ſemidiameter campi apparentis.

Coroll.

Coroll. 1.

230. Si ergo $E\varepsilon = z$ fuerit ſemidiameter campi apparentis, erit ε punctum obiecti extremum, ſed ab axe maxime remotum, ex quo adhuc radii in centrum A lentis obiectiuae incidentes per omnes lentes transmittuntur.

Coroll. 2.

231. Determinatur igitur magnitudo campi apparentis per aperturam ſequentium facierum refringentium, ac fortaſſe per aperturam vnius, ſi ſcilicet radii a puncto quodam magis remoto quam ε venientes nullum tranſitum per eam inuenirent, etiamſi per reliquas facies transmitterentur.

Scholion.

232. Si radii ex ſolo puncto E, quod eſt centrum campi apparentis, conſiderentur, ii quidem qui in A incidunt, quoniam perpetuo ſecundum axem progrediuntur, per omnes reliquas facies refringentes, quantumuis parua fuerit earum apertura, certe transmittentur; hincque campus apparens nunquam penitus euaneſcere poteſt. At quo magis punctum ε ab axe diſtans accipitur, vt radii ab eo per omnes facies transmittantur, eo maior earum apertura requiritur, quae cum ab earum curuatura pendeat, neque certum limitem ſuperare debeat, hinc vltimum punctum ε, vnde radii etiam nunc transmittuntur, ac propterea ſemidiameter campi apparentis determinatur.

natur. In ſequentibus quidem propoſitionibus campum apparentem ſeu eius ſemidiametrum $E\varepsilon = z$ vt datum aſſumam, et quanta eſſe debeat cuiusque faciei apertura, inueſtigabo: hinc enim facile viciſſim, ſi quaeque apertura fuerit cognita, campum apparentem ipſum definire licebit. Ceterum in hac definitione aſſumſi aperturam primae faciei eſſe euaneſcentem: ex quo facile intelligitur ea aucta etiam campum aliquantum extendi oportere; verum hoc augmentum poſtea in lucrum cedet, quod cum nunquam ſoleat eſſe notabile, eius rationem hic non habendam cenſui; quemadmodum etiam in ſequente definitione aperturae pupillae rationem non habebo.

Definitio 2.

233. Locus oculi idoneus eſt id punctum in axe, in quo radii ab extremitate campi apparentis per lentes transmiſſi axem interſecant. Oculus ſcilicet in hoc loco conſtitutus totum campum apparentem conſpiciet.

Coroll. 1.

234. Hinc igitur idoneus locus oculo aſſignabitur, ſi illa interſectio radiorum extremorum cum axe poſt lentem vltimam cadat, ſin autem haec interſectio ante lentem vltimam reperiatur, fieri nequit vt oculus in eo loco teneatur neque propterea totum campum contueri poterit.

Coroll. 2.

235. At ſi iſta interſectio pone lentem vltimam cadat, oculus totum campum apparentem perſpiciet, etiamſi pupilla maxime eſſet contracta: neque tamen ob maiorem pupillae amplitudinem maiorem campum percipere valet.

Coroll. 3.

236. Verum ob amplitudinem pupillae hoc commodi aſſequimur, vt oculus etiamſi extra locum idoneum conſtituatur, dummodo diſtantia non ſit nimis magna, tamen totum campum apparentem conſpicere poſſit: id quod egregie vſu veniet iis caſibus, quibus locus idoneus oculi ante faciem refringentem extremam cadit. Tum enim fieri poterit, vt oculus huic faciei immediate applicatus tamen totum campum percipiat.

Scholion.

237. Quando hic de viſione loquor, id ita in genere eſt interpretandum, vt a puncto viſo radius in oculum ingrediatur, neque hic curo, vtrum viſio ſit diſtincta nec ne? in ſequentibus enim docebitur, quomodo lentes diſponi conueniat, vt oculus in loco idoneo poſitus etiam in iuſta ab vltima imagine diſtantia reperiatur, quo viſio diſtincta reddatur. Hic igitur ſine vllo reſpectu ad viſionem diſtinctam habito, eum oculo locum aſſigno, vbi ab omnibus punctis

ctis in campo apparente contentis radios recipiat, et quoniam singula momenta, quae ad visionem pertinent, seorsim expediri conuenit; hic etiam non ad spatium diffusionis respicio, quod quidem semper per se euanescit, si apertura faciei primae euanescens statuatur.

Problema 2.

238. Si obiectum per vnicam lentem aspiciatur, determinare tam campum apparentem, quam locum idoneum oculi. Tab. III. Fig. 12.

Solutio.

Sint vt in problemate superiori distantiae determinatrices huius lentis, scilicet distantia obiecti ante lentem $AE = a$, et imaginis post lentem $aF = \alpha$, tum vero lentis crassities $Aa = v$, et distantia arbitraria constructionem lentis plene determinans $= k$. Deinde ponamus semidiametrum campi apparentis $E\varepsilon = z$, ita vt posita faciei anterioris PAP apertura infinite parua etiamnum a puncto ε radii per lentem transmittantur. Sit porro breuitatis gratia $\frac{k - v}{k + v} = i$, atque supra demonstrauimus campum apparentem ad ε vsque extendi, si pro facie posteriori PaP fuerit semidiameter aperturae $aN = \frac{vz}{na}$ existente $n = \frac{31}{20}$; perinde enim est, siue hic semidiameter affirmatiue accipiatur siue negatiue. Hinc ergo vicissim si semidiameter huius aperturae ponatur $= \omega$, erit semidiameter campi apparentis $E\varepsilon = z = \frac{na}{v}\omega$.

 Quod

Quod ad locum idoneum oculi attinet, qui ſit in O, quoniam inuenimus $FO = \frac{n\alpha\alpha}{n\alpha - iv}$, erit diſtantia $aO = \frac{-i\alpha v}{n\alpha - iv}$; ideoque negatiua, niſi ſit vel i numerus negatiuus, vel $iv > n\alpha$. Sin autem haec diſtantia aO fuerit poſitiua, oculus in O poſitus totum campum apparentem perſpiciet.

Coroll. 1.

239. Si craſſities lentis v euaneſcat, ob $z = \frac{n\alpha}{v}\alpha$, campus apparens euadet infinitus, ſeu potius indeterminatus; diſtantia vero aO euaneſcet. Oculus igitur lenti immediate applicatus tantum ſpatium conſpiciet, quantum per propriam indolem complecti valebit.

Coroll. 2.

240. Sin autem ob lentis craſſitiem $Aa = v$ diſtantia $aO = \frac{-i\alpha v}{n\alpha - iv}$ prodeat poſitiua, oculus in O poſitus totum campum apparentem aſpicere valebit, ſeu in obiecto ſpatium circulare ſpectabit, cuius ſemidiameter $E\varepsilon = z = \frac{n\alpha}{v}\alpha = \frac{31\alpha\alpha}{20v}$, quod ergo eo erit maius, quo tenuior fuerit lens.

Coroll. 3.

241. At ſi diſtantia aO reſultet negatiua, oculum in loco idoneo conſtitui non licet, quoniam is neceſſario poſt lentem teneri debet. Vbicunque autem is poſt lentem collocetur, non vniuerſum campum apparentem contuebitur, ſed tantum eius partem, et quidem

quidem eo minorem, quo magis poſt lentem remoueatur, propterea quod hoc modo magis a loco idoneo recedit.

Coroll. 4.

242. Hoc igitur caſu conueniet oculum immediate ad faciem lentis poſteriorem applicare, quo ſitu eatenus tantum radios accipiet, quatenus pupilla patet; ſicque campus viſus ab apertura pupillae pendebit; quae ſi eſſet nulla etiam campus apparens euaneſceret.

Coroll. 5.

243. Hinc patet ſi pupilla excedat aperturam faciei P*a*P ſeu ſi ſit $\omega > \alpha$, denotante ω ſemidiametrum pupillae, quia tum oculus huic faciei applicatus omnes radios transmiſſos recipit, eum totum campum apparentem eſſe viſurum. Sin autem ſit $\omega < \alpha$, partem tantum totius campi perſpiciet, cuius ſemidiameter erit $= \frac{na}{v}\omega = \frac{31\,a\,\omega}{20\,v}$, ſcilicet non maiorem quam ſi apertura faciei P*a*P aequalis eſſet pupillae.

Scholion.

244. Interim tamen ſi hoc caſu poſtremo apertura faciei lentis P*a*P, cui oculus eſt applicatus maior ſit quam pupilla, nihil obſtat quominus ea ſucceſſiue totam aperturam peragret, ſicque pedetentim totum campum apparentem con-

fpicere poterit, etiamfi eum fimul contueri non valeat. Ceterum notandum eft, fi etiam faciei anteriori apertura tribuatur, inde campum apparentem aliquantum augeri, fed partes obiecti vlteriores, quia non per medium lentis radios transmittunt, obfcuriores apparebunt, vnde merito a campo apparente excluduntur. At fi faciei anteriori tribuatur apertura, cuius femidiameter $= x$, vt facies pofterior omnes radios in illam incidentes transmittat, eius apertura tanto maior effe debet, fecundum regulas fupra traditas; fcilicet eius femidiametrum effe oportet $= ix + a$ feu $= ix + \frac{vz}{na}$.

Problema. 3.

245. Si inftrumentum dioptricum duabus conftet lentibus, definire campum apparentem, et locum oculi idoneum.

Solutio.

Tab. III. Fig. 13. Sit pro his lentibus vt hactenus:

pro PP: $AE = a$, $aF = \alpha$, $Aa = v$; dift. arb: $= k$, et $\frac{k-v}{k+v} = i$

pro QQ: $BF = b$, $bG = \mathfrak{b}$, $Bb = v'$; dift. arb: $= k'$ et $\frac{k'-v'}{k'+v'} = i'$

ac pofito femidiametro campi apparentis $E\varepsilon = z$, in imaginibus erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z$ et $G\eta = \frac{1}{ii'} \cdot \frac{\alpha\mathfrak{b}}{ab} z$. Confideretur iam radius a puncto ε in centrum lentis primae A incidens et per lentes transmiffus qui vtique per extremitates imaginum ζ et η transibit.

Iam

Iam ex problemate praecedente patet esse $a\mathrm{N} = \frac{vz}{na}$, vnde in altera lente punctum M' ita definietur, vt sit

$$\mathrm{F}\zeta - a\mathrm{N} : a\mathrm{F} = \mathrm{BM}' - \mathrm{F}\zeta : \mathrm{BF} \text{ siue}$$

$$\mathrm{BM}' = \mathrm{F}\zeta + \frac{\mathrm{BF}(\mathrm{F}\zeta - a\mathrm{N})}{a\mathrm{F}} = \frac{a\mathrm{B}.\mathrm{F}\zeta - \mathrm{BF}.a\mathrm{N}}{a\mathrm{F}}$$

vnde fit

$$\mathrm{BM}' = \frac{1}{i} \cdot \frac{\alpha + b}{a} z - \frac{bvz}{na\alpha}.$$

Nunc punctum N' hinc perinde definietur, atque ex problemate primo ex puncto M determinabatur punctum N; erit quippe

$$b\mathrm{N}' = i' . \mathrm{BM}' - \frac{v'}{n'b} . \mathrm{F}\zeta \text{ ideoque}$$

$$b\mathrm{N}' = \frac{i'}{i} \cdot \frac{\alpha + b}{a} z - \frac{i'bv}{na\alpha} z - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab} z$$

Hinc autem punctum O, vbi est locus oculi idoneus facile assignabitur erit enim $b\mathrm{N}' + \mathrm{G}\eta : b\mathrm{G} = b\mathrm{N}' : b\mathrm{O}$, indeque

$$b\mathrm{O} = \frac{b\mathrm{G}.b\mathrm{N}'}{b\mathrm{N}' + \mathrm{G}\eta} = \frac{\frac{i'}{i} \cdot \frac{\alpha + b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{nab}}{\frac{i'}{i} \cdot \frac{\alpha + b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{nab} + \frac{1}{ii'} \cdot \frac{\alpha\,\mathfrak{e}}{ab}}\,\mathfrak{e}$$

Quod si ergo ponamus semidiametrum aperturae

pro lente PP $\begin{cases} \text{faciei anterioris} = \mathfrak{A} = x \\ \text{faciei posterioris} = \mathfrak{a} \end{cases}$

pro lente QQ $\begin{cases} \text{faciei anterioris} = \mathfrak{B} \\ \text{faciei posterioris} = \mathfrak{b} \end{cases}$

habe-

habebimus

$$\mathfrak{A} = 0$$

$$\mathfrak{a} = \frac{v}{n\,a} z$$

$$\mathfrak{B} = \left(\frac{1}{i} \cdot \frac{\alpha + b}{a} - \frac{b\,v}{n\,a\,\alpha}\right) z$$

$$\mathfrak{b} = \left(\frac{i'}{i} \cdot \frac{\alpha + b}{a} - \frac{i'\,b\,v}{n\,a\,\alpha} - \frac{1}{i} \cdot \frac{\alpha\,v'}{n\,a\,b}\right) z$$

ac ſi diſtantia oculi poſt lentem QQ ponatur $bO = O$ erit

$$O = \frac{\mathfrak{b}}{\mathfrak{b} + \frac{1}{ii'} \cdot \frac{\alpha\,\mathfrak{C}}{a\,b} z} \cdot \mathfrak{C}$$

Coroll. 1.

246. Si ambarum lentium craſſities euaneſcat erit $v = 0$. $v' = 0$, et $i = i' = 1$; quo ergo caſu noſtrae formulae in ſequentes abibunt:

$$\mathfrak{A} = 0;\ \mathfrak{a} = 0;\ \mathfrak{B} = \frac{\alpha + b}{a} z \text{ et } \mathfrak{b} = \frac{\alpha + b}{a} z$$

Coroll. 2.

247. Datis ergo viciſſim aperturis lentium ex aequationum traditarum ea, pro qua quantitas z minimum valorem adipiſcitur, definietur campus adparens.

Coroll. 3.

248. Si igitur craſſities lentium euaneſcat, campus apparens ex apertura lentis poſterioris facillime determinatur. Erit enim $z = \frac{a\,\mathfrak{B}}{\alpha + b}$; id quod intelligendum eſt, ſi diſtantia $bO = O$ fuerit poſitiua oculusque in O collocetur.

Coroll.

Coroll. 4.

249. Sin autem diſtantia $bO = O$ prodeat negatiua oculusque vltimae lenti immediate applicetur, tum eius apertura plus non praeſtat quam ſi amplitudini pupillae eſſet aequalis. Quare ſi $\mathfrak{b}$ maior fuerit ſemidiametro pupillae ω loco $\mathfrak{b}$ ſcribatur ω et ex vltima aequatione verus valor ipſius z elicietur niſi forte ex aliqua reliquarum aequationum adhuc minor valor pro z eſſet proditurus.

Problema 4.

250. Si inſtrumentum dioptricum ex tribus conſtet lentibus determinare cum campum apparentem, tum locum oculi idoneum. Tab. III. Fig. 14.

Solutio.

Exiſtentibus imaginibus per has lentes ſucceſſiue repraeſentatis in $F\zeta$, $G\eta$ et $H\theta$, obiecto vero ipſo in $E\varepsilon$, ponamus vt hactenus:

Pro Lente	Diſtantias	craſſitiem	diſt. arb:	et
Prima PP..	$AE = a$; $aF = \alpha$;	$Aa = v$;	k;	$\frac{k-v}{k+v} = i$
Secunda QQ..	$BF = b$; $bG = \beta$;	$Bb = v'$;	k';	$\frac{k'-v'}{k'+v'} = i'$
Tertia RR..	$CG = c$; $cH = \gamma$;	$Cc = v''$;	k'';	$\frac{k''-v''}{k''+v''} = i''$

Semidiametros vero aperturarum

Pro Lente

$$PP \text{ faciei} \begin{cases} \text{anterioris} = \mathfrak{A} = \mathfrak{o} \\ \text{poſterioris} = \mathfrak{a} \end{cases}$$

$$QQ \text{ faciei} \begin{cases} \text{anterioris} = \mathfrak{B} \\ \text{posterioris} = \mathfrak{b} \end{cases}$$

$$RR \text{ faciei} \begin{cases} \text{anterioris} = \mathfrak{C} \\ \text{posterioris} = \mathfrak{c} \end{cases}$$

Tum vero ſit ſemidiameter campi apparentis $E\varepsilon = z$, ſupraque oſtendimus fore:

$$F\zeta = \frac{1}{i}\frac{\alpha}{a}z;\ G\eta = \frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z;\ H\theta = \frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}z.$$

His poſitis habebimus $aN = \mathfrak{a} = \frac{vz}{na}$; vnde ſi per F recta ipſi NM' parallela ducta intelligatur erit

$$F\zeta - aN : aF = BM' - F\zeta : BF \text{ ſiue } BM' = \frac{aB.F\zeta - BF.aN}{aF}$$

ideoque

$$BM' = \frac{\alpha + b}{\alpha}\cdot\frac{1}{i}\cdot\frac{\alpha}{a}z - \frac{bvz}{na\alpha} \text{ ſeu}$$

$$\mathfrak{B} = \frac{1}{i}\cdot\frac{\alpha + b}{a}z - \frac{bv}{na\alpha}z,$$

Porro vero eſt ex problemate primo $bN' = i'.BM' - \frac{v'}{nb}.F\zeta$, hincque $\mathfrak{b} = \frac{i'}{i}\cdot\frac{\alpha + b}{a}z - \frac{i'bv}{na\alpha}z - \frac{1}{i}\cdot\frac{\alpha v'}{nab}z$;

ſimili modo per G ducta intelligatur recta ipſi $N'M''$ parallela, eritque $bG : bN' + G\eta = CG : CM'' - G\eta$; ſiue

$$CM'' = \mathfrak{C} = G\eta + \frac{c}{\beta}(\mathfrak{b} + G\eta) = \frac{\beta + c}{\beta}\cdot G\eta + \frac{c}{\beta}\mathfrak{b},$$

vnde fit

$$\mathfrak{C} = \frac{1}{ii'}\cdot\frac{\alpha(\beta + c)}{ab}z + \frac{i'}{i}\cdot\frac{c(\alpha + b)}{a\beta}z - \frac{i'bcv}{na\alpha\beta}z - \frac{1}{i}\cdot\frac{\alpha cv'}{nab\beta}z.$$

Deinde

Deinde ex CM'' ita definitur cN'' per problema primum, vt sit

$$cN'' = i''.CM'' - \frac{v''}{nc}.G\eta \text{ seu } \mathfrak{c} = i''.\mathfrak{C} - \frac{1}{ii'}.\frac{\alpha\beta v''}{nabc}z$$

hincque

$$\mathfrak{c} = \frac{i''}{ii'}.\frac{\alpha(\beta+c)}{ab}z + \frac{i'i''}{i}.\frac{c(\alpha+b)}{a\beta}z - i'i''.\frac{bcv'}{na\alpha\beta}z - \frac{i''}{i}.\frac{\alpha cv'}{nab\beta}z - \frac{1}{ii'}.\frac{\alpha\beta v''}{nabc}z.$$

Isti ergo valores sequenti modo determinantur:

$$\mathfrak{A} = 0$$

$$\mathfrak{a} = \frac{v}{na}.E\varepsilon$$

$$\mathfrak{B} = \frac{\alpha+b}{\alpha}.F\zeta - \frac{b}{\alpha}.\mathfrak{a}$$

$$\mathfrak{b} = i'.\mathfrak{B} - \frac{v'}{nb}.F\zeta$$

$$\mathfrak{C} = \frac{\beta+c}{\beta}.G\eta + \frac{c}{\beta}.\mathfrak{b}$$

$$\mathfrak{c} = i''.\mathfrak{C} - \frac{v''}{nc}.G\eta$$

Cum iam punctum O praebeat locum oculi iustum, si ponamus $cO = O$, erit $cN'' + H\theta : cH = cN'' : cO$ vnde reperitur

$$O = \frac{\gamma\mathfrak{c}}{\mathfrak{c}+H\theta}, \text{ vel } \frac{1}{O} = \frac{1}{\gamma} + \frac{1}{ii'i''}.\frac{\alpha\beta}{ab}.\frac{z}{\mathfrak{c}c}.$$

Coroll. I.

251. Ex his aequationibus sequitur fore

$$\frac{\mathfrak{B}}{b} + \frac{\mathfrak{a}}{\alpha} = \frac{\alpha+b}{\alpha b}.F\zeta = \frac{1}{i}\left(\frac{\alpha}{b}+1\right)\frac{z}{a} \text{ et}$$

$$\frac{\mathfrak{C}}{c} - \frac{\mathfrak{b}}{\beta} = \frac{\beta+c}{\beta c}.G\eta = \frac{1}{ii'}\left(\frac{\alpha\beta}{bc}+\frac{\alpha}{b}\right)\frac{z}{a}.$$

hincque porro:

$$\frac{i\mathfrak{a}}{\alpha} + \frac{i\mathfrak{B}}{b} + \frac{ii'\mathfrak{b}}{\beta} - \frac{ii'\mathfrak{C}}{c} = \left(1 - \frac{\alpha\beta}{bc}\right)\frac{z}{a}.$$

Coroll. 2.

252. Si ergo craſſities lentium euaneſcat, cum ſit $\mathfrak{a}=0$, $\mathfrak{b}=\mathfrak{B}$ et $\mathfrak{c}=\mathfrak{C}$, definitio campi apparentis reducitur ad has duas aequationes.

$$\text{I. } \mathfrak{B}.\tfrac{1}{b}=\left(1+\tfrac{\alpha}{b}\right)\tfrac{z}{a}$$

$$\text{II. } \mathfrak{B}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)-\mathfrak{C}.\tfrac{1}{c}=\left(1-\tfrac{\alpha\beta}{bc}\right)\tfrac{z}{a}$$

vnde minor valor ipſius z praebet ſemidiametrum campi apparentis.

Coroll. 3.

253. Deinde ſi diſtantia $cO=O$ prodeat negatiua, vt oculum cogamur lenti vltimae immediate applicare, pro $\mathfrak{c}$ ſcribi oportet ſemidiametrum pupillae ω, et ex vltima aequatione definietur ſemidiameter campi z, niſi forte ex alia aequatione adhuc minor valor pro z prodeat.

Problema 5.

254. Si inſtrumentum Dioptricum ex quatuor lentibus ſuper eodem axe diſpoſitis conſtet, determinare cum campum apparentem, tum locum oculi idoneum.

Solutio.

Tab. III. Fig. 15. Exiſtente obiecto $E\varepsilon=z$, ſint imagines per lentes ſucceſſiue repraeſentatae $F\zeta$, $G\eta$, $H\theta$, et $I\iota$, ponamusque

musque pro lentium ſingularum determinatione vt hactenus:

Pro lente	diſtantias	craſſitiem	diſt. arb.	et	
Prima PP	$AE = a$;	$aF = \alpha$;	$Aa = v$;	$..k$;	$\frac{k-v}{k+v} = i$
Secunda QQ	$BF = b$;	$bG = \beta$;	$Bb = v'$;	$..k'$;	$\frac{k'-v'}{k'+v'} = i'$
Tertia RR	$CG = c$;	$cH = \gamma$;	$Cc = v''$;	$..k''$;	$\frac{k''-v''}{k''+v''} = i''$
Quarta SS	$DH = d$;	$dI = \delta$;	$Dd = v'''$;	$..k'''$;	$\frac{k'''-v'''}{k'''+v'''} = i'''$

Semidiametri vero aperturarum ſint:

Pro Lente

Prima PP faciei $\begin{cases} \text{anterioris} = \mathfrak{A} = \mathfrak{o} \\ \text{poſterioris} = \mathfrak{a} \end{cases}$

Secunda QQ faciei $\begin{cases} \text{anterioris} = \mathfrak{B} \\ \text{poſterioris} = \mathfrak{b} \end{cases}$

Tertia RR faciei $\begin{cases} \text{anterioris} = \mathfrak{C} \\ \text{poſterioris} = \mathfrak{c} \end{cases}$

Quarta SS faciei $\begin{cases} \text{anterioris} = \mathfrak{D} \\ \text{poſterioris} = \mathfrak{d} \end{cases}$

Si iam $E\varepsilon = z$ exhibeat ſemidiametrum campi apparentis erit, vt iam ſupra oſtendimus:

$$F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z; \; G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z; \; H\theta = \frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma}{abc} z$$

$$\text{et } I\iota = \frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta}{abcd} z$$

Quod ſi ratiocinium nunc vt ante inſtituamus, obtinebimus ſequentes aequationes:

$$\mathfrak{A}=0; \qquad \mathfrak{a}=\frac{v}{na}.\,E\varepsilon$$

$$\mathfrak{B}=(1+\tfrac{b}{\alpha})F\zeta-\tfrac{b}{\alpha}\mathfrak{a}: \qquad \mathfrak{b}=i'\mathfrak{B}-\frac{v'}{nb}.\,F\zeta$$

$$\mathfrak{C}=(1+\tfrac{c}{\beta})G\eta+\tfrac{c}{\beta}.\mathfrak{b}: \qquad \mathfrak{c}=i''\mathfrak{C}-\frac{v''}{nc}.\,G\eta$$

$$\mathfrak{D}=(1+\tfrac{d}{\gamma})H\theta+\tfrac{d}{\gamma}.\mathfrak{c}: \qquad \mathfrak{d}=i'''\mathfrak{D}-\frac{v'''}{nd}.\,H\theta$$

Ex quarum ordine priori conſequimur:

$$\frac{\mathfrak{B}}{b}+\frac{\mathfrak{a}}{\alpha}=(\tfrac{1}{\alpha}+\tfrac{1}{b})F\zeta=\tfrac{1}{i}(1+\tfrac{\alpha}{b})\tfrac{z}{a}$$

$$\frac{\mathfrak{C}}{c}-\frac{\mathfrak{b}}{\beta}=(\tfrac{1}{\beta}+\tfrac{1}{c})G\eta=\tfrac{1}{ii'}(\tfrac{\alpha}{b}+\tfrac{\alpha\beta}{bc})\tfrac{z}{a}$$

$$\frac{\mathfrak{D}}{d}-\frac{\mathfrak{c}}{\gamma}=(\tfrac{1}{\gamma}+\tfrac{1}{d})H\theta=\tfrac{1}{ii'i''}(\tfrac{\alpha\beta}{bc}+\tfrac{\alpha\beta\gamma}{bcd})\tfrac{z}{a}$$

ex ordine vero poſteriori

$$\mathfrak{a}=\frac{vz}{na}$$

$$\mathfrak{b}=i'\mathfrak{B}-\tfrac{1}{i}.\tfrac{\alpha}{b}.\tfrac{v'z}{na}$$

$$\mathfrak{c}=i''\mathfrak{C}-\tfrac{1}{ii'}.\tfrac{\alpha\beta}{bc}.\tfrac{v''z}{na}$$

$$\mathfrak{d}=i'''\mathfrak{D}-\tfrac{1}{ii'i''}.\tfrac{\alpha\beta\gamma}{bcd}.\tfrac{v'''z}{na}$$

Si denique pro loco oculi idoneo ponamus $dO=O$ erit

$$O=\frac{\delta\mathfrak{d}}{\mathfrak{d}+1^i} \text{ ſeu } \frac{1}{O}=\frac{1}{\delta}+\frac{1}{ii'i''i'''}.\frac{\alpha\beta\gamma}{bcd}.\frac{z}{a\mathfrak{d}}.$$

Coroll.

Coroll. 1.

255. Ex aequationibus prioribus deducimus sequentes:

$$\frac{i\mathfrak{a}}{\alpha}+\frac{i\mathfrak{B}}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\frac{i\mathfrak{a}}{\alpha}+i\frac{\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\beta}-\frac{ii'\mathfrak{C}}{c}=\left(1-\frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\frac{i\mathfrak{a}}{\alpha}+\frac{i\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\beta}-\frac{ii'\mathfrak{C}}{c}-\frac{ii'i''\mathfrak{c}}{\gamma}+\frac{ii'i''\mathfrak{D}}{d}=\left(1+\frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a}.$$

quae quomodo ad plures lentes sint continuandae, facile perspicitur.

Coroll. 2.

256. Si lentium crassities euanescat, fiet $\mathfrak{a}=0$; $\mathfrak{b}=\mathfrak{B}$; $\mathfrak{c}=\mathfrak{C}$ et $\mathfrak{d}=\mathfrak{D}$, porro $i=i'=i''=i'''=1$, vnde hae aequationes in sequentes formas abibunt:

$$\mathfrak{B}.\frac{1}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)-\mathfrak{C}\frac{1}{c}=\left(1-\frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)-\mathfrak{C}\left(\frac{1}{c}+\frac{1}{\gamma}\right)+\mathfrak{D}\frac{1}{d}=\left(1+\frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a}$$

ex quibus tribus aequationibus, vti in genere, valor ipsius z, qui prodierit minimus verum semidiametrum campi apparentis praebebit.

Coroll. 3.

257. Totus iste campus apparens reuera spectabitur ab oculo in puncto O constituto, dummodo distantia dO$=$O fuerit positiua. Sed si ea sit negatiua, oculusque lenti vltimo SS immediate applicetur, pona-

ponatur $\mathfrak{d} = \omega$, ſcilicet ſemidiametro pupillae, et ex vltima aequatione elicietur ſemidiameter ſpatii in obiecto reuera conſpicui.

Scholion 1.

258. Hinc igitur perſpicitur, quomodo campus apparens a ſingularum lentium apertura pendeat; ſimulque patet quanta eſſe debeat cuiusque lentis apertura, vt campus apparens datae magnitudinis obtineatur. Si enim quantitas z cum quantitatibus ad lentium determinationem pertinentibus pro data aſſumatur, per noſtras formulas ſucceſſiue ſemidiametri aperturarum pro ſingulis lentibus definiuntur: vbi quidem deinceps eſt diſpiciendum, num lentes tantae aperturae ſint capaces. Hinc ſcilicet campo apparenti limites praefiniuntur, quos transgredi non liceat; vnde ſequitur campum apparentem maiorem aſſumi non poſſe, quam vt aperturae inde pro ſingulis lentibus oriundae admitti queant. His autem definitis perinde eſt ſiue cuique lenti ea ipſa, quae fuerit inuenta apertura tribuatur, ſiue maior, dum ne ſit minor quandoquidem hic aperturam lentis obiectiuae euaneſcentem aſſumſimus. Verum ſi inſuper claritatis ratio habeatur, neceſſe eſt vt vera apertura cuiusque lentis eam, quam hic aſſignauimus, aliquantum ſuperet, et quidem ea quantitate, quam ſupra pro limitibus ob claritatem requiſit s exhibuimus, niſi enim hoc augmentum acceſſerit, extremitas in campo apparente minore lumine praedita erit quam

quam medium. Tum autem campus apparens latius patebit oramque obſcuriorem complectetur; quamobrem ſi circa extremitates minori lumine contenti eſſe velimus, ne opus quidem eſt, vt lentibus maior apertura, quam quidem per formulas noſtras definitur, tribuatur; ſuperfluumque foret aperturas vltra hos limites augere, ita vt hinc cuique lenti conueniens apertura conſtituatur.

Scholion 2.

259. Etſi pro caſu, quo lentium craſſities negligitur formulae noſtrae non multo ſimpliciores euadunt, tamen in iis percommode vſu venit, vt litterarum $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, coefficientes, ſcilicet $\frac{1}{b}+\frac{1}{\beta}$; $\frac{1}{c}+\frac{1}{\gamma}$; $\frac{1}{d}+\frac{1}{\delta}$; ipſam diſtantiam focalem, inuoluant cuiuſque lentis; in praxi autem apertura ſatis tuto ex diſtantia focali colligi ſolet. Nam ſi lentis QQ diſtantia focalis ponatur $= q$, erit $\frac{1}{b}+\frac{1}{\beta}=\frac{1}{q}$, ſicque $\mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})=\frac{\mathfrak{B}}{q}$; ac ne arcus nimis magni in apertura comprehendantur, neceſſe eſt vt ſit $\mathfrak{B}<\frac{1}{2}q$; et pro varia lentis forma valor fractionis $\frac{\mathfrak{B}}{q}$ vſque ad $\frac{1}{4}$, vel $\frac{1}{5}$ diminui debet. Quare ſi ponamus

$$\mathfrak{B}(\tfrac{1}{b}+\tfrac{1}{\beta})=\pi;\quad \mathfrak{C}(\tfrac{1}{c}+\tfrac{1}{\gamma})=\pi';\quad \mathfrak{D}(\tfrac{1}{d}+\tfrac{1}{\delta})=\pi''$$

hae litterae π, π', π'' eiusmodi denotabunt fractiones, quarum valor vt plurimum erit vel $\frac{1}{3}$ vel $\frac{1}{4}$ vel $\frac{1}{5}$, id ſcilicet tantum cauendum eſt, ne his litteris nimis magnus valor tribuatur. Quo obſeruato cum arbitrio noſtro relinquantur, imprimis conueniet has ipſas litteras

teras in calculum introduci, ex iisque reliquas determinari; earum enim beneficio campus apparens facillime definitur. Quin etiam ipſe campus apparens ſtatim quoque in calculum induci poterit, quippe cuius determinatio deinceps per formulam ſimpliciſſimam expedietur. Hunc in finem vt tota inueſtigatio ad meros numeros redigatur, ponam $\frac{z}{a} = \Phi$ ita vt Φ ſit angulus, ſub quo ſemidiameter campi apparentis ab oculo ad lentem obiectiuam collocato ſpectaretur. Videamus ergo quomodo per hos numeros π, π', π'', etc. et Φ reliquae quantitates definiantur.

Definitio 3.

260. Ratio aperturae cuiuſque lentis mihi vocabitur quotus, qui oritur, ſi ſemidiameter aperturae diuidatur per diſtantiam focalem lentis; eius craſſitie pro nihilo habita.

Coroll. 1.

261. Ita ſi b et β ſint diſtantiae determinatrices lentis, et $\mathfrak{B}$ ſemidiameter aperturae eiuſdem, quia diſtantia focalis eſt $= \frac{b\beta}{b+\beta}$, ratio aperturae erit $= \mathfrak{B}(\frac{1}{b} + \frac{1}{\beta})$

Coroll. 2.

262. Ratio igitur aperturae cuiuſuis lentis eſt fractio minor quam $\frac{1}{2}$, quandoquidem hanc legem ſanciuimus vt neutrius faciei arcus 60 gradibus maior in apertura contineatur.

Coroll.

Coroll. 3.

263. Si ſcilicet ambae facies fuerint aeque curuae ratio aperturae per hanc legem vsque ad $\frac{1}{2}$ augeri poterit; ſin autem altera facies fuerit plana, ratio aperturae $\frac{1}{4}$ ſuperare vix poterit: ac ſi lens ſit meniscus, ea adhuc minor ſtatui debebit.

Coroll. 4.

264. Cum autem nihil ſit, quo apertura lentium accuratius definiatur, ea fere arbitrio noſtro relinquitur, et quouis caſu commodiſſime per experientiam determinatur, ſufficietque notaſſe, eam fractioni ſiue $\frac{1}{3}$ ſiue $\frac{1}{4}$ ſiue etiam $\frac{1}{5}$ pro forma lentis aequalem ſtatui debere.

Scholion.

265. Vt ſcilicet quouis caſu ratio aperturae recte definiatur, radios vtriusque faciei lentis contemplari oportet, qui ſi fuerint f et g, erit diſtantia focalis $= \frac{fg}{(n-1)(f+g)} = \frac{20}{11} \cdot \frac{fg}{f+g}$. Iam ſemidiameter aperturae minor eſſe debet quam $\frac{1}{2}f$ vel quam $\frac{1}{2}g$, prout vel f vel g fuerit minor. Sit $g < f$, et cum ſemidiameter aperturae minor eſſe debeat quam $\frac{1}{2}g$, ratio aperturae minor accipienda eſt quam $\frac{11}{40}(1+\frac{g}{f})$. Vnde patet, ſi lens ſit vtrinque aeque conuexa, ſeu $g=f$, rationem aperturae capi debere infra $\frac{11}{20}$; ſin autem ſit altera facies plana ſeu $f=\infty$, illum limitem eſſe $\frac{11}{40}$, qui adhuc minor fiet, ſi lens ſit meni-

meniſcus, ſeu $\frac{g}{f}$ numerus negatiuus. Ceterum ſi ratio aperturae ſit $= \pi$, eique hoc modo idoneus valor tribuatur, perinde eſt ſiue is negatiue ſiue poſitiue accipiatur: ſemper autem conducet rationi aperturae minorem valorem tribui, quam ſecundum hanc regulam; partim vt obliquitas radiorum incidentium diminuatur, partim vero potiſſimum, vt ob claritatem aperturas lentium adhuc vltra augere liceat.

Problema 6.

266. Si inſtrumentum dioptricum ex quotcunque lentibus, quarum craſſitiem vt nullam ſpectare liceat, ſit compoſitum, dataque ſit ratio aperturae pro ſingulis lentibus vna cum campo apparente, definire diſtantias determinatrices ſingularum lentium.

Solutio.

Sit diſtantia obiecti ante lentem primam $AE = a$, imaginisque per eam repraeſentatae $aF = \alpha$, ac pro ſequentibus lentibus ponatur.

Pro Lente	Diſt. determinatrices	ratio aperturae
Secunda . .	$BF = b$; $bG = \mathfrak{G}$;	π
Tertia . .	$CG = c$; $cH = \gamma$;	π'
Quarta . .	$DH = d$; $dI = \delta$;	π''
Quinta . .	$EI = e$; $eK = \varepsilon$;	π'''

etc.

Hinc

Hinc ergo si semidiametri aperturarum harum lentium vt ante indicentur literis $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. erit $\pi = \mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})$; $\pi' = \mathfrak{C}(\frac{1}{c}+\frac{1}{\gamma})$; $\pi'' = \mathfrak{D}(\frac{1}{d}+\frac{1}{\delta})$; $\pi''' = \mathfrak{E}(\frac{1}{e}+\frac{1}{\varepsilon})$ etc. Tum vero si sit semidiameter campi apparentis $= z$ ponatur etiam $\frac{z}{a} = \Phi$. Cum igitur hinc sit:

$$\mathfrak{B} = \frac{\pi b \beta}{b+\beta};\ \mathfrak{C} = \frac{\pi' c \gamma}{c+\gamma};\ \mathfrak{D} = \frac{\pi'' d \delta}{d+\delta};\ \mathfrak{E} = \frac{\pi''' e \varepsilon}{e+\varepsilon} \text{ etc.}$$

habebimus ex §. 256 sequentes aequationes:

$$\frac{\pi \beta}{b+\beta} = (1+\frac{\alpha}{b})\Phi$$

$$\pi - \frac{\pi' \gamma}{c+\gamma} = (1-\frac{\alpha\beta}{bc})\Phi$$

$$\pi - \pi' + \frac{\pi'' \delta}{d+\delta} = (1+\frac{\alpha\beta\gamma}{bcd})\Phi$$

$$\pi - \pi' + \pi'' - \frac{\pi''' \varepsilon}{e+\varepsilon} = (1-\frac{\alpha\beta\gamma\delta}{bcde})\Phi$$

etc.

Quo iam facilius hinc per π, π', π'', π''' etc. et Φ distantiae determinatrices lentium definiri queant, ponatur $\alpha = Aa$; $\beta = Bb$; $\gamma = Cc$; $\delta = Dd$; $\varepsilon = Ee$ etc.

Ita vt litterae A, B, C, D, E etc. denotent numeros absolutos, ac nostrae aequationes induent has formas:

$$\frac{B\pi}{B+1} = (1+\frac{Aa}{b})\Phi$$

$$\pi - \frac{C\pi'}{C+1} = (1-\frac{ABa}{c})\Phi$$

$$\pi - \pi' + \frac{D\pi''}{D+1} = (1+\frac{ABCa}{d})\Phi$$

$$\pi - \pi' + \pi'' - \frac{E\pi'''}{E+1} = (1-\frac{ABCDa}{e})\Phi$$

etc.

vnde eliciuntur ſequentes determinationes:

$$b = \frac{A(B+1)a\Phi}{B\pi-(B+1)\Phi}$$

$$c = \frac{AB(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)}$$

$$d = \frac{ABC(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$$

$$e = \frac{ABCD(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Datis ergo praeter numeros $\Phi, \pi, \pi', \pi'', \pi'''$ etc. numeris A, B, C, D, E etc. cum diſtantia obiecti $AE = a$, per has formulas diſtantiae b, c, d, e etc. determinantur indeque inſuper alterae $\alpha, \beta, \gamma, \delta, \varepsilon$ etc. hoc modo:

$$\alpha = Aa$$

$$\beta = \frac{AB(B+1)a\Phi}{B\pi-(B+1)\Phi}$$

$$\gamma = \frac{ABC(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)}$$

$$\delta = \frac{ABCD(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$$

$$\varepsilon = \frac{ABCDE(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Hinc nanciſcimur diſtantias focales lentium

Primae $PP = \frac{Aa}{A+1}$

Secundae $QQ = \frac{ABa\Phi}{B\pi-(B+1)\Phi}$

Tertiae $RR = \frac{ABCa\Phi}{C\pi'-(C+1)(\pi-\Phi)}$

Quartae $SS = \frac{ABCDa\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$

Quintae $TT = \frac{ABCDEa\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$

etc.

Coroll.

Coroll. 1.

267. Ex angulo Φ cum distantia obiecti ante lentem primam $AE = a$, ita definitur semidiameter campi apparentis z, vt sit $z = a\Phi$: neque tamen campus apparens pro lubitu assumi potest, sed is per multiplicationem determinabitur, vt mox videbimus.

Coroll. 2.

268. Cum omnes numeri hic in calculum introducti aeque negatiue ac positiue accipi queant, obseruandum est eos perpetuo ita assumi debere, vt interualla lentium quae sunt $\alpha + b$; $\beta + c$; $\gamma + d$; $\delta + e$; etc. omnia prodeant positiua.

Coroll. 3.

269. Quod ad aperturam cuiusque lentis attinet eius semidiameter habebitur si eius distantia focalis multiplicetur per rationem aperturae littera π insignitam.

Scholion.

270. Quo formulas hic inuentas simpliciores reddamus quoniam litteris $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$; etc. non amplius indigebimus, ponamus ad abbreuiandum;

$$\frac{A}{A+1} = \mathfrak{A}; \quad \frac{B}{B+1} = \mathfrak{B}; \quad \frac{C}{C+1} = \mathfrak{C}; \quad \frac{D}{D+1} = \mathfrak{D}; \quad \frac{E}{E+1} = \mathfrak{E} \text{ etc.}$$

vt sit

$$A = \frac{\mathfrak{A}}{1-\mathfrak{A}}; \quad B = \frac{\mathfrak{B}}{1-\mathfrak{B}}; \quad C = \frac{\mathfrak{C}}{1-\mathfrak{C}}; \quad D = \frac{\mathfrak{D}}{1-\mathfrak{D}}; \quad E = \frac{\mathfrak{E}}{1-\mathfrak{E}} \text{ etc.}$$

at-

atque habebimus:

$$\alpha = Aa; \qquad b = \frac{Aa\Phi}{\mathfrak{B}\pi - \Phi}$$

$$\mathfrak{b} = \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi} \qquad c = \frac{ABa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$\gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \qquad d = \frac{ABCa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$\delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \qquad e = \frac{ABCDa\Phi}{\mathfrak{E}\pi'' - \pi'' + \pi' - \pi + \Phi}$$

$$\varepsilon = \frac{ABCDEa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \qquad \text{etc.}$$

Hincque porro definientur distantiae focales lentium:

$$\text{Primae } PP = \mathfrak{A}a$$

$$\text{Secundae } QQ = \frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi - \Phi}$$

$$\text{Tertiae } RR = \frac{AB\mathfrak{C}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$\text{Quartae } SS = \frac{ABC\mathfrak{D}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$\text{Quintae } TT = \frac{ABCD\mathfrak{E}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

etc.

et lentium interualla:

$$\text{I et II} = \frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi - \Phi}$$

$$\text{II et III} = \frac{ABa\Phi(\mathfrak{C}\pi' - (1 - \mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

$$\text{III et IV} = \frac{ABCa\Phi(\mathfrak{D}\pi'' - (1 - \mathfrak{C})\pi')}{(\mathfrak{C}\pi' - \pi + \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$$

$$\text{IV et V} = \frac{ABCDa\Phi(\mathfrak{E}\pi''' - (1 - \mathfrak{D})\pi'')}{(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$$

Quae interualla debent esse positiua.

Problema 7.

271. Positis iisdem, quae in problemate praecedente sunt assumta, definire locum idoneum oculi, vnde totus campus apparens conspici queat.

So-

Solutio.

Maneant omnes denominationes vt ante, et quia apertura lentis PP vt nulla ſpectatur, pro reliquis lentibus ex data aperturae ratione, ſemidiameter aperturae cuiusque ita ſe habebit

Lentis	ſemidiameter aperturae
Secundae QQ . .	$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}\cdot\Phi$
Tertiae RR . .	$\frac{AB\mathfrak{C}a\pi'}{\mathfrak{C}\pi'-\pi'+\Phi}\Phi$
Quartae SS . .	$\frac{ABC\mathfrak{D}a\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\Phi$
Quintae TT . .	$\frac{ABCD\mathfrak{E}a\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\Phi$

vbi litterae $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. valores in praecedente Scholio aſſignatos obtinent.

Deinde magnitudines ſingularum imaginum conſiderari conuenit, quae ob $E\varepsilon = z = a\Phi$ et

$$\alpha = Aa,\ \beta = Bb;\ \gamma = Cc;\ \delta = Dd \text{ etc.}$$

erunt

$$F\zeta = \alpha\Phi = Aa\Phi$$

$$G\eta = ABa\Phi$$

$$H\theta = ABCa\Phi$$

$$I\iota = ABCDa\Phi$$

etc.

Iam pro quolibet lentium numero locus oculi idoneus ſeorſim definiri debet; denotante ergo O diſtantiam oculi poſt vltimam lentem

I. Pro vnica lente

Quia craſſities lentis vt nulla ſpectatur, euidens eſt pro loco oculi idoneo fore $O = o$.

II. Pro duabus Lentibus

Tab. III. Fig. 13. Cum hic ſit $bN' + G\eta : bG = bN' : bO$ erit $bO = O = \frac{bN'}{bN' + G\eta} \cdot \mathfrak{C}$. Sed eſt $bN' = \frac{A\mathfrak{B}\pi}{\mathfrak{B}\pi - \Phi} a\Phi$ et $G\eta = ABa\Phi$ vnde fit

$$bN' + G\eta = Aa\Phi \cdot \frac{(B+1)\mathfrak{B}\pi - B\Phi}{\mathfrak{B}\pi - \Phi} = \frac{ABa\Phi(\pi - \Phi)}{\mathfrak{B}\pi - \Phi}$$

ob $(B+1)\mathfrak{B} = B$. Erit ergo $\frac{bN'}{bN' + G\eta} = \frac{\mathfrak{B}\pi}{B(\pi - \Phi)}$, quae fractio per $\mathfrak{C} = \frac{ABa\Phi}{(\mathfrak{B}\pi - \Phi)}$ multiplicata dat locum oculi idoneum

$$O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi - \Phi)(\mathfrak{B}\pi - \Phi)}$$

III. Pro tribus Lentibus

Fig. 14. Cum hic ſit $cN'' + H\theta : cH = cN'' : cO$, erit $cO = O = \frac{cN''}{cN'' + H\theta} \cdot \gamma$. Sed eſt

$$cN'' = \frac{AB\mathfrak{C}a\pi'\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \text{ et } H\theta = ABCa\Phi$$

hincque ob $(C+1)\mathfrak{C} = C$ fiet

$$cN'' + H\theta = \frac{ABCa\Phi(\pi' - \pi + \Phi)}{\mathfrak{C}\pi' - \pi + \Phi}$$

et $\frac{cN''}{cN'' + H\theta} = \frac{\mathfrak{C}\pi'}{C(\pi' - \pi + \Phi)}$. Nunc igitur ob $\gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ habebimus diſtantiam oculi idoneam:

$$O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi' - \pi + \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

IV.

IV. Pro quatuor Lentibus

Cum sit $dN'''' + Ii : dI = dN'''' : dO$, erit $dO = O = \frac{dN''''}{dN'''' + Ii} \cdot \delta$. Sed est Tab. III. Fig. 15.

$$dN'''' = \frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \text{ et } Ii = ABCDa\Phi$$

hincque ob $(D+1)\mathfrak{D} = D$ fiet

$$dN'''' + Ii = \frac{ABCDa\Phi(\pi'' - \pi' + \pi - \Phi)}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \text{ et}$$

$$\frac{dN''''}{dN'''' + Ii} = \frac{\mathfrak{D}\pi''}{D(\pi'' - \pi' + \pi - \Phi)}.$$ Ergo ob $\delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$

prodit distantia oculi idonea

$$O = \frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi'' - \pi' + \pi - \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}.$$

V. Pro quinque Lentibus

Si ratiocinium simili modo ad casum quinque lentium extendatur, reperiemus distantiam oculi idoneam

$$O = \frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi''' - \pi'' + \pi' - \pi + \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}.$$

VI. Pro sex Lentibus

Eodemque modo progrediendo colligitur fore pro casu sex lentium distantiam oculi idoneam

$$O = \frac{ABCDE\mathfrak{F}a\pi''''\Phi}{(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)(\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \Phi)}$$

sicque vlterius, quousque libuerit progredi licet.

Coroll. 1.

272. Quouis ergo caſu neceſſe eſt, vt diſtantia oculi idonea prodeat poſitiua: ſi enim fieret negatiua totus campus apparens nusquam conſpici poſſet.

Coroll. 2.

273. Iis autem caſibus, quibus diſtantia O fit negatiua oculum immediate vltimae lenti applicari conueniet; Tum vero oculus plus non cernet, quam ſi vltimae lentis apertura aequalis eſſet amplitudini pupillae.

Coroll. 3.

274. Hoc ergo caſu ſtatuatur ſemidiameter aperturae vltimae lentis $= \omega$ ſemidiametro pupillae, ex eaque aequatione eliciatur valor ipſius Φ, quo invento erit $a\Phi$ ſemidiameter campi apparentis, qui in obiecto reuera conſpicietur.

Problema 8.

275. Poſitis iisdem atque in problematibus praecedentibus, eam conditionem in lentium diſpoſitione definire, vt oculus in loco idoneo poſitus obiectum ſimul diſtincte videat.

Solutio.

Quia aperturam lentis obiectiuae euaneſcentem aſſumimus, in viſione alia confuſio locum habere nequit niſi quatenus oculus non in diſtantia iuſta ab vltima

vltima imagine, quam intuetur exiſtit, quae ergo tolletur, ſi lentes ita diſponantur, vt imago vltima ante oculum in O ſitum in diſtantia iuſta, quam littera l deſignauimus, verſetur. Cum igitur in figuris locus oculi ante imaginem vltimam cadat haec diſtantia negatiue ſumta ipſi l aequalis eſt ponenda; vnde pro quouis lentium numero ſequentes habebimus determinationes.

I. Pro vnica lente

Cum hic ſit $O = o$, et $OF = \alpha = Aa$, oportet eſſe $Aa = -l$, ideoque $A = -\frac{l}{a}$: et $\alpha = -l$, vnde indoles huius lentis determinatur, ita vt eius diſtantia focalis eſſe debeat $= \frac{al}{l-a}$. Tab. III. Fig. 12.

II. Pro duabus lentibus

Ex inuenta diſtantia $bO = O$, erit $OG = \frac{O}{bN'}$. $G\eta$. Eſt vero $\frac{O}{bN'} = \frac{1}{\pi - \Phi}$; vnde fit $\frac{AB a \Phi}{\pi - \Phi} = -l$, hincque pro ſecunda lente $B = -\frac{(\pi - \Phi)l}{Aa\Phi}$. et $\mathfrak{B} = \frac{B}{B+1}$. Vel cum ſit $Aa\Phi = -\frac{(\pi - \Phi)l}{B}$, erit pro loco oculi Fig. 13.

$$O = \frac{-\mathfrak{B}\pi}{B(\mathfrak{B}\pi - \Phi)} l$$

III. Pro tribus lentibus

Hic eſt $OH = \frac{O}{cN''}$. $H\theta = \frac{H\theta}{\pi' - \pi + \Phi}$, vnde obtinetur: Fig. 14.

$$OH = \frac{ABCa\Phi}{\pi' - \pi + \Phi} = -l$$

ſicque pro vltima lente habebitur:

$$C = \frac{-(\pi' - \pi + \Phi)l}{ABa\Phi}$$

 At

At ſi pro determinatione primae lentis capiatur $Aa\Phi = \frac{-(\pi' - \pi + \Phi)l}{BC}$, erit diſtantia oculi

$$O = \frac{-\mathfrak{C}\pi'}{C(\mathfrak{C}\pi' - \pi + \Phi)} l$$

IV. Pro quatuor lentibus.

Tab. III. Fig. 15. Cum ſit $OI = \frac{O}{a\aleph'''}$. $I\iota = \frac{I\iota}{\pi'' - \pi' + \pi - \Phi}$, habebitur

$$OI = \frac{ABCDa\Phi}{\pi'' - \pi' + \pi - \Phi} = -l.$$

Vnde pro vltima lente

$$D = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{ABCa\Phi}$$

Cum autem ſit $ABCa\Phi = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{D}$ erit in loco oculi hoc valore ſurrogando

$$O = \frac{-\mathfrak{D}\pi''}{D(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} l.$$

V. Pro quinque Lentibus

Simili modo pro quinque lentibus vltima ita comparata eſſe debet, vt ſit

$$E = \frac{-(\pi''' - \pi'' + \pi' - \pi + \Phi)l}{ABCDa\Phi}$$

Prima autem inde definita fit

$$O = \frac{-\mathfrak{E}\pi'''}{E(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)} l.$$

VI. Pro ſex Lentibus

Eodem modo patet pro ſex lentibus fore

$$F = \frac{-(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)l}{ABCDEa\Phi}$$

atque

atque ſi hinc Aa definiatur:

$$O = \frac{-\mathfrak{F}\pi''''}{F(\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)}\, l$$

quas formulas quousque libuerit, continuare licet.

Coroll. 1.

276. Si diſtantia oculi iuſta l fuerit infinita erit vt ſequitur:

I. Pro vna lente $A = \infty$ et $\mathfrak{A} = 1$

II. Pro duabus lentibus $B = \infty$ et $\mathfrak{B} = 1$

III. Pro tribus lentibus $C = \infty$ et $\mathfrak{C} = 1$

IV. Pro quatuor lentibus $D = \infty$ et $\mathfrak{D} = 1$

etc.

Coroll. 2.

277. Caſu ergo, quo diſtantia oculi iuſta l eſt infinita, diſtantia oculi poſt lentem vltimam erit pro quouis lentium numero:

I. Pro vnica lente $O = 0$

II. Pro duabus lentibus $O = \frac{Aa\pi\Phi}{(\pi - \Phi)}$

III. Pro tribus lentibus $O = \frac{ABa\pi'\Phi}{(\pi' - \pi + \Phi)^2}$

IV. Pro quatuor lentibus $O = \frac{ABCa\pi''\Phi}{(\pi'' - \pi' + \pi - \pi)^2}$

etc.

Scholion.

278. Hactenus campum apparentem Φ vt datum conſideraui, ex eoque tam lentium indolem quam

quam earum dispositionem determinaui, vt campus datae amplitudinis appareat, nihilque obstare deprehendimus, quominus huic conditioni satisfiat; cum numeri A, B, C, D, etc. penitus arbitrio relinquantur, aperturarum vero rationes π, π', π'', etc. infra $\frac{1}{3}$ vel $\frac{1}{4}$ accipi debeant. Verum hic multiplicationis ratio nondum in computum est ducta qua simul campus apparens ita adstringitur, vt certum limitem excedere nequeat. Quoniam igitur in omnibus instrumentis dioptricis multiplicatio imprimis proposita esse solet, quemadmodum per eam campus apparens definiatur in sequente problemate exponamus.

Problema 9.

279. Si instrumentum dioptricum ex quotcunque lentibus fuerit compositum, quarum quidem crassities vt nulla spectetur, simul vero multiplicationis ratio sit proposita, determinare campum apparentem.

Solutio.

Manentibus omnibus denominationibus, quibus hactenus sumus vsi, ita vt $a\Phi$ semidiametrum campi apparentis denotet, sit b distantia, ad quam multiplicationem referamus. Magnitudo igitur $a\Phi$ in distantia hac $= b$ nudo oculo cerneretur sub angulo, cuius tangens est $= \frac{a\Phi}{b}$. Quare si multiplicationis ratio statuatur $= m$, necesse est vt eadem magnitudo $a\Phi$ per lentes spectetur sub angulo, cuius tangens sit $= \frac{m a \Phi}{b}$:

Iam

Iam vero ex iis, quae in problematibus praecedentibus ſunt tradita iſte angulus facile aſſignatur, ſicque obtinebitur angulus Φ, indeque ſemidiameter campi apparentis $a\Phi$. Cum autem haec multiplicatio non ad ipſos angulos ſed eorum tangentes referatur euidens eſt tantum partes obiecti minimas circa centrum E ſitas in ratione propoſita multiplicari, remotiores vero in ratione minore. Quo notato hanc multiplicationis rationem m pro quouis lentium numero contemplemur.

I. Pro vnica Lente

Tangens anguli, quo imago $F\zeta$ ab oculo in O conſtituto conſpicitur, eſt $\frac{F\zeta}{OF} = \frac{F\zeta}{aF}$ ob $aO = o$, Erit ergo $\frac{ma\Phi}{b} = \Phi$, ſeu $ma = b$. Hoc ergo caſu campus apparens non determinatur ſed multiplicationis ratio eſt $m = \frac{b}{a}$. Verum vt viſio ſit diſtincta, per ſuperius problema debet eſſe $A = -\frac{l}{a}$ et diſtantia oculi poſt lentem $O = o$. At obiectum ſitu erecto cernetur: Tab. III. Fig. 12.

II. Pro duabus Lentibus

Tangens anguli, quo imago $G\eta$ ab oculo in O conſtituto cernetur, eſt $= \frac{bN'}{bO} = \pi - \Phi = \frac{ma\Phi}{b}$; vnde ſequitur ſemidiameter campi apparentis Fig. 13.

$$\Phi = \frac{\pi b}{ma + b} \text{ pro ſitu inuerſo}$$

Quo inuento, vt viſio ſit diſtincta oportet eſſe

$$B = \frac{-ml}{Ab}, \text{ et } \mathfrak{B} = \frac{-ml}{Ab - ml}$$

hincque prodit diſtantia oculi poſt lentem ocularem

$$O = \frac{Abl(ma+b)}{mmal + Abb}.$$

III. Pro tribus Lentibus

Tab. III. Fig. 14. Tangens anguli, quo imago $H\theta$ ab oculo in O conſtituto cernitur eſt $\frac{cN''}{cO} = \pi' - \pi + \Phi = \frac{ma\Phi}{b}$, vnde fit ſemidiameter campi apparentis:

$$\Phi = \frac{(\pi'-\pi)b}{ma-b} \text{ pro ſitu erecto}$$

Deinde vt viſio ſit diſtincta, oportet eſſe

$$C = \frac{-ml}{ABb} \text{ et } \mathfrak{C} = \frac{-ml}{ABb-ml}$$

Cum igitur ſit $\pi' - \pi + \Phi = \frac{ma(\pi'-\pi)}{ma-b}$ erit

$$\mathfrak{C}\pi' - \pi + \Phi = \pi' - \pi + \Phi - \frac{ABb\pi'}{ABb-ml}l = \frac{ma(\pi'-\pi)}{ma-b} - \frac{ABb\pi'}{ABb-ml}$$

hincque pro loco oculi

$$O = \frac{-ABbl\pi'}{(ABb-ml)(\mathfrak{C}\pi'-\pi+\Phi)} = \frac{ABbl(ma-b)\pi'}{(mmal-ABbb)\pi' + ma(ABb-ml)\pi}$$

IV. Pro quatuor lentibus

Fig. 15. Tangens anguli, quo imago $I\iota$ ab oculo in O conſtituto cernitur eſt $= \frac{dN'''}{dO} = \pi'' - \pi' + \pi - \Phi = \frac{ma\Phi}{b}$;

vnde elicitur:

$$\Phi = \frac{(\pi''-\pi'+\pi)b}{ma+b} \text{ pro ſitu inuerſo}$$

Hinc porro pro viſione diſtincta eſſe debet

$$D = \frac{-ml}{ABCb}, \text{ et } \mathfrak{D} = \frac{-ml}{ABCb-ml}$$

vnde fit

$$\mathfrak{D}\pi'' - \pi' + \pi - \Phi = \frac{ma(\pi''-\pi'+\pi)}{ma+b} - \frac{ABCb\pi''}{ABCb-ml}$$

et

et pro loco oculi

$$O = \frac{ABCbl(ma+b)\pi''}{(mmal+ABCbb)\pi''+ma(ABCb-ml)(\pi'-\pi)}.$$

V. Pro quinque lentibus

Eodem modo progrediendo pro campo apparente reperitur:

$$\Phi = \frac{(\pi'''-\pi''+\pi'-\pi)b}{ma-b} \text{ pro fitu erecto}$$

et vt vifio euadat diftincta

$$E = \frac{-ml}{ABCDb} \text{ et } \mathfrak{E} = \frac{-ml}{ABCDb-ml}$$

vnde pro loco oculi idoneo concluditur

$$O = \frac{ABCDbl(ma-b)\pi'''}{(mmal-ABCDbb)\pi'''+ma(ABCDb-ml)(\pi''-\pi'+\pi)}.$$

VI. Pro fex lentibus

Hic campus apparens ita definitur, vt fit:

$$\Phi = \frac{(\pi''''-\pi'''+\pi''-\pi'+\pi)b}{ma+b} \text{ pro fitu inuerfo}$$

vifio vero diftincta exigit

$$F = \frac{-ml}{ABCDEb};\ \mathfrak{F} = \frac{-ml}{ABCDEb-ml}.$$

vnde pro loco oculi idoneo

$$O = \frac{ABCDEbl(ma+b)\pi''''}{(mmal+ABCDEbb)\pi''''+ma(ABCDEb-ml)(\pi'''-\pi''+\pi'-\pi)}$$

ficque progreffio ad plures lentes eft manifefta.

Coroll. I.

280. Datis ergo rationibus aperturarum fingularum lentium π, π', π'', etc. vna cum ratione multiplicationis m, diftantia b, ad quam multiplicatio

 refer-

refertur, et diſtantia obiecti ante inſtrumentum a, determinatur campus apparens.

Coroll. 2.

281. Vt ergo campus apparens pro data multiplicatione maximus obtineatur litteris π, π', π'', etc. ita valores maximos tribui conueniet, vt alternatim ſint poſitiui et negatiui.

Coroll. 3.

282. Si igitur valores π, π', π'', etc. vsque ad $\frac{1}{3}$ augeri liceat, maximus valor ipſius Φ pro quouis lentium numero erit vt ſequitur:

Pro caſu duarum lentium $\Phi = \frac{b}{3(ma+b)}$

Pro caſu trium lentium $\Phi = \frac{2b}{3(ma-b)}$

Pro caſu quatuor lentium $\Phi = \frac{3b}{3(ma+b)}$

Pro caſu quinque lentium $\Phi = \frac{4b}{3(ma-b)}$

etc.

Coroll. 4.

283. Quo plures ergo lentes adhibentur eo magis campus apparens augeri poteſt ſimul vero patet, quo maior multiplicatio deſideretur, eo minorem fieri campum apparentem.

Coroll.

Coroll. 5.

284. Ratio multiplicationis m tam poſitiue quam negatiue capi poteſt. Si poſitiue accipitur pro lentium numero pari ſitum inuerſum, pro impari autem ſitum erectum declarat. Contrarium vero euenit, ſi m fuerit numerus negatiuus.

Scholion 1.

285. Hic autem imprimis notandum eſt, valorem ipſius Φ tum ſolum angulum $EA\varepsilon$ praebere, quando fuerit tam exiguus, vt aliquot gradus non ſuperet; ſi enim valor ipſius Φ prodeat multo maior, tum tangentem huius anguli $EA\varepsilon$ exprimit. Plerumque autem, ſi quidem multiplicatio ſit modica, iſte valor ipſius Φ tam paruus reperitur, vt ſine errore pro ipſo angulo $EA\varepsilon$ accipi poſſit. Hic igitur ob aliam cauſam amplitudo campi apparentis reſtringitur, vt certum limitem ſuperare nequeat; cum enim angulus, quo radii in oculum incidentes in O ad axem inclinantur, nunquam poſſit eſſe rectus, neque fortaſſe vix $60°$ ſuperare queat, quandoquidem ne nudo quidem oculo ſpatium in coelo maius quam $120°$ conſpicere valeamus; ſi illum angulum maximum, quem oculus capere valeat, circiter $63°$ ſtatuamus, vt eius tangens ſit $= 2$ pro quouis lentium numero habebimus $\frac{ma\Phi}{b} = 2$, vnde fit $\Phi = \frac{2b}{ma}$, et $a\Phi = \frac{2b}{m}$. Data ergo multiplicatione m et diſtantia b ad quam refertur, ſemidiameter ſpatii in obiecto

conſpicui nunquam maior exiſtere poteſt quam $\frac{2b}{m}$ quotcunque etiam adhibeantur lentes, eaeque ita diſponantur vt maximum campum patefaciant. In Teleſcopiis ergo, vbi ſumitur $b = a$, et ſemidiameter campi ex ipſo angulo Φ aeſtimatur, eius tangens nunquam maior eſſe poteſt quam $\frac{2}{m}$: vnde ſequentem tabellam adiungo, quae pro quauis multiplicatione ſemidiametrum campi apparentis maximi oſtendit, quem nunquam ſuperare liceat.

Multiplicatio m	Semidiam: campi app: maximi	Multiplicatio m	Semidiam: campi app: maximi
5	21°, 48′	60	1°, 54′, 33″
10	11, 18	70	1, 38, 13
15	7, 35	80	1, 25, 57
20	5, 42	90	1, 16, 24
25	4, 34	100	1, 8, 45
30	3, 49	150	0, 45, 50
35	3, $16\frac{1}{2}$	200	0, 34, 22
40	2, 52	250	0, 27, 30
45	2, 33	300	0, 22, 55
50	2, $17\frac{1}{2}$	400	0, 17, 11
		500	0, 13, 45

Quod ſi ergo numerum lentium multiplicando iam fere ad tantum campum apparentem pertigerimus, is vlterius nullo modo augeri poterit.

Scho-

Scholion 2.

286. Quod ad locum oculi idoneum attinet, eum ideo in O constituimus, vt omnes radios per lentes transmissos accipiat, etiamsi pupilla maxime esset constricta: ex quo patet ob amplitudinem pupillae oculum de hoc loco sine vllo detrimento aliquantillum remoueri posse, ita vt superfluum foret hunc locum nimis sollicite obseruare, nisi forte apertura vltimae lentis fuerit admodum magna. Sin ea autem pupillam non superet, eaque adeo sit minor, manifestum est oculum ei immediate applicatum aeque omnes radios excipere, et eundem campum contueri, ac si in loco idoneo esset constitutus. His igitur casibus, si forte distantia O pro loco oculi prodeat negatiua, nihil de campo apparente perit, dummodo oculus lenti vltimae immediate applicetur. His itaque, quae ad visionem per instrumenta dioptrica in genere pertinent, expeditis, superest, vt inuestigemus, quantum visio ob diuersam radiorum refrangibilitatem turbetur, et quemadmodum hanc perturbationem euitare queamus.

CAPVT

CAPVT VI.

DE CONFVSIONE A DIVERSA RADIORVM INDOLE ORIVNDA.

Problema I.

287.

Si a puncto dato E radii per lentem PP transmittantur definire variationem in loco imaginis F, quae a diuersa radiorum refrangibilitate oritur.

Solutio.

Sit distantia puncti E ante lentem $AE = a$, facierum autem lentis radius anterioris $= f$ posterioris $= g$ et crassities $Aa = v$, quae quantitates sunt constantes. Posita nunc refractionis ratione ex aere in vitrum $= n : 1$, ob diuersam radiorum naturam numerus n erit variabilis, ideoque etiam locus imaginis F post lentem expressae, cuius distantia si ponatur $aF = \alpha$, erit ex supra inuentis

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+v} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-v}$$

vbi

vbi quantitas k etiam pro variabili eſt habenda, quia tantum a, f, g et v ſunt conſtantes. Quaeſtio ergo huc redit, vt ſi numerus n differentiali ſuo dn creſcere ſumatur, definiatur differentiale diſtantiae α. Quare differentientur ambae aequationes illae:

$$\frac{dn}{f} = \frac{2dn}{k+v} - \frac{2ndk}{(k+v)^2}$$

$$\frac{dn}{g} = \frac{-d\alpha}{\alpha\alpha} - \frac{2dn}{k-v} + \frac{2ndk}{(k-v)^2}.$$

indeque eliminato dk habebitur

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = 2\,dn(k+v) - 2dn(k-v) - \frac{d\alpha(k-v)^2}{\alpha\alpha}$$

Reſtituantur pro f et g valores initio poſiti, ac peruenietur ad hanc aequationem:

$$\frac{dn(k+v)^2}{a} + \frac{dn(k-v)^2}{\alpha} + 4vdn + \frac{(n-1)d\alpha}{\alpha\alpha}(k-v)^2 = 0$$

vnde reperitur:

$$\frac{d\alpha}{\alpha\alpha} = \frac{-dn}{(n-1)a}\left(\frac{k+v}{k-v}\right)^2 - \frac{dn}{(n-1)\alpha} - \frac{4vdn}{(n-1)(k-v)^2} \text{ ſeu}$$

$$d\alpha = \frac{-\alpha dn}{n-1}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$$

Tum vero cum etiam k ſit quantitas variabilis erit

$$dk = \frac{-(k+v)dn}{n(n-1)}\left(1 + \frac{k+v}{2a}\right).$$

Cum ergo poſuerimus $\frac{k-v}{k+v} = i$, ob $di = \frac{2vdk}{(k+v)^2}$ erit

$$di = \frac{-vdn}{n(n-1)}\left(\frac{1}{a} + \frac{2}{k+v}\right).$$

Coroll. I.

288. Si $n:1$ denotet rationem refractionis radiorum mediae naturae, vt ſit $n = \frac{31}{20} = 1,55$. erit

pro radiis rubris ſeu minime refractis $n = 1,54$, et violaceis $n = 1,56$. quorum valorum diſcrimen a medio, cum ſit $= \frac{1}{100}$, pro differentiali dn haberi poterit.

Coroll. 2.

289. Quare ſi α denotet diſtantiam imaginis a radiis mediis formatae, pro ea quae a rubris formatur, erit $dn = -\frac{1}{100}$ et $\frac{dn}{n-1} = \frac{-1}{55}$. Hinc diſtantia imaginis rubrae poſt lentem erit $= \alpha + \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$ Diſtantia autem imaginis violaceae poſt lentem erit

$$\alpha - \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right).$$

Coroll. 3.

290. Si craſſities lentis euaneſcat, vt ſit $v = 0$, ob variabilitatem numeri n erit

$$d\alpha = \frac{-\alpha dn}{n-1}\left(1 + \frac{\alpha}{a}\right) = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a} + \frac{1}{\alpha}\right)$$

Ac ſi diſtantia focalis lentis ponatur $= p$, cum ſit $\frac{1}{a} + \frac{1}{\alpha} = \frac{1}{p}$, erit $d\alpha = \frac{-\alpha\alpha dn}{(n-1)p} = \frac{-100\alpha\alpha dn}{11p}$.

Scholion.

291. Hoc ergo modo ob diuerſam radiorum naturam valor diſtantiae α immutatur, vnde ſi ab ea tanquam ab obiecto radii porro ad lentem ſecundam emittantur, fiet etiam reſpectu huius lentis diſtantia obiecti variabilis. Quam ob cauſam in loco imaginis ab ea formatae duplex variatio orietur: id quod deinceps etiam in lentibus ſequentibus multo

magis

magis eueniet. Hanc igitur variationem, quae pro quauis lente in loco imaginis nafcitur, in fequente problemate determinemus.

Problema 2.

292. Si locus imaginis F, quae refpectu lentis QQ vicem obiecti gerit, ob diuerfam radiorum naturam ipfe fit variabilis, determinare variationem, quam ob eandem caufam imago fequens in G patietur.

Solutio.

Sit pro radiis mediae naturae, quibus refpondet numerus n diftantia obiecti F ante lentem $BF = b$; imaginisque inde proiectae diftantia poft lentem $bG = \mathfrak{b}$; dum autem n abit in $n + dn$, hae diftantiae ambae b et $\mathfrak{b}$ capiant fua incrementa differentialia db et $d\mathfrak{b}$. Ad quae inuenienda fit lentis QQ radius faciei anterioris $= f$, pofterioris $= g$, et craffities $Bb = v$, eritque vt ante:

$$\frac{n-1}{f} = \frac{1}{b} + \frac{2n}{k+v} \text{ et } \frac{n-1}{g} = \frac{1}{\mathfrak{b}} - \frac{2n}{k-v}$$

vbi k cum b et $\mathfrak{b}$ pro variabili eft habenda. Differentiatione ergo inftituta habebitur.

$$\frac{dn}{f} = \frac{-db}{bb} + \frac{2dn}{k+v} - \frac{2ndk}{(k+v)^2}$$
$$\frac{dn}{g} = -\frac{d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}} - \frac{2dn}{k-v} + \frac{2ndk}{(k-v)^2}$$

vnde eliminato dk, fit

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = \frac{-db}{bb}(k+v)^2 - \frac{d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}}(k-v)^2 + 4vdn$$

 quae

quae multiplicata per $n-1$, ſi pro f et g valores dati ſubſtituantur prodit

$$\frac{dn(k+v)^2}{b}+2ndn(k+v)+\frac{dn(k-v)^2}{\beta}-2ndn(k-v)$$
$$=-\frac{(n-1)db}{bb}(k+v)^2-\frac{(n-1)d\beta}{\beta\beta}(k-v)^2+4(n-1)vdn.$$

ſeu

$$\frac{dn(k+v)^2}{(n-1)b}+\frac{dn(k-v)^2}{(n-1)\beta}+\frac{4vdn}{n-1}+\frac{db}{bb}(k+v)^2+\frac{d\beta}{\beta\beta}(k-v)^2=0$$

Atque hinc elicitur:

$$d\beta=\frac{-\beta\beta db}{bb}\left(\frac{k+v}{k-v}\right)^2-\frac{\beta dn}{n-1}\left(1+\frac{\beta}{b}\left(\frac{k+v}{k-v}\right)^2+\frac{4\beta v}{(k-v)^2}\right)$$

Pro variabilitate autem ipſius k reperietur

$$\frac{dk}{(k+v)^2}=-\frac{db}{2nbb}-\frac{dn}{2n(n-1)b}-\frac{dn}{n(n-1)(k+v)}$$

Quare ſi ponatur $\frac{k-v}{k+v}=i$, ob $di=\frac{2vdk}{(k+v)^2}$ erit

$$di=\frac{-vdb}{nbb}-\frac{vdn}{n(n-1)b}-\frac{2vdn}{n(n-1)(k+v)}$$

ac ſi loco k numerus i introducatur, erit

$$d\beta=\frac{-\beta\beta db}{iibb}-\frac{bdn}{n-1}\left(1+\frac{\beta}{iib}+\frac{(1-i)^2\beta}{iiv}\right) \text{ et}$$

$$di=\frac{-vdb}{nbb}-\frac{vdn}{n(n-1)}\left(\frac{1}{b}+\frac{1-i}{v}\right).$$

Coroll. 1.

293. Inuenta aequatio differentialis etiam hac forma repraeſentari poteſt vt ſit

$$\frac{id\beta}{\beta\beta}+\frac{db}{ibb}=\frac{-dn}{n-1}\left(\frac{i}{\beta}+\frac{1}{ib}+\frac{(1-i)^2}{iv}\right)$$

ſiue

ſiue reſtituendo k

$$\frac{d\mathfrak{G}}{\mathfrak{G}\mathfrak{G}}\left(\frac{k-v}{k+v}\right)+\frac{db}{bb}\left(\frac{k+v}{k-v}\right)=\frac{-dn}{n-1}\left(\frac{1}{\mathfrak{G}}\left(\frac{k-v}{k+v}\right)+\frac{1}{b}\left(\frac{k+v}{k-v}\right)+\frac{4v}{kk-vv}\right)$$

vbi haec obſeruanda eſt analogia, vt quemadmodum ad b refertur $\frac{k+v}{k-v}$, ita ad $\mathfrak{G}$ referatur $\frac{k-v}{k+v}$.

Coroll. 2.

294. Si lentis huius craſſities euaneſcat, fit $v=0$, et $i=1$ vbi figura lentis non amplius in computum ingreditur ſed ſola diſtantia focalis, vnde variatio in loco imaginis G ita erit comparata, vt ſit

$$\frac{d\mathfrak{G}}{\mathfrak{G}\mathfrak{G}}+\frac{db}{bb}=\frac{-dn}{n-1}\left(\frac{1}{\mathfrak{G}}+\frac{1}{b}\right) \text{ ideoque}$$

$$d\mathfrak{G}=\frac{-\mathfrak{G}\mathfrak{G}}{bb}db-\frac{\mathfrak{G}\mathfrak{G}dn}{n-1}\left(\frac{1}{\mathfrak{G}}+\frac{1}{b}\right).$$

Coroll. 3.

295. Si caſum a radiis mediae naturae, ad quos formulae hactenus traditae ſunt accommodatae, ad radios rubros transferre velimus, poni oportet $dn=-\frac{1}{100}$, ſin autem ad radios violaceos $dn=+\frac{1}{100}$.

Problema 3.

296. Si radii ab obiecto E per lentes quotcunque transmittantur, determinare variationem in locis ſingularum imaginum, quae a diuerſa radiorum refrangibilitate proficiſcitur.

 Solutio.

Solutio.

Retineantur omnes denominationes, quibus in ſuperioribus capitibus ſumus vſi, ac ſint $a, \alpha, b, \beta, c, \gamma$ etc. diſtantiae determinatrices lentium pro radiis mediae naturae. Variata ergo ratione refractionis etiam hae diſtantiae variabuntur, quarum variationes differentialibus indicemus. Cum autem diſtantiae inter binas lentes maneant conſtantes, illae variationes ita erunt comparatae vt ſit

$$d\alpha + db = 0,\ d\beta + dc = 0,\ d\gamma + d.d = 0 \text{ etc.}$$

Cum iam diſtantia obiecti AE $= a$ ſit inuariabilis erit ex problemate primo ſi ponatur $\frac{k-v}{k+v} = i$

$$d\alpha = -db = \frac{-\alpha\alpha dn}{i(n-1)}\left(\frac{1}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right) = \frac{-\alpha\alpha dn}{n-1} \cdot \frac{k+v}{k-v}\left(\frac{k-v}{\alpha(k+v)} + \frac{k+v}{a(k-v)} + \frac{4v}{kk-vv}\right)$$

$$\text{et } di = \frac{-v\,dn}{n(n-1)}\left(\frac{1}{a} + \frac{1-i}{v}\right)$$

Deinde pro ſecunda lente, ad quam referuntur diſtantiae determinatrices b, et β cum arbitraria k' et craſſitie v', vnde fecimus $\frac{k'-v'}{k'+v'} = i'$, habebimus

$$d\beta = -dc = \frac{-\beta\beta db}{i'i'bb} - \frac{\beta\beta dn}{i'(n-1)}\left(\frac{i'}{\beta} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)$$

$$\text{et } di' = \frac{-v'db}{nbb} - \frac{v'dn}{n(n-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

Simili modo pro tertia lente, ad quam referuntur diſtantiae determinatrices c et γ cum arbitraria k'' et craſſitie v'' poſito $\frac{k''-v''}{k''+v''} = i''$ adipiſcemur:

$$d\gamma = -d.d = \frac{-\gamma\gamma dc}{i''i''cc} - \frac{\gamma\gamma dn}{i''(n-1)}\left(\frac{i''}{\gamma} + \frac{1}{i''c} + \frac{(1-i'')^2}{i''v''}\right)$$

$$\text{et } di'' = \frac{-v''dc}{ncc} - \frac{v''dn}{n(n-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

atque

atque vlterius progrediendo obtinebimus ſequentes formulas:

$$d\delta = -de = \frac{-\delta\delta\, dd}{i'''\, i'''\, dd} - \frac{\delta\delta\, dn}{i'''(n-1)}\left(\frac{i'''}{\delta} + \frac{1}{i'''d} + \frac{(1-i''')^2}{i'''\, v'''}\right)$$

$$\text{et } di'''' = \frac{-v'''\, dd}{n\, dd} - \frac{v'''\, dn}{n(n-1)}\left(\frac{1}{d} + \frac{1-i'''}{v'''}\right)$$

vnde haec differentialia facile ad quotcunque lentes extenduntur. Atque ſi hic ſucceſſiue valores differentialium dd, dc, db, iam ante definiti ſubſtituantur omnia haec differentialia tam diſtantiarum determinatricium, quam numerorum i, i', i'', i''' etc. per differentiale dn exprimentur.

Si ratio refractionis pro ſingulis lentibus ſit diuerſa pro iisque ordine exprimatur numeris, n, n', n'' etc. perſpicuum eſt, differentialia hic inuenta ſequenti modo expreſſum iri:

$$\text{I. } d\alpha = -db = \frac{-\alpha\alpha\, dn}{i(n-1)}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right)$$

$$di = \frac{-v\, dn}{n(n-1)}\left(\frac{1}{a} + \frac{1-i}{v}\right)$$

$$\text{II. } d\beta = -dc = \frac{-\beta\beta\, db}{i'\, i'.\, bb} - \frac{\beta\beta\, dn'}{i'(n'-1)}\left(\frac{i'}{\beta} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)$$

$$di' = \frac{-v'\, db}{n' \cdot bb} - \frac{v'\, dn'}{n'(n'-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

$$\text{III. } d\gamma = -dc = \frac{-\gamma\gamma\, dc}{i''.\, i''.\, cc} - \frac{\gamma\gamma\, dn''}{i''(n''-1)}\left(\frac{i''}{\gamma} + \frac{1}{i''c} + \frac{(1-i'')^2}{i''.\, v''}\right)$$

$$di'' = \frac{-v''\, dc}{n''\, cc} - \frac{v''\, dn''}{n''(n''-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

etc.

ex quibus formulis etiam mutationes ſingularum imaginum ac proinde etiam tandem vltimae imaginis facile

facile definiri poterunt, seu potius loco imaginum angulos, sub quibus eae oculo ad iustam distantiam l positae sint adpariturae, consideremus.

Pro vna lente $\frac{1}{i} \cdot \frac{\alpha}{a} \cdot \frac{z}{l}$

Pro duabus lentibus $\frac{1}{i.i'} \cdot \frac{\alpha \beta}{a b} \cdot \frac{z}{l}$

Pro tribus lentibus $\frac{1}{i.i'.i''} \cdot \frac{\alpha \beta \gamma}{a b c} \cdot \frac{z}{l}$

etc.

quatenus scilicet praeter distantias $a, \alpha; b, \beta; c, \gamma$ etc. etiam litterae i, i', i'', i''' etc. sunt variabiles.

Coroll. 1.

297. Hinc igitur per differentiationem definire licet, quanta mutatio in loco vltimae imaginis, quae obiectum visionis constituit, ob diuersam radiorum refrangibilitatem oriri debeat.

Coroll. 2.

298. Deinde cum etiam magnitudinem cuiusque imaginis supra per distantias determinatrices et numeros i, i', i'' etc. definiuerimus, pro magnitudine imaginis habetur $\frac{1}{i\, i'\, i''\, i'''} \cdot \frac{\alpha \beta \gamma \delta}{a b c d} \cdot \frac{z}{l}$ (189). simili modo mutatio assignari potest, quam magnitudo vltimae imaginis ob diuersam refrangibilitatem radiorum patietur.

Coroll. 3.

299. Cognita autem vtraque mutatione, quam vltima imago tam respectu loci, quam magnitudinis

subit,

ſubit, non difficulter colligetur, quanta confuſione ipſa viſio ob diuerſam radiorum refrangibilitatem perturbetur.

Coroll. 4.

300. Si craſſities lentium euaneſcat fiet

$$d\alpha = -db = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$d\mathfrak{b} = -dc = -\frac{\mathfrak{b}\mathfrak{b}db}{bb} - \frac{\mathfrak{b}\mathfrak{b}dn}{n-1}\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)$$

$$d\gamma = -dd = \frac{-\gamma\gamma dc}{cc} - \frac{\gamma\gamma dn}{n-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

$$d\delta = -de = \frac{-\delta\delta dd}{dd} - \frac{\delta\delta dn}{n-1}\left(\frac{1}{d}+\frac{1}{\delta}\right)$$

etc.

numeri autem i, i', i'' etc. abeunt in vnitatem nulliqne mutationi amplius ſunt obnoxii.

Si ergo craſſities lentium euaneſcat, pro diuerſa refractione ſingularum lentium formulae ſuperiores abibunt in ſequentes:

I. $d\alpha = -db = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$

II. $d\mathfrak{b} = -dc = -\frac{\mathfrak{b}\mathfrak{b}db}{bb} - \frac{\mathfrak{b}\mathfrak{b}dn'}{n'-1}\left(\frac{1}{\mathfrak{b}}+\frac{1}{b}\right)$

III. $d\gamma = -dd = \frac{-\gamma\gamma dc}{cc} - \frac{\gamma\gamma dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$

etc.

Ceterum per ſe manifeſtum eſt, quando §. 288 dn vel $+\frac{1}{100}$ vel $-\frac{1}{100}$ ſignificare dicitur, id tantum de illa vitri ſpecie, pro qua eſt refractio radiorum

 mé-

mediorum $n=\frac{31}{20}$ esse intelligendum; et pro aliis vitri speciebus differentialia dn', dn'', dn''', etc. haud mediocriter ab $\frac{1}{100}$ discrepare posse. Quanta autem futura sit haec diuersitas, optandum esset, vt ea potius experimentis, quam ex theoria quapiam definiretur.

Scholion

301. Cum igitur ob diuersam radiorum refrangibilitatem cuique imagini duplex alteratio inducatur, quarum altera eius magnitudinem, altera vero eius locum afficit, duplex inde confusio in visionem infertur. Si enim formulae in superioribus capitibus exhibitae ad radios mediae naturae restringantur, pro quibus est $n=\frac{31}{20}$, posito $dn=-\frac{1}{100}$, ex formulis hic traditis differentialibus locus et magnitudo imaginis a radiis rubris formatae definietur, posito autem $dn=+\frac{1}{100}$, locus et magnitudo imaginis violaceae declarabitur. Scilicet si Ii fuerit imago vltima visioni obiecta, quae a radiis mediae naturae formatur, per formulas modo inuentas, prout vel $dn=-\frac{1}{100}$ vel $dn=+\frac{1}{100}$ ponatur, definietur tam imago rubra $R\varrho$ quam violacea Vv; atque ex natura differentialium manifestum est cum interualla IR et IV inter se aequalia esse debere, tum etiam differentias $Ii-R\varrho$ et $Vv-Ii$, ita vt oculo series innumerabilium imaginum inter extremas $R\varrho$ et Vv sitarum simul cernenda offeratur, vnde eo maior confusio oriatur necesse est, quo maior fuerit differentia tam ratione loci quam magnitudinis. Quare haec confusio penitus tolleretur, si eiusmodi lentium dispositio definiri posset, vt tam interuallum RV,

Tab. III. Fig. 16.

quam

quam differentia inter imagines Rϱ et Vv ad nihilum redigeretur, quod vtrumque nisi simul praestari queat, confusionem perfecte tollere non licet Verumtamen etiamsi neutri harum conditionum satisfieri possit, tamen dabitur pro oculo eiusmodi locus O, vbi confusio minime sensibilis percipiatur, qui erit in concursu rectae $v\varrho$ productae cum axe: ibi enim omnes extremitates $\varrho\iota v$ communibus radiis cernentur, neque propterea extremitas obiecti colore tincta apparebit. Quare si simul punctum O conueniat cum loco oculi idoneo alia confusio non percipietur, nisi quae inde originem trahit, quod forte imagines extremae Rϱ et Vv nimis a distantia iusta discrepent, siquidem media Iι ad distantiam iustam ob oculo fuerit remota. Neque tamen hinc ora obiecti coloribus iridis cincta apparebit, cui confusionis speciei maxime est occurrendum; ideoque ea quae adhuc adfuerit confusio facile tolerari poterit, quae vero etiam omnino tolleretur si modo interuallum RV vel in nihilum redigi vel saltem satis paruum reddi posset. Hinc ergo intelligimus vitium illud, quo obiecta coloribus iridis circumdata saepe repraesentantur, non tam necessario cum instrumentis dioptricis esse coniunctum, vt nullo pacto ab iis separari queat, quamobrem eo magis operae erit pretium, vt investigemus quomodo haec instrumenta ab isto vitio liberari possint. Quae tota investigatio huc redit, vt determinetur punctum O, vbi recta per terminos imaginum v, ι, ϱ ducta cum axe concurrit hocque punctum cum loco oculi iam

ſupra definito conueniens reddatur, ſi quidem fieri poteſt: vnde perſpicitur locum oculi O hac proprietate praeditum eſſe oportere, vt angulus, ſub quo vltima imago cernitur ob variabilitatem numero n tributam nullam mutationem patiatur. Tum vero inſuper videndum erit, num interualla IR et IV vel ad nihilum reduci, vel minima reddi queant.

Problema 4.

Tab. III. Fig. 12. 302. Propoſita vnica lente definire locum oculi, vnde obiectum ſine margine colorato cernatur.

Solutio.

Sit obiecti $E\varepsilon$ ante lentem diſtantia $EA = a$, imago vero per radios mediae naturae in $F\zeta$ repraeſentetur, ponaturque $aF = \alpha$. Pro lente vero ſit eius craſſities $Aa = v$, et quantitas arbitraria $= k$, vnde capiatur $\frac{k-v}{k+v} = i$. Hinc poſito $E\varepsilon = z$ erit $F\zeta = \frac{1}{i}.\frac{\alpha}{a}z$ (86): quare ſi pro loco oculi ſtatuatur diſtantia $aO = O$, quae eſt fixa, erit $OF = \alpha - O$, et anguli $FO\zeta$ tangens $= \frac{1}{i}\frac{\alpha}{a}.\frac{z}{\alpha - O}$, quae formula ob diuerſam radiorum refrangibilitatem nullam mutationem ſubire debet. Inde autem quantitates α et i tantum variantur, dum reliquae manent conſtantes. Quare iſtius formulae differentiale logarithmicum nihilo aequale poſitum praebet hanc aequationem

$$-\frac{di}{i} + \frac{d\alpha}{\alpha} - \frac{d\alpha}{\alpha - O} = 0 \text{ ſeu } \frac{-di}{i} - \frac{O\,d\alpha}{\alpha(\alpha - O)} = 0$$

vbi

vbi ſi valores ſupra inuenti ſubſtituantur, prodit

$$\frac{v\,dn}{in(n-1)}\left(\frac{1}{a}+\frac{1-i}{v}\right)+\frac{O\,\alpha\,dn}{i(n-1)(\alpha-O)}\left(\frac{i}{\alpha}+\frac{1}{ia}+\frac{(1-i)^2}{iv}\right)=0$$

quae aequatio per $\frac{dn}{i(n-1)}$ diuiſa praebet:

$$\frac{v}{n\alpha}+\frac{1-i}{n}+\frac{O\alpha}{\alpha-O}\left(\frac{i}{\alpha}+\frac{1}{ia}+\frac{(1-i)^2}{iv}\right)=0$$

vnde locus oculi definiri poterit: qui ſi debeat conuenire cum ſupra inuento (238), vbi inuenimus $O=\frac{-i\alpha v}{n\alpha-iv}$, erit

$\alpha-O=\frac{n\alpha\alpha}{n\alpha-iv}$ et $\frac{O\alpha}{\alpha-O}=\frac{-iv}{n}$, hincque noſtra aequatio per n multiplicata abit in:

$$\frac{v}{a}+1-i-\frac{iiv}{\alpha}-\frac{v}{a}-(1-i)^2=0$$

ſeu $i-ii-\frac{iiv}{\alpha}=0$, ideoque $i=\frac{\alpha}{\alpha+v}=\frac{k-v}{k+v}$. Quamobrem quantitatem arbitrariam k ita definiri conueniet vt ſit $k=2\alpha+v$: et cum ſit $i=\frac{\alpha}{\alpha+v}$ pro loco oculi habebimus $O=\frac{-\alpha v}{n\alpha+(n-1)v}=\frac{-20\,\alpha v}{31\alpha+11v}$ ob $n=\frac{31}{20}$.

Quod ſi porro hinc variationem in loco imaginis deſideremus definiri oportet differentiale $d\alpha$, quod fiet;

$$d\alpha=\frac{-\alpha\alpha\,dn}{i(n-1)}\left(\frac{i}{\alpha}+\frac{1}{ia}+\frac{(1-i)^2}{iv}\right)$$

et pro i poſito valore $\frac{\alpha}{\alpha+v}$

$$d\alpha=\frac{-(\alpha+v)dn}{n-1}\left(1+\frac{\alpha+v}{a}\right)$$

qui valor ſi ad nihilum redigi poſſet, confuſio omnis a diuerſa radiorum refrangibilitate oriunda perfecte tolleretur.

Coroll. 1.

303. Pro lentis ergo conſtructione quantitas arbitraria k ita accipi debet, vt ſit $k = 2\alpha + v$; atque tum oculus in eo loco conſtitutus, vbi totum campum apparentem percipiat, ſimul nullam confuſionem a diuerſa radiorum indole ſentiet.

Coroll. 2.

304. Vt autem oculus ſimul imaginem in diſtantia iuſta aſpiciat, oportet ſit $\alpha - O = -l$, ideoque $l = \frac{-31\alpha(\alpha+v)}{31\alpha+11v}$. Vnde colligitur: $\alpha = \frac{-l}{2} - \frac{v}{2} - \sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$, hincque

$$O = \frac{1}{2}l - \frac{1}{2}v - \sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)} \text{ et } k = -l - 2\sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$$

Coroll. 3.

305. Poteſt vero inſuper effici, vt etiam $d\alpha$ euaneſcat, quod euenit ſi $a + \alpha + v = 0$, hoc eſt $a = \frac{1}{2}l - \frac{1}{2}v + \sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$. Verum cum hoc caſu ob $\alpha = -a - v$ imago in ipſum obiectum cadet, ita, vt radii nullam refractionem pati ſint cenſendi, viſio per lentem perinde erit comparata atque nudis oculis.

Coroll. 4.

306. Si craſſities lentis v plane euaneſcat, tam ob campum apparentem, quam diuerſam radiorum refrangibilitatem fit $O = 0$ hoc ergo caſu oculus lenti immediate applicatus nullam confuſionem ob diuerſam radiorum naturam percipiet. Dum ergo fuerit $\alpha = -l$ viſio erit diſtincta.

Scho-

Scholion.

307. Hic ſcilicet penitus mentem abſtrahimus a confuſione iam ſupra determinata, quae a lentium apertura oritur, ideoque aperturam primae lentis vt euaneſcentem ſpectamus. Eam igitur hic tantum confuſionis ſpeciem contemplamur, quae a diuerſa radiorum refrangibilitate originem ducit; quam plerumque tolli obſeruauimus, ſi angulus ad O inuariabilis reddatur; tum enim ora obiecti ſatis bene terminata conſpicietur, neque coloribus iridis cincta. Interim tamen adhuc aliqua confuſio ſentiri poterit inde oriunda, quod ſi imago media iuſtam ab oculo diſtantiam teneat, imaginum extremarum altera ſit nimis propinqua alter animis remota, verum ſi earum interuallum non ſit admodum magnum, confuſio haec parum erit ſenſibilis. Ita hic inuenimus, quod experientia ſatis comprobatur, ſi obiecta per vnicam lentem ſpectemus, ea margine colorato deſtituta apparere dummodo oculus immediate applicetur quod ſi quando ſecus euenire videatur, cauſa aperturae lentis ſine dubio erit tribuenda, cui conditioni rationes hic allegatae refragantur.

Problema 5.

308. Si inſtrumentum dioptricum duabus inſtructum ſit lentibus, defini re locum oculi, vnde obiectum ſine margine colorato videatur. Tab. III. Fig. 13.

Solutio

Solutio.

Posita obiecti distantia $AE = a$, sint pro radiis mediae naturae reliquae distantiae determinatrices $aF = \alpha$, $BF = b$ et $bG = \beta$ crassities vero lentium $Aa = v$, $Bb = v'$, et distantiae arbitrariae k et k', ponaturque $\frac{k-v}{k+v} = i$ et $\frac{k'-v'}{k'+v'} = i'$. His positis si magnitudo obiecti $E\varepsilon$ vocetur $= z$, erit imago $G\eta = \frac{1}{ii'}\frac{\alpha\beta}{ab}z$, vnde si oculi distantia ponatur $bO = O$,

ob $GO = \beta - O$, erit anguli $GO\eta$ tangens $\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}\cdot\frac{z}{\beta-O}$ cuius differentiale logarithmicum nihilo aequatum praebet,

$$-\frac{di}{i}-\frac{di'}{i'}+\frac{d\alpha}{\alpha}-\frac{db}{b}+\frac{d\beta}{\beta}-\frac{d\beta}{\beta-O}=0$$

quae valoribus supra (296) inuentis substitutis abit in:

$$\frac{v\,dn}{in(1-i)}\left(\frac{1}{a}+\frac{1-i}{v}\right)+\frac{v'\,db}{n'i'bb}+\frac{v'\,dn'}{i'n'(n'-1)}\left(\frac{1}{b}+\frac{1-i'}{v'}\right)$$
$$-\frac{db}{\alpha}-\frac{db}{b}+\frac{O}{\beta-O}\left(\frac{\beta\,db}{i'i'bb}+\frac{\beta\,dn'}{i'(n'-1)}\left(\frac{i'}{\beta}+\frac{1}{i'b}+\frac{(1-i')^2}{i'v'}\right)\right)=0$$

Verum conditio campi exigebat $O = \frac{\beta\beta}{\beta+\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z}$, existente

$$\beta = \left(\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{n'a\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{n'ab}\right)z, \quad (245), \text{ vnde fit}$$

$$\frac{O}{\beta-O} = \frac{\beta\beta}{\frac{1}{ii'}\cdot\frac{\alpha\beta\beta}{ab}z} = \frac{ii'ab}{\alpha\beta}\left(\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{na\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{n'ab}\right)$$

Nunc

Nunc vero eſt $db = \frac{\alpha\alpha dn}{i(n-1)}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right)$, quem valorem antequam ſubſtituamus, transformemus aequationem noſtram in hanc formam

$$\frac{dn}{n-1}\left(\frac{v}{ina} + \frac{1-i}{in} + \frac{v'}{i'nb} + \frac{1-i'}{i'n} + \frac{\mathfrak{b}O}{i'(\mathfrak{b}-O)}\left(\frac{i'}{\mathfrak{b}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)\right.$$
$$\left.+ db\left(\frac{v'}{ni'bb} - \frac{1}{\alpha} - \frac{1}{b} + \frac{\mathfrak{b}O}{i'i'bb(\mathfrak{b}-O)}\right)\right) = 0$$

vbi poſterius membrum abit in $-\frac{iv}{n\alpha\alpha}db$, tum vero erit

$$0 = \frac{dn}{n-1}\left\{\begin{array}{l} 1 + \frac{b}{\alpha} + \frac{i'i'b}{\mathfrak{b}}\left(1 + \frac{b}{\alpha}\right) + \frac{(1-i')^2 b(\alpha+b)}{\alpha v'} + \frac{2-i-i'}{n} \\ \frac{-iv}{n\alpha} - \frac{i'v'}{n\mathfrak{b}} - \frac{ibv}{n\alpha\alpha} - \frac{ii'i'bbv}{n\alpha\alpha\mathfrak{b}} - \frac{i(1-i')^2 bbv}{n\alpha\alpha v'} \end{array}\right\}$$

diſtinguendo n' ab n erit

$$\frac{dn}{n-1}\left(\frac{iv}{\alpha n} - \frac{(1-i)}{n}\right) = \frac{dn'}{n'-1}\left\{\begin{array}{l} 1 + \frac{b}{\alpha} + \frac{i'i'b}{\mathfrak{b}}\left(1 + \frac{b}{\alpha}\right) - \frac{ii'i'bbv}{n\alpha\alpha\mathfrak{b}} - \frac{i'v'}{n'\mathfrak{b}'} + \frac{1-i'}{n'} \\ -\frac{ibv}{n\alpha\alpha} + \frac{b(1-i')^2}{v'}\left(1 + \frac{b}{\alpha}\right) - \frac{i(1-i')^2 bbv}{n\alpha\alpha v'} \end{array}\right\}$$

cuius aequationis complicatio obſtat, quominus quicquam commode inde concludi poſſit.

Coroll. I.

309. Si ambae lentes craſſitie careant, vt ſit $v=0$, $v'=0$, et $i=i'=1$, aequatio differentialis prima eſt

$$\frac{d\alpha}{\alpha} - \frac{db}{b} - \frac{O\,d\mathfrak{b}}{\mathfrak{b}(\mathfrak{b}-O)} = 0$$

tum vero:

$$d\alpha = -db = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{\alpha} + \frac{1}{a}\right) \text{ et}$$

$$d\mathfrak{b} = \frac{-\mathfrak{b}\mathfrak{b}db}{bb} - \frac{\mathfrak{b}\mathfrak{b}dn}{n-1}\left(\frac{1}{\mathfrak{b}} + \frac{1}{b}\right) = \frac{-dn}{n-1}\left(\frac{\alpha\alpha\mathfrak{b}\mathfrak{b}}{bb}\left(\frac{1}{\alpha} + \frac{1}{a}\right) + \mathfrak{b}\mathfrak{b}\left(\frac{1}{\mathfrak{b}} + \frac{1}{b}\right)\right.$$

quibus valoribus ſubſtitutis et per $\frac{dn}{n-1}$ diuiſione facta fit

$$-\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)\left(\frac{1}{\alpha}+\frac{1}{b}\right)+\frac{O\,\beta}{\beta-O}\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\beta}+\frac{1}{b}\right)=0$$

Vnde ſi oculus lenti poſteriori immediate applicaretur vt eſſet

$O=0$, deberet eſſe $\left(\frac{1}{\alpha}+\frac{1}{a}\right)\left(\frac{1}{\alpha}+\frac{1}{b}\right)=0$.

Coroll. 2.

310. Verum in eadem hypotheſi, vt locus oculi congruat cum eo, quem viſio campi exigit, debet eſſe $O=\frac{b\beta(\alpha+b)}{b(\alpha+b)+\alpha\beta}$, vnde fit $\beta-O=\frac{\alpha\beta\beta}{b(\alpha+b)+\alpha\beta}$, ideoque $\frac{O}{\beta-O}=\frac{b(\alpha+b)}{\alpha\beta}=\frac{bb}{\beta}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$ quo valore ſubſtituto noſtra aequatio erit

$$0=\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(-\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)+bb\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\beta}+\frac{1}{b}\right)\right)$$

quae reducitur ad hanc formam

$$0=bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\beta}+\frac{1}{b}\right)$$

diſtinguendo n' ab n, membra a dn pendentia ſe deſtruunt et oritur

$$0=\frac{dn'}{n'-1}bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\beta}+\frac{1}{b}\right)$$

quod ita oſtenditur:

Cum aequatio prima differentialis praebeat:

$$\frac{d\alpha}{\alpha}-\frac{db}{b}-\frac{O\,d\beta}{\beta(\beta-O)}=0 \text{ ſiue}$$

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{O\,d\beta}{\beta(\beta-O)}=0$$

Cum

Cum igitur ex conditione campi apparentis fit

$\frac{O}{\mathfrak{E}-O} = -\frac{bb}{\mathfrak{E}}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$ erit

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right) - \frac{bb}{\mathfrak{E}^2}\left(\frac{1}{\alpha}+\frac{1}{b}\right)d\mathfrak{E} = 0$$

ideoque loco $d\mathfrak{E}$ fuum valorem fuftituendo

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right) - \frac{bb}{\mathfrak{E}^2}\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{\mathfrak{E}\mathfrak{E}d\alpha}{bb} - \frac{\mathfrak{E}^2 dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\mathfrak{E}}\right)\right) = 0$$

vbi membra, quae $d\alpha$ continent, manifefto fe deftruunt, et tota quaeftio ad hanc aequationem perducitur

$$\frac{dn'}{n'-1}\cdot bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\mathfrak{E}}+\frac{1}{b}\right) = 0.$$

Coroll. 3.

311. Quod fi ergo huic conditioni fatisfieri poffit, obiectum fine margine colorato apparebit: praeterea vero confufio penitus tolleretur, fi reddi liceret $d\mathfrak{E} = 0$, quod fit per hanc aequationem

$$\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\mathfrak{E}}+\frac{1}{b} = 0 \text{ fiue}$$

$$+\frac{dn}{n-1}\cdot\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)$$

$$+\frac{dn'}{n'-1}\left(\frac{1}{\mathfrak{E}}+\frac{1}{b}\right) = 0$$

$$\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)+bb\left(\frac{1}{\mathfrak{E}}+\frac{1}{b}\right) = 0.$$

Coroll. 4.

312. Priori autem aequationi fatisfieri nequit nifi fuerit vel $\alpha+b=0$ vel $\frac{1}{\mathfrak{E}}+\frac{1}{b}=0$. Illo cafu ambae lentes coniungerentur, vt vnicam conftituerent, hoc vero pofterioris diftantia focalis fieret infinita, qui

 cafus

casus iterum ad casum vnicae lentis rediret; foret enim $O = -\alpha - b$, ob $\mathfrak{E} = -b$, et oculus priori lenti immediate applicari deberet.

Scholion.

313. Si simili modo has inuestigationes ad plures lentes extendere vellemus, non neglecta earum crassitie in formulas plane inextricabiles delaberemur vnde vix quicquam concludi posset. Verum quia in omnibus fere instrumentis dioptricis, praecipue quae pluribus lentibus constant, iis tam exigua crassities tribui solet, vt sine notabili errore pro nihilo haberi possit, tam taediosae indagationi facile supersedere poterimus. Ad quod accedit, quod hic non de summo rigore geometrico agatur, sed contenti esse queamus, dummodo hanc confusionem satis prope cognouerimus: ex quo sufficiet in consideratione plurium lentium earum crassitiem prorsus neglexisse.

Supplementum IV.

Si ratio refractionis in singulis lentibus sit diuersa solutio sequenti modo absoluetur:

I. Prima aequatio differentialis prorsus se habebit, vt in problemate, ita, vt sit

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)+d\mathfrak{E}\left(\frac{1}{\mathfrak{E}}+\frac{1}{c}\right)-\frac{O\,d\gamma}{\gamma(\gamma-O)}=0$$

et quia etiam, vt ante, est

$$\frac{O}{\gamma-O}=\frac{bbcc}{\alpha\mathfrak{E}^2\gamma}\left(1+\frac{\alpha}{b}\right)+\frac{cc}{\mathfrak{E}\gamma}\left(1+\frac{\mathfrak{E}}{c}\right)$$

erit

erit noſtra aequatio

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(d\alpha-\frac{bbcc}{\mathfrak{G}^2\gamma^2}d\gamma\right)+\left(\frac{1}{\mathfrak{G}}+\frac{1}{c}\right)\left(d\mathfrak{G}-\frac{cc}{\gamma\gamma}d\gamma\right)=0.$$

II. Nunc autem ratio diuerſae refractionis eſt habenda; vnde in ſuperioribus additamentis inuenimus eſſe

$$d\alpha=-\frac{\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$d\mathfrak{G}=\frac{\mathfrak{G}\mathfrak{G}d\alpha}{bb}-\frac{\mathfrak{G}\mathfrak{G}dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\mathfrak{G}}\right)$$

$$d\gamma=\frac{\gamma\gamma d\mathfrak{G}}{cc}-\frac{\gamma\gamma dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

Hincque ergo fiet

$$d\alpha-\frac{bbcc}{\mathfrak{G}^2\gamma^2}d\gamma=d\alpha-\frac{bbd\mathfrak{G}}{\mathfrak{G}^2}+\frac{bbccdn''}{\mathfrak{G}^2(n''-1)}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

$$=\frac{bb.dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\mathfrak{G}}\right)+\frac{bbccdn''}{\mathfrak{G}^2(n''-1)}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

Deinde

$$d\mathfrak{G}-\frac{cc}{\gamma\gamma}d\gamma=\frac{ccdn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

III. Ex his ergo noſtra aequatio differentialis abibit in hanc formam:

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{bbdn'}{n'-1}\left[\frac{1}{b}+\frac{1}{\mathfrak{G}}\right]+\frac{bbc^2dn''}{\mathfrak{G}^2(n''-1)}\left[\frac{1}{c}+\frac{1}{\gamma}\right]\right)$$

$$+\left(\frac{1}{\mathfrak{G}}+\frac{1}{c}\right)\left(\frac{ccdn''}{n''-1}\left[\frac{1}{c}+\frac{1}{\gamma}\right]\right)$$

ſiue

$$\frac{dn'}{n'-1}.bb.\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\mathfrak{G}}+\frac{1}{b}\right)$$

$$+\frac{dn''}{n''-1}.cc\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\frac{bb}{\mathfrak{G}\mathfrak{G}}\left(\frac{1}{\alpha}+\frac{1}{b}\right)+\frac{1}{\mathfrak{G}}+\frac{1}{c}\right)=0$$

IV. Pro caſu autem illo ſingulari, quo oculus lenti vltimae immediate, debet adplicari, ob $O=0$, habebitur ſimpliciter haec aequatio

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)+d\beta\left(\frac{1}{\mathfrak{G}}+\frac{1}{c}\right)=0$$

 ſiue

ſiue, ſubſtituto valore $d\beta$

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)+\frac{\beta\beta\, d\alpha}{bb}\left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)$$
$$-\frac{\beta\beta\, dn'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)=0.$$
$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}+\tfrac{\beta\beta}{bb}\left[\tfrac{1}{\beta}+\tfrac{1}{c}\right]\right)$$
$$-\frac{\beta^2\, dn'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)=0,$$

ſeu tandem

$$+\frac{dn}{n-1}\alpha\alpha\left(\tfrac{1}{b}+\tfrac{1}{\alpha}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{b}+\tfrac{\beta^2}{b^2}\left[\tfrac{1}{\beta}+\tfrac{1}{c}\right]\right)$$
$$+\frac{dn'}{n'-1}\beta\beta\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)=0.$$

V. Hoc modo tantum margo coloratus tollitur; vt autem tota confuſio tollatur, quod fit ſi $d\gamma=0$, inſuper ſatisfieri debet huic aequationi

$$0=\frac{dn}{n-1}\cdot\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbcc}\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)$$
$$+\frac{dn'}{n'-1}\cdot\frac{\beta\beta\gamma\gamma}{cc}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)$$
$$+\frac{dn''}{n''-1}\cdot\gamma\gamma\cdot\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right).$$

Problema 6.

Tab. III. Fig. 14. 314. Si inſtrumentum dioptricum tribus conſtet lentibus, quarum craſſities euaneſcat, eam definire diſpoſitionem, vt oculus in eo loco, quem campus poſtulat, conſtitutus obiectum ſine margine colorato conſpiciat.

Solutio.

Poſita ergo diſtantia obiecti ante lentem obiectiuam $AE=a$, eiusque magnitudine $E\varepsilon=z$, vocentur

centur diſtantiae imaginum a radiis mediae naturae formatarum, vt hactenus

$aF = \alpha$, $BF = b$; $bG = \mathfrak{b}$, $CG = c$, et $cH = \gamma$.

eritque imago vltima $H\theta = \frac{\alpha\mathfrak{b}\gamma}{abc}z$, et poſita oculi poſt lentem vltimam diſtantia $cO = O$, erit $OH = \gamma - O$ et anguli $HO\theta$ tangens $= \frac{\alpha\mathfrak{b}\gamma}{abc} \cdot \frac{z}{\gamma - O}$, quae debet eſſe invariabilis. Poſito ergo eius differentiali logarithmico $= 0$ habebimus hanc aequationem:

$$\frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\mathfrak{b}}{\mathfrak{b}} - \frac{dc}{c} + \frac{d\gamma}{\gamma} - \frac{d\gamma}{\gamma - O} = 0$$

At et formulis ſupra (300) erutis habemus

$$d\alpha = -db = \frac{-dn}{n-1} \cdot \alpha\alpha\left(\frac{1}{a} + \frac{1}{\alpha}\right)$$

$$d\mathfrak{b} = -dc = -\frac{\mathfrak{b}\mathfrak{b}dn}{n-1}\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{a} + \frac{1}{\alpha}\right) + \frac{1}{b} + \frac{1}{\mathfrak{b}}\right)$$

$$d\gamma = -\frac{\gamma\gamma dn}{n-1}\left(\frac{\alpha\alpha\mathfrak{b}\mathfrak{b}}{bbcc}\left(\frac{1}{a} + \frac{1}{\alpha}\right) + \frac{\mathfrak{b}\mathfrak{b}}{cc}\left(\frac{1}{b} + \frac{1}{\mathfrak{b}}\right) + \frac{1}{c} + \frac{1}{\gamma}\right)$$

vnde noſtra aequatio erit

$$d\alpha\left(\frac{1}{\alpha} + \frac{1}{b}\right) + d\mathfrak{b}\left(\frac{1}{\mathfrak{b}} + \frac{1}{c}\right) - \frac{O\,d\gamma}{\gamma(\gamma - O)} = 0$$

Sed ob campum apparentem ſupra 254 inuenimus:

$$\mathfrak{B} = \mathfrak{b} = b\left(1 + \frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{C} = \mathfrak{c} = \frac{bc}{\mathfrak{b}}\left(1 + \frac{\alpha}{b}\right)\frac{z}{a} + \frac{\alpha c}{b}\left(1 + \frac{\mathfrak{b}}{c}\right)\frac{z}{a}$$

hincque

$O = \frac{\gamma\mathfrak{c}}{\mathfrak{c} + H\theta}$ et $\frac{O}{\gamma - O} = \frac{\mathfrak{c}}{H\theta}$ vnde fit ob

$H\theta = \frac{\alpha\mathfrak{b}\gamma}{bc} \cdot \frac{z}{a}$

$$\frac{O}{\gamma - O} = \frac{bbcc}{\alpha\mathfrak{b}\mathfrak{b}\gamma}\left(1 + \frac{\alpha}{b}\right) + \frac{cc}{\mathfrak{b}\gamma}\left(1 + \frac{\mathfrak{b}}{c}\right) \text{ ſeu}$$

$$\frac{O}{\gamma(\gamma - O)} = \frac{bbcc}{\mathfrak{b}\mathfrak{b}\gamma\gamma}\left(\frac{1}{\alpha} + \frac{1}{b}\right) + \frac{cc}{\gamma\gamma}\left(\frac{1}{\mathfrak{b}} + \frac{1}{c}\right)$$

Valo-

Valoribus iam his ſubſtitutis habebimus:

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(d\alpha-\frac{bbcc}{\beta\beta\gamma\gamma}d\gamma\right)+\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(d\beta-\frac{cc}{\gamma\gamma}d\gamma\right)=0$$

At eſt

$$d\alpha-\frac{bbcc}{\beta\beta\gamma\gamma}d\gamma=\frac{dn}{n-1}\left(bb\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{bbcc}{\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)$$

$$d\beta-\frac{cc}{\gamma\gamma}d\gamma=\frac{dn}{n-1}\left(cc\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)$$

Quare facta diuiſione per $\frac{dn}{n-1}$ nanciſcemur hanc aequationem:

$$bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{b}+\frac{1}{\beta}+\frac{cc}{\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)+cc\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(\frac{1}{c}+\frac{1}{\gamma}\right)=0$$

Quodſi vero oculus lenti poſtremae immediate applicetur, ſeu ſit $O=0$, conditio praeſcripta hanc poſtulat aequationem:

$$\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\beta\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{1}{b}+\frac{1}{\beta}\right)=0$$

Confuſio vero a diuerſa radiorum refrangibilitate oriunda perfecte tolletur ſi praeterea fuerit $d\gamma=0$ ſeu

$$\frac{\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{\beta\beta}{cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{1}{c}+\frac{1}{\gamma}=0.$$

Coroll. I.

315. Si rationes aperturarum lentium in computum ducantur, eaque pro lente ſecunda ponatur $=\pi$, pro tertia $=\pi'$ erit $bN'=\frac{\pi b\beta}{b+\beta}$ et $CM''=cN''=\frac{\pi' c\gamma}{c+\gamma}$. Tum vero poſito $\frac{z}{a}=\Phi$, erit $G\eta=\frac{\alpha\beta}{ab}a\Phi$, et $H\theta=\frac{\alpha\beta\gamma}{abc}\cdot a\Phi$. Hinc fiet $\frac{O}{\gamma-O}=\frac{cN''}{H\theta}=\frac{\pi' abcc}{\alpha\beta(c+\gamma)a\Phi}$ et $\frac{O}{\gamma(\gamma-O)}=\frac{\pi' abcc}{\alpha\beta\gamma(c+\gamma)}\cdot\frac{1}{a\Phi}$.

Coroll.

Coroll. 2.

316. Quodſi porro vt ſupra ponatur

$\alpha = Aa$, $\mathfrak{E} = Bb$, $\gamma = Cc$ erit $\frac{O}{\gamma(\gamma-\delta)} = \frac{\pi'}{ABC(C+1)a\Phi}$;
tum vero $bN' = \frac{\pi Bb}{B+1}$ et $CM'' = \frac{\pi' Cc}{C+1}$: ac $G\eta = ABa\Phi$;
Cum iam ſit

$bN' + G\eta : bG = CM'' - G\eta : CG$ erit $\frac{bN'+G\eta}{bG} = \frac{CM''-G\eta}{CG}$

ideoque

$\frac{1}{bG} + \frac{1}{CG} = \frac{CM''}{CG.G\eta} - \frac{bN'}{bG.G\eta}$ hoc eſt $\frac{1}{\mathfrak{E}} + \frac{1}{c} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$

ſimili vero modo eſt $\frac{1}{\alpha} + \frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}$.

Coroll. 3.

317. Per easdem ſubſtitutiones fit

$\frac{1}{a} + \frac{1}{\alpha} = \frac{A+1}{Aa}$; $\frac{1}{b} + \frac{1}{\mathfrak{E}} = \frac{B+1}{Bb}$; $\frac{1}{c} + \frac{1}{\gamma} = \frac{C+1}{Cc}$ etc.

hincque:

$\alpha a\,(\frac{1}{a} + \frac{1}{\alpha}) = A(A+1)a$

$\mathfrak{E}\mathfrak{E}\,(\frac{1}{b} + \frac{1}{\mathfrak{E}}) = B(B+1)b$

$\gamma\gamma(\frac{1}{c} + \frac{1}{\gamma}) = C(C+1)c$ ſicque porro.

Coroll. 4.

318. His ergo nouis denominationibus introductis differentialia ex §. 300 ita exprimentur

$d\alpha = -db = -\frac{dn}{n-1}.\ A(A+1)a$

$d\mathfrak{E} = -dc = -BBdb - \frac{dn}{n-1}.\ B(B+1)b$

$d\gamma = -d.d = -CCdc - \frac{dn}{n-1}.\ C(C+1)c$

quae formulae commodius in calculum introducentur.

 Quem-

Quemadmodum hic nouae formae adhibentur in ſequentibus vſurpandae; ita et pro caſu diuerſae refractionis ſequentibus formulis in poſterum vti licebit:

$$\frac{O}{\gamma(\gamma-O)} = \frac{\pi'}{ABC(C+1)a\Phi}$$

$$\frac{1}{\alpha}+\frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}$$

$$\frac{1}{\beta}+\frac{1}{c} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$$

$$\frac{1}{\gamma}+\frac{1}{d} = \frac{\pi'' D}{ABC(D+1)a\Phi} - \frac{\pi'}{ABC(C+1)a\Phi}$$

etc.

tum vero formulae differentiales erunt

$$d\alpha = -db = \frac{-dn}{n-1}.\ A(A+1)a$$

$$d\beta = -dc = B^2 d\alpha\ \frac{-dn'}{n'-1}.\ B(B+1)b$$

$$d\gamma = -dd = C^2 d\beta\ \frac{-dn''}{n''-1} C(C+1)c$$

$$d\delta = -de = D^2 d\gamma \frac{-dn'''}{n'''-1}.\ D(D+1)d$$

etc.

Atque ex his formulis vt margo coloratus euaneſcat, ſatisfieri debet huic aequationi

$$0 = \frac{dn'}{n'-1}.\ \frac{\pi b}{Aa\Phi} + \frac{dn''}{n''-1}.\ \frac{\pi' c}{ABa\Phi}$$

Vt autem haec confuſio penitus tollatur, fieri debet

$$d\gamma = -\frac{dn}{n-1}.\ A(A+1)B^2C^2a$$
$$-\frac{dn'}{n'-1}.\ B(B+1)C^2.b$$
$$-\frac{dn''}{n''-1}.\ C(C+1)c = 0.$$

ſiue

fiue

$$\frac{dn}{n-1}\cdot\frac{(A+1)a}{A} + \frac{dn'}{n'-1}\cdot\frac{(B+1)b}{AAB} = 0 + \frac{dn''}{n''-1}\cdot\frac{(C+1)c}{A^2B^2C}.$$

Problema 7.

319. Si inftrumentum dioptricum quatuor conftet lentibus quarum craffities negligi queat, eam definire difpofitionem vt oculus in eo loco quem campus apparens poftulat, conftitutus obiectum fine confufione a diuerfa radiorum indole oriunda confpiciat. Tab. III. Fig. 15.

Solutio.

Pofita diftantia obiecti ante inftrumentum $AE = a$, eiusque magnitudine $E\varepsilon = z$, vocentur diftantiae imaginum a radiis mediae naturae formatarum vt hactenus.

$aF = \alpha$, $BF = b$; $bG = \beta$, $CG = c$; $cH = \gamma$; $DG = d$; $dI = \delta$

Tum vero ponamus praeterea

$\alpha = Aa$, $\beta = Bb$; $\gamma = Cc$; $\delta = Dd$

atque introducantur rationes aperturarum pro fingulis lentibus poft primam, quae fint π pro fecunda QQ, π' pro tertia RR et π'' pro quarta SS, fumto pro campo $\frac{z}{a} = \Phi$. His pofitis erunt imagines $F\zeta = Aa\Phi$; $G\eta = ABa\Phi$; $H\theta = ABCa\Phi$ et $I\iota = ABCDa\Phi = \frac{\alpha\beta\gamma\delta}{abcd}z$.

 Iam

Iam posita oculi distantia post instrumentum $dO = O$, vt sit $OI = \delta - O$ erit anguli $IO\iota$ tangens $\frac{\alpha\beta\gamma\delta}{abcd}\cdot\frac{z}{\delta - O}$, quae cum ob diuersam radiorum refrangibilitatem immutata manere debeat, differentiata dabit hanc aequationem

$$\frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\beta}{\beta} - \frac{dc}{c} + \frac{d\gamma}{\gamma} - \frac{dd}{d} - \frac{O\,d\delta}{\delta(\delta - O)} = 0$$

quae ob $db = -d\alpha$, $dc = -d\beta$ et $dd = -d\gamma$ abit in hanc

$$d\alpha\left(\tfrac{1}{\alpha} + \tfrac{1}{b}\right) + d\beta\left(\tfrac{1}{\beta} + \tfrac{1}{c}\right) + d\gamma\left(\tfrac{1}{\gamma} + \tfrac{1}{d}\right) - \frac{O\,d\delta}{\delta(\delta - O)} = 0$$

Verum modo ante notauimus fore (318)

$$d\alpha = \frac{-dn}{n-1}\cdot A(A+1)a$$

$$d\beta = BB\,d\alpha - \frac{dn}{n-1}\cdot B(B+1)b$$

$$d\gamma = CC\,d\beta - \frac{dn}{n-1}\cdot C(C+1)c$$

$$d\delta = DD\,d\gamma - \frac{dn}{n-1}\cdot D(D+1)d$$

vbi pro b, c, d, valores §. 266 assignati substitui debent. Porro vero iam animaduertimus esse (316)

$$\frac{1}{\alpha} + \frac{1}{b} = \frac{1}{Aa\Phi}\cdot\frac{\pi B}{B+1}$$

$$\frac{1}{\beta} + \frac{1}{c} = \frac{1}{ABa\Phi}\left(\frac{\pi' C}{C+1} - \frac{\pi}{B+1}\right)$$

$$\frac{1}{\gamma} + \frac{1}{d} = \frac{1}{ABCa\Phi}\left(\frac{\pi'' D}{D+1} - \frac{\pi'}{C+1}\right)$$

atque

$$\frac{O}{\delta(\delta - O)} = \frac{\pi''}{ABCD(D+1)a\Phi}.$$

Quod

Quodsi iam priores valores in posterioribus successiue substituantur, habebimus

$$d\alpha = \frac{-dn}{n-1} \cdot A(A+1)a$$

$$d\beta = \frac{-dn}{n-1} B(AB(A+1)a+(B+1)b$$

$$d\gamma = \frac{-dn}{n-1} \cdot C(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$d\delta = \frac{-dn}{n-1} \cdot D(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)$$

His igitur valoribus in aequatione differentiali substitutis, et diuisione per $\frac{dn}{(n-1)z\Phi}$ facta aequatio nostra secundum singula membra distributa erit

$$-\frac{\pi B}{(B+1)} \cdot (A+1)a$$

$$-\frac{1}{A}\left(\frac{\pi' C}{C+1}-\frac{\pi}{B+1}\right)(AB(A+1)a+(B+1)b)$$

$$-\frac{1}{AB}\left(\frac{\pi'' D}{D+1}-\frac{\pi'}{C+1}\right)(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$+\frac{1}{ABC} \cdot \frac{\pi''}{D+1}(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)=0$$

Terna autem priora membra negatiua sola collecta praebent

$$-\frac{BCD(A+1)\pi''}{D+1}a-\frac{CD(B+1)\pi''}{A(D+1)}b-\frac{D(C+1)\pi''}{AB(D+1)}c$$

$$+\frac{\pi}{A}b+\frac{\pi'}{AB}c$$

quibus si quartum addatur prodit

$$\frac{\pi}{A}b+\frac{\pi'}{AB}c+\frac{\pi''}{ABC}d$$

Iam duo hic casus considerari oportet, alterum quo punctum O post lentem vltimam cadit, alterum vero

vero, quo ob diſtantiam O negatiuam oculus lenti vltimae immediate applicatur. Pro priori caſu, quo diſtantia $dO = O$ prodit poſitiua habetur iſta aequatio ſi quidem in aequatione modo inuenta pro b, c, d valores §. 266 inuenti ſubſtituantur

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

Pro caſu poſteriori, quo diſtantia O euaneſcens aſſumitur facta multiplicatione per $\frac{D+1}{a\ \Phi}$.

$$\frac{BCD(A+1)\pi''}{\Phi}+\frac{CD(B+1)^2\pi''}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\pi''}{C\pi'-(C+1)(\pi-\Phi)}$$

$$=\frac{(B+1)(D+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)(D+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}.$$

Hoc modo efficitur, vt obiectum ſine margine colorato appareat at omnis confuſio tolletur ſi praeterea fuerit

$$AB^2C^2D(A+1)+\frac{ABC^2D(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{ABCD(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{ABC(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

ſeu per ABC diuidendo:

$$BCD(A+1)+\frac{CD(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

$$BCD(A+1)+\frac{CD(B+1)b}{Aa}+\frac{D(C+1)c}{ABa}+\frac{(D+1)\ d}{A\ B\ C\ a}=0$$

Coroll. I.

320. Si vtrique conditioni ſatisfieri poteſt, vt nec locus nec magnitudo imaginis vllam mutationem patiatur locus oculi non amplius in computum ingreditur ſed imago, vndecunque cernatur, ab omni confuſione prorſus libera erit.

Coroll. 2.

321. Vt ergo hunc summum perfectionis gradum consequamur, binis his aequationibus satisfieri oportet.

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

et

$$A+1+\frac{(B+1)^2\Phi}{B(B\pi-(B+1)\Phi)}+\frac{(C+1)^2\Phi}{BC(C\pi'-(C+1)(\pi-\Phi))}+\frac{(D+1)^2\Phi}{BCD(D\pi''-(D+1)(\pi'-\pi+\Phi))}=0.$$

Coroll. 3.

322. Si porro vt supra fecimus ponamus

$$\frac{A}{A+1}=\mathfrak{A},\ \frac{B}{B+1}=\mathfrak{B},\ \frac{C}{C+1}=\mathfrak{C} \text{ et } \frac{D}{D+1}=\mathfrak{D}$$

istae aequationes sequenti modo simplicius exprimentur:

$$+\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0 \text{ et}$$

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

hic prima membra multiplicari debent per $\frac{dn}{n-1}$

secunda per $\frac{dn'}{n'-1}$

tertia per $\frac{dn''}{n''-1}$

etc.

Coroll. 4.

323. Sin autem non liceat has ambas aequationes adimplere, curandum est, vt saltem priori, qua magnitudo apparens a confusione liberatur, satisfiat

fiat hoc enim modo obiectum sine margine colorato apparebit vbi quidem ad duos casus respici conuenit prout distantia oculi dO prodierit positiua vel negatiua.

Scholion.

324. Introducendis ergo rationibus aperturarum quibus supra iam commode sumus vsi ad campum apparentem definiendum aequationes etiam istae confusionem a diuersa radiorum refrangibilitate oriundam tollentes satis fiunt simplices, vt sine molestia tractari queant; si quidem crassities lentium negligatur. Hoc ergo modo problema generale, quicunque fuerit lentium numerus, expediri conueniet.

Supplementum V.

Si ratio refractionis in singulis lentibus discrepet, prodit primo quidem eadem aequatio differentialis

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)+d\beta\left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)+d\gamma\left(\tfrac{1}{\gamma}+\tfrac{1}{d}\right)\frac{-O\,d\delta}{\delta(\delta-O)}=0$$

in qua ergo casu $O=0$ vltimum membrum abiici debet.

I) Si autem O habeat valorem positiuum, erit, vt ante,

$$\frac{O}{\delta(\delta-O)}=\frac{\pi''}{ABCD(D+1)\alpha\Phi}$$

atque etiam valores $\frac{1}{\alpha}+\frac{1}{b}$; $\frac{1}{\beta}+\frac{1}{c}$; $\frac{1}{\gamma}+\frac{1}{d}$ manent iidem, vt ante.

At

At vero ob diuerſitatem refractionis habebimus

$$d\alpha = \frac{-dn}{n-1} . A(A+1)a$$

$$d\beta = BBd\alpha - \frac{dn'}{n'-1} . B(B+1)b$$

$$d\gamma = CCd\beta - \frac{dn''}{n''-1} C. (C+1)c$$

$$d\delta = DDd\gamma - \frac{dn'''}{n'''-1} . D(D+1)d$$

Hinc ergo aequatio noſtra ſucceſſiue ita formetur:

$$\frac{-Od\delta}{\delta(\delta-O)} = \frac{-\pi''d\delta}{ABCD(D+1)a\Phi}$$

$$= \frac{-\pi''Dd\gamma}{ABC(D+1)a\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi''.d}{ABC.a\Phi}$$

Addatur $d\gamma\left(\frac{1}{\gamma}+\frac{1}{d}\right) = \frac{d\gamma.\pi''D}{ABC(D+1)a\Phi} - \frac{\pi'd\gamma}{ABC(C+1)a\Phi}$

et prodibit $\frac{-\pi'd\gamma}{ABC(C+1)a\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi''.d}{ABC.a\Phi}$

et pro $d\gamma$ ſubſtituto valore

$$\frac{-\pi'Cd\beta}{AB(C+1)a\Phi} + \frac{dn''}{n''-1} . \frac{\pi'c}{ABa\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi''.d}{ABC.a\Phi}$$

Iam addatur

$$d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right) = \frac{\pi'Cd\beta}{AB(C+1)a\Phi} - \frac{\pi d\beta}{AB(B+1)a\Phi}$$

proditque

$$\frac{-\pi d\beta}{AB(B+1)a\Phi} + \frac{dn''}{n''-1} . \frac{\pi'c}{AB.a\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi''d}{ABCa\Phi}$$

et ſubſtituto valore ipſius $d\beta$

$$\frac{-\pi Bd\alpha}{A(B+1)a\Phi} + \frac{dn'}{n'-1} . \frac{\pi.b}{A.a\Phi} + \frac{dn''}{n''-1} . \frac{\pi'c}{ABa\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi''d}{ABC.a\Phi}$$

denique addatur $d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right) = \frac{\pi Bd\alpha}{A(B+1)a\Phi}$

ac aequatio quaeſita, qua margo coloratus euaneſcit, erit

$$\frac{dn'}{n'-1} . \frac{\pi b}{Aa\Phi} + \frac{dn''}{n''-1} . \frac{\pi'c}{ABa\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi''d}{ABCa\Phi} = 0$$

II) Sin autem O habeat valorem negatiuum: tunc ſumi debet O=o et pro eodem ſcopo aequatio ita formabitur:

Cum ſit

$$d\gamma\left(\frac{1}{\gamma}+\frac{1}{d}\right)=\frac{\pi'' D\, d\gamma}{ABC(D+1)a\Phi}-\frac{\pi' d\gamma}{ABC(C+1)a\Phi}$$

ſubſtituto valore $d\gamma$ fiet

$$CC\,d\beta\left(\frac{\pi''\ D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$
$$-\frac{d\,n''}{n''-1}.\ C(C+1)c\left(\frac{\pi''\ D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$

Addatur

$$d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)=\frac{\pi' C d\beta}{AB(C+1)a\Phi}-\frac{\pi\, d\beta}{AB(B+1)a\Phi}$$

eritque

$$d\beta\left(\frac{CD\pi''}{AB(D+1)a\Phi}-\frac{\pi}{AB(B+1)a\Phi}\right)-\frac{d n''}{n''-1}(C+1)c\left(\frac{\pi''\ D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

et ſubſtituto valore ipſius $d\beta$

$$d\alpha\left(\frac{BCD\pi''.}{A(D+1\,a\Phi}-\frac{B\pi}{A(B+1)a\Phi}\right)$$
$$-\frac{d n'}{n'-1}\left(\frac{(B+1)CD.b\pi''}{A(D+1)a\Phi}-\frac{\pi b}{A.\,a\Phi}\right)-\frac{d n''}{n''-1}(C+1)c\left(\frac{\pi''\ D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

Addatur

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)=\frac{\pi B d\alpha}{A(B+1)a\Phi}$$

eritque

$$-\frac{dn}{n-1}.\left(\frac{A+1)BCDa,\pi''}{(D+1)a\Phi}\right.$$
$$-\frac{dn'}{n'-1}\left(\frac{(B+1)CDb\pi''-(D+1)b\pi}{A(D+1)a\Phi}\right)$$
$$-\frac{dn''}{n''-1}\left(\frac{(C+1)Dc\pi''-(D+\ )c\pi'}{AB(D+1)a\Phi}\right)$$

Vnde

Vnde pro caſu $O=o$ aequatio, qua margo coloratus deſtruitur, erit

$$o=\frac{adn}{n-1}\cdot(A+1)BCD\pi''$$
$$+\frac{bdn'}{n'-1}\cdot\left(\frac{(B+1)CD\pi''-(D+1)\pi}{A}\right)$$
$$+\frac{cdn''}{n''-1}\left(\frac{(C+1)D\pi''-(D+1)\pi'}{AB}\right)$$

III) Vt autem praeterea omnis confuſio tollatur, inſuper reddi oportet $d\delta=o$; vnde haec aequatio naſcitur

$$o=\begin{cases}\frac{adn}{n-1}\cdot A(A+1)B^2C^2D^2\\ +\frac{bdn'}{n'-1}\cdot B(B+1)C^2D^2\\ +\frac{cdn''}{n''-1}\cdot C(C+1)D^2\\ +\frac{d.dn'''}{n'''-1}\cdot D(D+1)\end{cases}$$

quae per $A^2B^2C^2D^2$ diuiſa dat

$$o=\begin{cases}\frac{adn}{n-1}\cdot\frac{A+1}{A}\\ +\frac{bdn'}{n'-1}\cdot\frac{B+1}{A^2B}\\ +\frac{cdn''}{n''-1}\cdot\frac{C+1}{A^2B^2C}\\ +\frac{d.dn'''}{n'''-1}\cdot\frac{D+1}{A^2B^2C^2D}\end{cases}$$

Circa hanc autem aequationem inprimis notandum eſt, ſi omnes lentes pari facultate refringente ſint praeditae, ei ſatisfieri haud poſſe; ex quo haec aequatio proprie pertinet ad caſum, quo diuerſae refractiones locum habent.

Problema 8.

325. Si instrumentum dioptricum ex quotcunque lentibus sit compositum, quarum crassitiem negligere liceat, eam determinare dispositionem, vt oculus in eo loco, quem campus apparens postulat positus nullam confusionem sentiat.

Solutio.

Sit obiecti ante lentem primam distantia $AE = a$, eiusque magnitudo $E\varepsilon = z$, quae quidem conspici queat, et statuatur $\frac{z}{a} = \Phi$. Deinde sint distantiae imaginum a radiis mediae naturae formatarum vt supra:

$$AE = a;\ BF = b;\ CG = c;\ DH = d;\ EI = e$$
$$aF = \alpha;\ bG = \beta;\ cH = \gamma;\ dI = \delta;\ eK = \varepsilon$$
etc.

ac vocemus breuitatis gratia

$$\alpha = Aa,\ \beta = Bb,\ \gamma = Cc,\ \delta = Dd,\ \varepsilon = Ee \text{ etc.}$$

tum vero etiam

$$\frac{A}{A+1} = \mathfrak{A};\ \frac{B}{B+1} = \mathfrak{B};\ \frac{C}{C+1} = \mathfrak{C};\ \frac{D}{D+1} = \mathfrak{D},\ \frac{E}{E+1} = \mathfrak{E} \text{ etc.}$$

Iam apertura primae lentis PP vt euanescente considerata sit ratio aperturae pro reliquis lentibus

$$QQ = \pi;\ RR = \pi';\ SS = \pi'';\ TT = \pi''' \text{ etc.}$$

His

His positis supra vidimus (266) fore praeter $\alpha = A a$

$b = \frac{A a \Phi}{\mathfrak{B}\pi - \Phi}$ $\qquad \beta = \frac{A B a \Phi}{\mathfrak{B}\pi - \Phi}$

$c = \frac{A B a \Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ $\qquad \gamma = \frac{A B C a \Phi}{\mathfrak{C}\pi' - \pi + \Phi}$

$d = \frac{A B C a \Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$ $\qquad \delta = \frac{A B C D a \Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$

$e = \frac{A B C D a \Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$ $\qquad \varepsilon = \frac{A B C D E a \Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$

etc. etc.

Deinde singularum imaginum magnitudo ita se habebit.

$F\zeta = A a \Phi$; $G\eta = A B a \Phi$; $H\theta = A B C a \Phi$; $I\iota = A B C D a \Phi$

etc.

et ipsi aperturarum semidiametri:

Lentis secundae $QQ = \frac{A \mathfrak{B} a \Phi}{\mathfrak{B}\pi - \Phi}\pi$

Lentis tertiae $RR = \frac{A B \mathfrak{C} a \Phi}{\mathfrak{C}\pi' - \pi + \Phi}\pi'$

Lentis quartae $SS = \frac{A B C \mathfrak{D} a \Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\pi''$

Lentis quintae $TT = \frac{A B C D \mathfrak{E} a \Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}.\pi'''$

Mutata iam refractionis lege $n : 1$ infinite parum, distantiae $\alpha, b, \beta, c; \gamma, d, \delta$ etc. tales mutationes recipiunt.

$$d\alpha = -\frac{dn}{n-1}. A a (A+1)$$

$$d\beta = \frac{-dn}{n-1}. A B a \left((A+1)B + \frac{(B+1)\Phi}{\mathfrak{B}\pi - \Phi}\right)$$

$$d\gamma = \frac{-dn}{n-1}. A B C a \left((A+1)BC + \frac{(B+1)C\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(C+1)\Phi}{\mathfrak{C}\pi' - \pi + \Phi}\right)$$

$$d\delta = \frac{-dn}{n-1}. A B C D a \left((A+1)BCD + \frac{(B+1)CD\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(C+1)D\Phi}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{(D+1)\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\right)$$

etc.

Porro vero habetur:

$$\frac{1}{\alpha}+\frac{1}{b}=\frac{1}{Aa\Phi}\cdot\mathfrak{B}\pi$$

$$\frac{1}{\mathfrak{S}}+\frac{1}{c}=\frac{1}{ABa\Phi}\left(\mathfrak{C}\pi'-\frac{\mathfrak{B}\pi}{B}\right)$$

$$\frac{1}{\gamma}+\frac{1}{d}=\frac{1}{ABCa\Phi}\left(\mathfrak{D}\pi''-\frac{\mathfrak{C}\pi'}{C}\right)$$

$$\frac{1}{\delta}+\frac{1}{e}=\frac{1}{ABCDa\Phi}\left(\mathfrak{E}\pi'''-\frac{\mathfrak{D}\pi''}{D}\right)$$

etc.

His pofitis pro quolibet lentium numero feorfim formulas quaefito fatisfacientes expediamus: pofita diftantia oculi poft lentem vltimam $=O$.

I. Pro vnica Lente

Habetur haec aequatio differentialis $\frac{-O\,d\alpha}{\alpha(\alpha-O)}=0$, pro quo cafu tam magnitudo imaginis quam eius locus manebit inuariatus fi fuerit $d\alpha=0$, hoc eft $A(A+1)=0$, vnde deberet effe vel $A=0$ vel $A=-1$: quorum prius vifio non admittit pofterius autem lentem tollit. Tum vero ob campum effe debet $O=0$.

II. Pro duabus lentibus

Habetur haec aequatio differentialis, qua margo coloratus tollitur:

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{O\,d\mathfrak{S}}{\mathfrak{S}(\mathfrak{S}-O)}=0$$

at ob campum apparentem eft $\frac{O}{\mathfrak{S}(\mathfrak{S}-O)}=\frac{1}{BB}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$, ita vt habeamus:

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{d\mathfrak{S}}{BB}\left(\frac{1}{\alpha}+\frac{1}{b}\right)=0$$

Verum

Verum eſt $d\mathfrak{E} = \mathrm{BB}\,d\alpha - \frac{dn}{n-1}\,.\,\mathrm{AB}a.\,\frac{(\mathrm{B}+1)\Phi}{\mathfrak{B}\pi-\Phi}$

Quare ſi O habeat valorem poſitiuum erit ob

$$\mathfrak{B}(\mathrm{B}+1)=\mathrm{B},\ \frac{\pi}{\mathfrak{B}\pi-\Phi}=0$$

Sin autem valor O prodeat negatiuus, quo caſu capitur $O=0$ erit $\frac{(\mathrm{A}+1)\mathfrak{B}\pi}{\Phi}=0.$

Omnis autem confuſio penitus tolletur, ſi inſuper fuerit

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{\mathrm{A}\mathfrak{B}(\mathfrak{B}\pi-\Phi)}=0$$

III. Pro tribus Lentibus

Si calculum eodem modo proſequamur, obiectum ſine margine colorato conſpicietur:

1. Si ex campo apparente diſtantia O prodeat poſitiua hanc aequationem adimplendo:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}=0.$$

2. Si ob diſtantiam O prodeuntem negatiuam capiatur $O=0$ huic aequationi erit ſatisfaciendum:

$$\frac{\pi'}{\mathfrak{A}\Phi}+\frac{\pi'}{\mathrm{A}\mathfrak{B}(\mathfrak{B}\pi-\Phi)}=\frac{\pi}{\mathrm{ABC}(\mathfrak{B}\pi-\Phi)}$$

Omnis autem confuſio penitus tolletur ſi fuerit inſuper

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{\mathrm{A}\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{\mathrm{AB}\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=0.$$

IV. Pro quatuor Lentibus

Vt obiectum ſine margine colorato ſpectetur:

1. Si

1. Si ex campo apparente distantia O prodeat positiua huic aequationi erit satisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0$$

2. Sin autem capiatur $O=0$, huic

$$\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=\frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi'-\pi+\Phi)}$$

Omnis vero confusio penitus tolletur si fuerit praeterea

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

V. Pro quinque Lentibus

Vt obiectum tantum sine margine colorato conspiciatur

1. Si ex campo apparente distantia O prodeat positiua, huic aequationi erit satisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=0$$

2. Sin autem capiatur $O=0$ huic

$$\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

$$\frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

Omnis autem confusio penitus tolletur, si praeterea satisfiat huic aequationi:

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}=0$$

Atque hinc manifesta est progressio ad maiorem lentium numerum.

Coroll.

Coroll. 1.

326. In casu ergo vnicae lentis licet quidem obiectum a margine colorato liberare neque vero confusionem penitus tollere. Casu autem duarum lentium ne margo quidem coloratus tolli potest, siquidem oculus in eo loco, quem campi apparentis conditio postulat, teneatur.

Coroll. 2.

327. Quodsi vero plures duabus habeantur lentes sufficiens quantitatum numerus adest, quarum determinatione non solum margo coloratus deleri, sed etiam forte omnis confusio penitus auferri posse videtur praecipue si lentium numerus ternarium superet.

Scholion.

328. Quod ergo incommodum a diuersa radiorum natura oriundum adeo graue vel summo Neutono est visum, vt instrumenta dioptrica nullo modo ab eo liberari posse sit arbitratus, id quidem saltem quod ad marginem coloratum attinet ad quem Neutonus inprimis spectabat, iam satis feliciter tolli posse certum est; ita vt saltem ob hanc causam non opus sit ad Telescopia Catoptrica confugere. Hoc autem vitio sublato si praeterea alterum confusionis fontem obstruamus lentes scilicet nullam confusionem parientes adhibendo, nullum est dubium quin instrumenta dioptrica ad summum perfectionis gradum euehi queant.

Quae igitur hactenus particulatim circa ſingulas horum inſtrumentorum affectiones propoſuimus, ea colligi conueniet, vnde in capite ſequente praecepta generalia pro omnium inſtrumentorum dioptricorum conſtructione tradere eſt viſum.

Supplementum VI.

Ex iis, quae ante ſunt adiecta, poterimus etiam problematis ſolutionem pro caſu exhibere, quo ſingulae lentes peculiari refractione ſunt praeditae, vbi quidem tantum poſtremae aequationes pro confuſione vitanda mutationem quandam poſtulant; interim tamen etiam priores formulas, quibus locus oculi, quem campus apparens requirit, diſtinctius repraeſentemus.

I. *Diſtantia Oculi poſt vltimam lentem* pro quouis lentium numero ſe habebit, vt ſequitur.

Num. lentium	O id eſt, diſtantia oculi poſt lentem vltimam
I.	o
II.	$\frac{A\mathfrak{B}a\pi\Phi}{(\pi-\Phi)(\mathfrak{B}\pi-\Phi)}$ ſeu $\frac{\mathfrak{B}b\pi}{\pi-\Phi}$
III.	$\frac{AB.\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$ ſeu $\frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi}$
IV.	$\frac{ABC\mathfrak{D}.a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$ ſeu $\frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi}$
V.	$\frac{ABCD\mathfrak{E}.a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$ ſeu $\frac{\mathfrak{E}e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi}$

etc.

II.

II. Si valor ipſius O ſit poſitiuus, ad marginem coloratum tollendum ſequentes aequationes ſunt adimplendae:

Num. lentium	
I.	$0 = 0$
II.	$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi}$
III.	$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi}$
IV.	$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi} + \frac{ddn'''}{n'''-1} \cdot \frac{\pi''}{ABCa\Phi}$

etc.

III. Si valor ipſius O prodeat negatiuus, quo caſu capi debet $O = 0$, ad marginem coloratum tollendum ſequentes aequationes ſunt adimplendae.

Num. lentium	
I.	$0 = 0$
II.	$0 = \frac{adn}{n-1} \cdot (A+1)B\pi$
III.	$0 = \frac{adn}{n-1}(A+1)BC\pi' + \frac{bdn'}{n'-1} \cdot \frac{(B+1)C\pi' - (C+1)\pi}{A}$
IV.	$0 = \frac{adn}{n-1} \cdot (A+1)BCD\pi'' + \frac{bdn'}{n'-1}\left(\frac{(B+1)CD\pi'' - (D+1)\pi}{A}\right) + \frac{cdn''}{n''-1}\left(\frac{(C+1)D\pi'' - (D+1)\pi'}{AB}\right)$

etc.

IV. Vt autem insuper omnis confusio huius generis tollatur, sequentes aequationes sunt adimplendae:

Num. lentium	
I.	$0 = \frac{adn}{n-1} \cdot \frac{A+1}{A}$
II.	$0 = \frac{adn}{n-1} \cdot \frac{A+1}{A} + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2 B}$
III.	$0 = \frac{adn}{n-1} \cdot \frac{A+1}{A} + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2 B} + \frac{cdn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C}$
IV.	$0 = \frac{adn}{n-1} \cdot \frac{A+1}{A} + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2 B} + \frac{cdn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C} + \frac{ddn'''}{n'''-1} \cdot \frac{D+1}{A^2 B^2 C^2 D}$.

etc.

Quarum formularum ordo hinc distinctius perspicitur, quam in problemate.

CAPVT

CAPVT VII.

DE CONSTRVCTIONE INSTRVMENTORVM DIOPTRICORVM IN GENERE.

Problema I.

329.

Si instrumentum dioptricum vnica constet lente crassitiei cuiuscunque, PP, definire omnia momenta ad visionem pertinentia. Tab. III. Fig. 12.

Solutio.

Quod primo ad ipsius lentis structuram attinet ponatur obiecti $E\varepsilon$ ante eam distantia $AE = a$, imaginisque $F\zeta$ post eam proiectae $aF = \alpha$; tum vero lentis crassities $Aa = v$ et quantitas arbitraria $= k$, vnde capiatur $\frac{k-v}{k+v} = i$. Hinc facies lentis ita erunt formatae, vt sit existente $n = \frac{31}{20}$, Conf. §. 68.

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei posterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$.

Ponatur porro semidiameter obiecti conspicui $E\varepsilon = z$, et semidiameter aperturae in facie anteriori $AM = x$, in facie autem posteriori semidiameter aperturae non

ſit minor quam ix. Denique oculi poſt lentem diſtantia vocetur $aO = O$, cuius diſtantia iuſta ſit $= l$. His poſitis ſequentia momenta perpendi oportet.

1. Debet eſſe $O = \alpha + l$ vt diſtantia imaginis viſae naturae oculi conueniat.

2. Conſideranda venit multiplicatio, quae ex ratione definitur, quam diameter obiecti viſus tenet ad eiusdem diametrum viſum ſi nudo oculo in diſtantia data $= h$ ſpectaretur. Quodſi ergo hic exponens multiplicationis ponatur $= m$ erit

$$m = \frac{1}{i} \cdot \frac{\alpha h}{a l} \text{ pro ſitu inuerſo}$$

3. Gradus claritatis determinatur ſemidiametro coni luminoſi, qui a quouis obiecti puncto in oculum immittitur, qui ſi ponatur $= y$ erit

$$y = i\,l.\frac{x}{\alpha} = \frac{hx}{ma}$$

4. Confuſio inquinans viſionem menſuratur ſemidiametro apparente circuli, qui nudo oculo aeque magnus cernitur, ac ſingula obiecti puncta per lentem ſpectata. Hanc menſuram vocaui (194) ſemidiametrum confuſionis, ad quem definiendum ſi ponatur:

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a} + \frac{2}{k+v}\right)\left(\frac{1}{ia} + \frac{2}{k-v}\right)^2 + \left(\frac{n}{\alpha} - \frac{2}{k-v}\right)\left(\frac{i}{\alpha} - \frac{2}{k+v}\right)^2\right)$$

erit ſemidiameter confuſionis $= \frac{1}{4} i . \frac{\alpha}{l} x^3 P = \frac{1}{4} i^2 . \frac{ma}{h} x^3 . P$.

5. Vt oculus maximum campum apparentem percipiat, debet eſſe $O = \frac{-i\alpha v}{n\alpha - iv}$

6. Quae

6. Quae diſtantia ſi fuerit poſitiua, ſemidiameter campi ſeu obiecti conſpicui $E\varepsilon = z$, ita pendet ab apertura faciei poſterioris, vt poſito huius ſemidiametro $= a$ ſit $z = \frac{na}{v}. a$.

7. Sin autem diſtantia illa pro O aſſignata prodierit negatiua, oculum lenti immediate applicari conueniet, vt ſit $O = o$; tum vero pro a poſito ſemidiametro pupillae ω erit pro campo apparente $z = \frac{na}{v}\omega$.

8. Vt obiectum ſine margine colorato appareat, exiſtente diſtantia O ante inuenta poſitiua, debet eſſe $i = \frac{\alpha}{\alpha + v}$. ſeu $k = 2\alpha + v$.

9. Sin autem ob illam diſtantiam negatiuam prodeuntem capiatur $O = o$, vt margo coloratus euitetur, oportet eſſe $\frac{v}{a} + 1 - i = o$, vnde fit $k = -2a - v$.

10. Omnis denique confuſio a diuerſa radiorum refrangibilitate oriunda prorſus tolletur, ſi inſuper fieri poſſet $(\alpha + v)(a + \alpha + v) = o$

Coroll. 1.

330. Si exponens multiplicationis m cum gradu claritatis y proponatur, erit $my = \frac{bx}{a}$. quae proprietas ad lentium numerum quantumuis magnum patet. Cum ergo $\frac{b}{a}$ ſit quantitas data, erit x vt my; tum vero y vt $\frac{x}{m}$, ac m vt $\frac{x}{y}$

Coroll. 2.

331. Tam maior ergo multiplicatio quam maior claritatis gradus poſtulat maiorem aperturam. Verum

Verum aucto x confusio augetur in ratione triplicata, siquidem quantitas P maneat eadem: quare si x ex confusione etiamnum tolerabili determinetur, simul quantitas my determinatur.

Coroll. 3.

332. Vt igitur tam multiplicationem m quam claritatem y salua confusione maxime augere liceat, quantitatem arbitrariam k ita definiri conueniet, vt litterae P minimus valor concilietur.

Cum autem posuerimus $\frac{k-v}{k+v} = i$, valor ipsius P fiet minimus, huic aequationi satisfaciendo:

$$2i^4\left(\frac{n}{\alpha}+\frac{1}{v}\right)\left(\frac{1}{\alpha}+\frac{1}{v}\right)^2-\frac{2ni^3}{\alpha v}\left(\frac{1}{\alpha}+\frac{1}{v}\right)-\frac{i^3}{v}\left(\frac{1}{\alpha}+\frac{2}{v}\right)^2$$
$$+\frac{2ni}{av}\left(\frac{1}{a}+\frac{1}{v}\right)+\frac{i}{v}\left(\frac{1}{a}+\frac{2}{v}\right)^2-2\left(\frac{n}{a}+\frac{1}{v}\right)\left(\frac{1}{a}+\frac{1}{v}\right)^2=0$$

vti §. 45 inuenimus.

Coroll. 4.

333. Si loco i valor substituatur suus $\frac{k-v}{k+v}$, aequatio transibit in hanc formam:

$$+\frac{2n}{a^3}(k+v)^4+\frac{(2n+1)}{aav}(k+v)^3(k+3v)+\frac{4(n+2)}{av}(k+v)^3$$
$$-\frac{2n}{\alpha^3}(k-v)^4-\frac{(2n+1)}{\alpha\alpha v}(k-v)^3(k-3v)+\frac{4(n+2)}{\alpha v}(k-v)^3+32k=0$$

quae per v multiplicata etiam ad casum accommodari potest quo est $v=0$: tum autem reperitur $k=\frac{4(n+2)a\alpha}{(2n+1)(a-\alpha)}$ et

$$P=\frac{n}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left((4n-1)\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{a\alpha}\right)$$

Scholion

Scholion

334. Hic non generatim omnem visionem, quae fit per vnicam lentem, considero, sed tantum eam, qua maximus campus apparens conspicitur; quamobrem locum oculi ita definiui, vt ipsi maximus campus adferatur. Sin autem minore campo velimus esse contenti oculus quoque alibi post lentem constitutus obiecta distincte cernere poterit, quemadmodum per lentes satis amplas, quae tabularum vitrearum nomine sunt notae, fieri solet. Hunc autem casum quoniam per se facile expeditur, atque in instrumentis dioptricis magnitudo campi potissimum spectatur, hic non attingo.

Problema 2.

335. Si instrumentum dioptricum duabus constet lentibus cuiuscunque crassitiei definire omnia momenta ad visionem pertinentia. Tab. III. Fig. 13.

Solutio.

Obiecto constituto in $E\epsilon$, eius imagines proiiciantur per istas lentes in $F\zeta$ et $G\eta$, ac ponantur quantitates vtramque lentem determinantes

$AE = a$, $aF = \alpha$, crassities $Aa = v$ et distantia arbitraria $= k$

$BF = b$, $bG = \beta$, crassities $Bb = v'$ et distantia arbitraria $= k'$

 ponatur-

ponaturque breuitatis gratia $\frac{k-v}{k+v}=i$ et $\frac{k'-v'}{k'+v'}=i'$. Hinc exiſtente $n=\frac{31}{20}$, conſtructio vtriusque lentis ita ſe habebit,

Radius faciei	anterioris	poſterioris
Pro Lente PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Pro Lente QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$

Sit porro ſemidiameter aperturae lentis primae PP in facie anteriori $=x$, in facie autem poſteriori maior ſit quam ix. Tum vero lentis ſecundae QQ ſemidiameter aperturae in facie anteriori maior eſſe debet quam $i.\frac{bx}{\alpha}$, in poſteriori vero maior quam $ii'.\frac{bx}{\alpha}$. Deinde ſit ſemidiameter obiecti conſpicui $E\varepsilon=z$, diſtantia oculi $bO=O$, eiusque diſtantia iuſta $=l$. His poſitis ad ſequentia momenta erit attendendum.

1. Vt oculus imaginem $G\eta$ in diſtantia iuſta conſpiciat, oportet eſſe $O=\beta+l$.

2. Poſita diſtantia $=h$ ad quam multiplicatio referatur, ſit exponens multiplicationis $=m$, ac ſupra inuenimus eſſe oportere $m=\frac{1}{ii'}.\frac{\alpha\beta h}{abl}$ pro ſitu erecto.

3. Denotante y gradum claritatis fiet

$$y=ii'l.\frac{bx}{\alpha\beta} \text{ ideoque } my=\frac{hx}{a}$$

4. Pro confuſione poſito brevitatis ergo

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}+\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$

$$Q=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

erit-

eritque

$$\text{semid. confusionis} = \tfrac{1}{4} ii' . \frac{\mathfrak{E}}{I} . \frac{b}{\alpha} x^3 \left(\frac{1}{i'i'} . \frac{\alpha\alpha}{bb} P + ii . \frac{bb}{\alpha\alpha} Q \right).$$

5. Pro campo apparente definiendo, qui pendet ab apertura singularum facierum lentium, si ponamus

Semidiametrum aperturae

Pro lente PP $\begin{cases} \text{faciei anterioris} = \mathfrak{A} = x \\ \text{faciei posterioris} = \mathfrak{a} \end{cases}$

Pro lente QQ $\begin{cases} \text{faciei anterioris} = \mathfrak{B} \\ \text{faciei posterioris} = \mathfrak{b} \end{cases}$

habebimus sequentes aequationes

$$\mathfrak{a} = \frac{v}{na} z$$

$$\mathfrak{B} = \left(\frac{1}{i} . \frac{\alpha + b}{a} - \frac{bv}{na\alpha} \right) z$$

$$\mathfrak{b} = \left(\frac{i'}{i} . \frac{\alpha + b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} . \frac{\alpha v'}{nab} \right) z$$

ex quibus tribus aequationibus valor ipsius z minimus praebet semidiametrum campi apparentis.

6. Quem campum vt oculus reuera perspiciat, eius locus ita debet sumi, vt sit

$$O = \frac{\frac{i'}{i} \frac{\alpha + b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \frac{\alpha v'}{nab}}{\frac{i'}{i} . \frac{\alpha + b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} . \frac{\alpha v'}{nab} + \frac{1}{ii'} . \frac{\alpha\mathfrak{E}}{ab}} . \mathfrak{E}$$

si quidem haec distantia fuerit positiua.

7. Sin autem haec distantia prodierit negatiua, oculum lenti QQ immediate applicari conuenit, vt sit $O = 0$; tum vero pro campo apparente inue-

 niendo

niendo in aequationibus n^o. 5 allatis loco β ſcribatur ſemidiameter pupillae ω; atque ex iisdem valor ipſius z minimus erutus dabit ſemidiametrum campi apparentis.

8. Obiectum porro ſine margine colorato cernetur, exiſtente diſtantia O n^o 6 inuenta poſitiua, ſi huic aequationi ſatisfiat:

$$o = 1 + \frac{b}{\alpha} + \frac{i'i'b}{\beta}\left(1 + \frac{b}{\alpha}\right) + \frac{(1-i')^2 b(\alpha + b)}{\alpha v'} + \frac{2-i-i'}{n}$$
$$- \frac{iv}{n\alpha} - \frac{i'v'}{n\beta} - \frac{ibv}{n\alpha\alpha} - \frac{ii'i'bbv}{n\alpha\alpha\beta} - \frac{i(1-i')^2 bbv}{n\alpha\alpha v'}$$

9. Sin autem capiatur diſtantia $O = o$, obiectum margine colorato carebit, ſi fuerit

$$o = \frac{1-i}{in} + \frac{1-i'}{i'n} + \frac{v}{ina} + \frac{v'}{inb} + \alpha\left(1 + \frac{\alpha}{i\alpha} + \frac{(1-i)^2\alpha}{i.i.v}\right)\left(\frac{v'}{ni'bb} - \frac{1}{\alpha} - \frac{1}{b}\right)$$

10. Omnis autem confuſio penitus tolletur, quae quidem a diuerſa radiorum refrangibilitate proficiſcitur, ſi huic aequationi ſatisfacere licuerit $d\beta = o$, ſeu huic

$$o = \frac{\alpha\alpha}{ii'bb}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right) + \frac{i'}{\beta} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}.$$

Coroll. 1.

336. Quia ambarum lentium diſtantia aB neceſſario eſt poſitiua, oportet ſit $\alpha + b > o$. Tum vero aperturae ita debent eſſe comparatae vt ſit $a > ix$; $\mathfrak{B} > i.\frac{bx}{\alpha}$ et $\beta > ii'.\frac{bx}{\alpha}$.

Coroll. 2.

337. Caſu igitur, quo ob valorem ipſius O prodeuntem negatiuum diſtantia $O = o$ aſſumitur, quia

quia tum $b = \omega$ ſtatuitur, etiam eſſe debet $\omega > ii'\frac{bx}{\alpha}$ ſeu $x < \frac{\alpha\omega}{ii'b}$. Scilicet hoc caſu inutile eſſet primam aperturam maiorem ſumere, quia radii non in oculum ingrederentur.

Scholion.

338. Si ſimili modo inſtrumenta dioptrica pluribus lentibus inſtructa proſequi vellemus, in calculos tantopere intricatos delaberemur, vt ex iis nihil fere concludere liceret. Oritur autem haec calculi complicatio a craſſitie lentium, qua neglecta omnia fiunt multo concinniora. Quare ſi pluribus lentibus vti velimus, craſſitiem tam paruam aſſumamus, vt ſine errore pro nihilo haberi poſſit, id quod in praxi etiam ſedulo obſeruari ſolet. Praeterea etiam campi apparentis locique oculi determinatio non capit rigorem geometricum, parumque refert ſi in ea aliquantum aberretur. Similis quoque ratio eſt conditionum, quibus effectus a diuerſa radiorum refrangibilitate oriundus tollitur; ſufficiet enim iis proxime ſatisfeciſſe, cum perfecta huius confuſionis deſtructione ſperari quidem poſſit. Quocirca in ſequentibus, vbi inſtrumenta pluribus lentibus inſtructa euoluemus, craſſitiem lentium in calculo penitus praetermittamus: in quo negotio ne eadem toties repetere opus habeamus, problema generale praemittamus in quo omnia momenta generatim tantum exponamus eaque deinceps pro quouis lentium numero accurate deſcribamus.

Problema 5.

339. Si inſtrumentum dioptricum compoſitum ſit ex lentibus quotcunque quarum craſſitiem negligere liceat, elementa eius conſtructionem continentia exponere, ex quibus deinceps regulae dirigentes conſtructionem ipſam ſtabiliri poſſint.

Solutio.

Obiecto in E conſtituto ponantur diſtantiae determinatrices ſingularum lentium vna cum numeris arbitrariis ad ſingulas pertinentibus vt ſequitur.

Pro Lente prima $EA = a$; $aF = \alpha$; num: arb: $= \lambda$

Pro Lente ſecunda $FB = b$; $bG = \beta$; num: arb: $= \lambda'$

Pro Lente tertia $GC = c$; $cH = \gamma$; num: arb: $= \lambda''$

Pro Lente quarta $HD = d$; $dI = \delta$; num: arb: $= \lambda'''$

Pro Lente quinta $IE = e$; $eK = \varepsilon$; num: arb: $= \lambda''''$

Pro Lente ſexta $KF = f$; $fL = \zeta$; num: arb: $= \lambda'''''$

etc.

ex quibus elementis quomodo ſingulae lentes debeant formari ſupra eſt expoſitum.

Nunc autem porro ponatur breuitatis gratia:

$\alpha = \mathfrak{A}a$; $\beta = \mathfrak{B}b$; $\gamma = \mathfrak{C}c$: $\delta = \mathfrak{D}d$; $\varepsilon = \mathfrak{E}e$; $\zeta = \mathfrak{F}f$

etc.

tum

tum vero ſtatuatur etiam

$$\frac{A}{A+1}=\mathfrak{A};\ \frac{B}{B+1}=\mathfrak{B};\ \frac{C}{C+1}=\mathfrak{C};\ \frac{D}{D+1}=\mathfrak{D};\ \frac{E}{E+1}=\mathfrak{E};\ \frac{F}{F+1}=\mathfrak{F}$$

etc.

ita vt ſint diſtantiae focales lentium:

$$\mathfrak{A}a;\ \mathfrak{B}b;\ \mathfrak{C}c;\ \mathfrak{D}d;\ \mathfrak{E}e;\ \mathfrak{F}f \quad \text{etc.}$$

Nunc ſit ſemidiameter aperturae primae lentis obiectiuae $=x$, pro reliquis vero lentibus ratio aperturarum litteris π, π', π'', π''', π'''' etc. exponatur; ita vt ſemidiameter aperturae, cuiusque maior accipi debeat quam ſecundum has rationes: ſcilicet capi oportebit:

	Semid: apert:
Lentis Secundae	$> \pi\mathfrak{B}b$
Lentis tertiae	$> \pi'\mathfrak{C}c$
Lentis quartae	$> \pi''\mathfrak{D}d$
Lentis quintae	$> \pi'''\mathfrak{E}e$
Lentis ſextae	$> \pi''''\mathfrak{F}f$ etc.

Quod ſi iam ſemidiameter ſpatii in obiecto conſpicui vocetur $=z$, fiatque $\frac{z}{a}=\Phi$, vt ſit $z=a\Phi$, atque hinc diſtantiae determinatrices ita exprimentur, vt ſit primo $\alpha=Aa$ tum vero:

$$b=\frac{Aa\Phi}{\mathfrak{B}\pi-\Phi};\qquad \beta=\frac{ABa\Phi}{\mathfrak{B}\pi-\Phi}$$

$$c=\frac{ABa\Phi}{\mathfrak{C}\pi'-\pi+\Phi};\qquad \gamma=\frac{ABCa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$

$$d=\frac{ABCa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi};\qquad \delta=\frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$$

$$e=\frac{ABCDa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi};\qquad \varepsilon=\frac{ABCDEa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}$$

$$f=\frac{ABCDEa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi};\qquad \zeta=\frac{ABCDEFa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi} \quad \text{etc.}$$

Circa

Circa has expreſſiones primum notandum eſt, aggregata $\alpha+b$; $\beta+c$; $\gamma+d$; $\delta+e$; $\varepsilon+f$ etc. eſſe oportere poſitiua quippe quibus lentium interualla exprimuntur. Erit nempe

Interuallum
Lentium

$$\text{I et II} = \frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi} > 0$$

$$\text{II et III} = \frac{AB a\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)} > 0$$

$$\text{III et IV} = \frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} > 0$$

$$\text{IV et V} = \frac{ABCDa\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)} > 0$$

$$\text{V et VI} = \frac{ABCDEa\Phi(\mathfrak{F}\pi''''-(1-\mathfrak{E})\pi''')}{(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)} > 0$$

etc.

Praeterea vt lentes ſequentes omnes radios a prima exceptos transmittant, debet eſſe

Semid: aperturae

Lentis ſecundae $> \frac{\Phi}{\mathfrak{B}\pi-\Phi}x$

Lentis tertiae $> \frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}x$

Lentis quartae $> \frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}x$

Lentis quintae $> \frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}x$

Lentis ſextae $> \frac{\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}x$

etc.

Denique

Denique ex §. 214 posito $\mu = 0,938191$ et $\nu = 0,232692$ erunt litterarum maiuscularum P, Q, R, etc. valores

$$P = \frac{\mu}{A^3 a^3}(A+1)(\lambda(A+1)^2 + \nu A)$$

$$Q = \frac{\mu}{B^3 b^3}(B+1)(\lambda'(B+1)^2 + \nu B)$$

$$R = \frac{\mu}{C^3 c^3}(C+1)(\lambda''(C+1)^2 + \nu C)$$

$$S = \frac{\mu}{D^3 d^3}(D+1)(\lambda'''(D+1)^2 + \nu D)$$

$$T = \frac{\mu}{E^3 e^3}(E+1)(\lambda''''(E+1)^2 + \nu E)$$

$$V = \frac{\mu}{F^3 f^3}(F+1)(\lambda'''''(F+1)^2 + \nu F)$$

etc.

His ita constitutis denominationibus, sit O distantia oculi post lentem vltimam, cuius distantia iusta ponatur $= l$. Deinde statuatur exponens multiplicationis $= m$, relatus ad distantiam h, ita vt diameter obiecti per instrumentum visi m vicibus maior cernatur, quam si idem obiectum a nudo oculo in distantia $= h$ aspiceretur: multiplicationi autem adiungi conuenit situm indicando, vtrum obiectum situ erecto an inuerso sit appariturum. Porro gradus claritatis y denotet semidiametrum coni luminosi, qui a quouis obiecti puncto in oculum transmittitur posito semidiametro pupillae $= \omega$. Semidiametrum denique confusionis voco semidiametrum apparentem circulorum qui in nudo oculo aeque magni depinguntur, atque singula obiecti puncta per instrumentum in oculo.

Coroll. 1.

340. Quia pro apertura ſingularum lentium geminos limites inuenimus, ſemidiameter aperturae cuiusque conuenientiſſime aggregato amborum limitum aequalis aſſumitur, vel ſaltem non minor. Vterque autem limes etſi forte alter prodeat negatiuus, affirmatiue accipi debet.

Coroll. 2.

341. Hinc ergo ſequentes conſequimur formulas pro ſingularum lentium aperturis.

Semidiam: apert:

Lentis ſecundae $= \frac{\mathfrak{A}\mathfrak{B} a \pi \pm x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$

Lentis tertiae $= \frac{\mathrm{AB}\mathfrak{C} a \pi' \pm x}{\mathfrak{C}\pi' - \pi + \Phi} \cdot \Phi$

Lentis quartae $= \frac{\mathrm{ABC}\mathfrak{D} a \pi'' \pm x}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \cdot \Phi$

Lentis quintae $= \frac{\mathrm{ABCD}\mathfrak{E} a \pi''' \pm x}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \cdot \Phi$

Lentis ſextae $= \frac{\mathrm{ABCDE}\mathfrak{F} a \pi'''' \pm x}{\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi} \cdot \Phi$

etc.

Coroll. 3.

342. Ex diſtantiis binis determinatricibus cum numero arbitrario quaelibet lens facile conſtruitur; id quod pro lente prima ex ſuperioribus repetamus. Nempe ſi ſit $\lambda > 1$ lens ſimplex ſatisfaciet cuius conſtructio poſito breuitatis gratia:

$\varrho = 0,190781$; $\sigma = 1,627401$ et $\tau = 0,905130$

ita

ita ſe habet:

$$\text{radius faciei} \quad \text{anterioris} \;\; \frac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\surd(\lambda-1)}; \quad \text{poſterioris} \;\; \frac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\surd(\lambda-1)}$$

ſeu numeris ſubſtitutis ſi ſit $\lambda = 1+v$

$$\text{radius faciei}\begin{cases}\text{anterioris} = \dfrac{a\alpha}{+0,190781\alpha+1,627401a\pm0,905133(a+\alpha)\surd v}\\[2ex] \text{poſterioris} = \dfrac{a\alpha}{+1,627401\alpha+0,190781a\mp0,905133(a+\alpha)\surd v}\end{cases}$$

Coroll. 4.

343. Sin autem ſit $\lambda < 1$, ſed tamen $> 0,191827$ lens eſt duplicanda ſeu ex duabus ſimplicibus componenda, quarum conſtructio, ſi ponatur $\lambda = 0,191827+v$ ita ſe habet

Pro Lente radius faciei

$$\text{priori}\begin{cases}\text{anterioris} = \dfrac{a\alpha}{-0,622915\alpha+0,813700a\pm0,905133(a+\alpha)\surd v}\\[2ex] \text{poſterioris} = \dfrac{a\alpha}{+1,532010\alpha+0,095390a\mp0,905133(a+\alpha)\surd v}\end{cases}$$

$$\text{poſteriori}\begin{cases}\text{anterioris} = \dfrac{a\alpha}{+0,095390\alpha+1,532010a\pm0,905133(a+\alpha)\surd v}\\[2ex] \text{poſterioris} = \dfrac{a\alpha}{+0,813700\alpha-0,622915a\mp0,905133(a+\alpha)\surd v}\end{cases}$$

Coroll. 5.

344. At ſi ſit $\lambda = 0,042165+v$, lente vtendum eſt triplicata ita conſtruenda:

Pro lente radius

$$\text{Priori}\left\{\begin{aligned}&\text{anterioris} = \frac{a\alpha}{-0,854153\alpha+0,542467a\pm0,905133(a+\alpha)\sqrt{v}}\\&\text{posterioris} = \frac{a\alpha}{+1,500214\alpha+0,063594a\mp0,905133(a+\alpha)\sqrt{v}}\end{aligned}\right.$$

$$\text{Media}\left\{\begin{aligned}&\text{anterioris} = \frac{a\alpha}{-0,415280\alpha+1,021340a\pm0,905133(a+\alpha)\sqrt{v}}\\&\text{posterioris} = \frac{a\alpha}{+1,021340\alpha-0,415280a\mp0,905133(a+\alpha)\sqrt{v}}\end{aligned}\right.$$

$$\text{Posteriori}\left\{\begin{aligned}&\text{anterioris} = \frac{a\alpha}{+0,063594\alpha+1,500214a\pm0,905133(a+\alpha)\sqrt{v}}\\&\text{posterioris} = \frac{a\alpha}{+0,542467\alpha-0,854153a\mp0,905133(a+\alpha)\sqrt{v}}\end{aligned}\right.$$

Coroll. 6.

345. Si denique sit $\lambda = -0,010216+v$ lens facienda est quadruplicata ita construenda:

Pro lente radius faciei

$$\text{Prima}\left\{\begin{aligned}&\text{anterioris} = \frac{a\alpha}{-1,029770\alpha+0,406850a\pm0,905133(a+\alpha)\sqrt{v}}\\&\text{posterioris} = \frac{a\alpha}{+1,484315\alpha+0,047655a\mp0,905133(a+\alpha)\sqrt{v}}\end{aligned}\right.$$

$$\text{Secunda}\left\{\begin{aligned}&\text{anterioris} = \frac{a\alpha}{-0,670615\alpha+0,766005a\pm0,905133(a+\alpha)\sqrt{v}}\\&\text{posterioris} = \frac{a\alpha}{+1,125160\alpha-0,311460a\mp0,905133(a+\alpha)\sqrt{v}}\end{aligned}\right.$$

$$\text{Tertia}\left\{\begin{aligned}&\text{anterioris} = \frac{a\alpha}{-0,311460\alpha+1,125160a\pm0,905133(a+\alpha)\sqrt{v}}\\&\text{posterioris} = \frac{a\alpha}{+0,766005\alpha-0,670615a\mp0,905133(a+\alpha)\sqrt{v}}\end{aligned}\right.$$

$$\text{Quarta}\left\{\begin{aligned}&\text{anterioris} = \frac{a\alpha}{+0,047655\alpha+1,484315a\pm0,905133(a+\alpha)\sqrt{v}}\\&\text{posterioris} = \frac{a\alpha}{+0,406850\alpha-1,029770a\mp0,905133(a+\alpha)\sqrt{v}}\end{aligned}\right.$$

Scholion

Scholion.

346. Quae in his corollariis de constructione lentis primae sunt allata, mutatis litteris a et α ad reliquas lentes facile accommodari manifestum est quarum loco, si vsus requirat, vt numeri λ', λ'' etc. sint vnitate minores, etiam lentes siue duplicatae siue triplicatae siue adeo quadruplicatae adhiberi debebunt. Ceterum hae formulae ad lentes quotcunque patent ita vt proposito lentium numero quocunque tantum literae, quae ad lentes sequentes pertinerent, sint omittendae. In his autem denominationibus statim introduximus campum apparentem numero Φ contentum, cum sit semidiameter spatii in obiecto conspicui $z = a\Phi$. Interim tamen campum apparentem non ad lubitum augere licet, siquidem is multiplicationis ratione et numeris π, π', π'' etc. determinatur. Hi autem numeri semper infra $\frac{1}{2}$ imo $\frac{1}{3}$ subsistunt, ac plerumque $\frac{1}{4}$ vel adeo $\frac{1}{6}$ superare nequeunt; quandoque etiam minores accipi debent, id quod formulae coroll. 2. quouis casu declarabunt, quibus semidiameter aperturae cuiusque lentis exprimitur. Quare retentis his denominationibus momenta constructionis pro quouis lentium numero determinato expendamus.

Problema 4.

347. Si instrumentum dioptricum duabus constet lentibus, quarum crassitiem negligere liceat, definire momenta quibus constructio continetur.

Solutio.

Cum hic lens secunda sit vltima, momenta constructionis ita se habebunt.

1. Vt oculus imaginem in distantia iusta conspiciat, debet esse $O = \mathfrak{E} + l$ ideoque

$O = l + \frac{AB\alpha\Phi}{\mathfrak{B}\pi - \Phi}$. et ex conditione campi apparentis

$O = \frac{\mathfrak{B}b\pi}{\pi - \Phi}$.

2. Exponens multiplicationis m ad distantiam h relatus praebet hanc aequationem

$m = AB\frac{h}{l}$ pro situ erecto

sin autem situs inuersus desideretur, numerus m negatiue est capiendus.

3. Pro gradu claritatis semper est $y = \frac{hx}{ma}$.

4. Pro campo apparente habebitur haec aequatio:

$\pi - \Phi = \frac{-ma\Phi}{h}$ vnde fit $\Phi = \frac{-\pi h}{ma - h}$.

5. Si distantia O hinc prodeat negatiua, capi debet $O = 0$ seu $\frac{AB\alpha\Phi}{\mathfrak{B}\pi - \Phi} = -l$ tum vero π eiusmodi valorem induit, vt apertura lentis ocularis non superet aperturam pupillae; fiet nempe $\pi\mathfrak{B}b = \omega$, hincque

$\Phi = \frac{\mathfrak{B}\pi\omega}{A\mathfrak{B}\alpha\pi + \omega}$. vel $(B+1)\omega = -\pi l$.

6. Semidiameter confusionis sequenti formula exprimetur.

$\frac{Bbb}{4A\alpha l}x^3\left(\frac{AA\alpha\alpha}{bb}P + \frac{bb}{AA\alpha\alpha}Q\right)$

quae

quae factis substitutionibus abit in hanc:

$$\frac{\mu m x^3}{4aab}\left(\frac{(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\Phi(B+1)(\lambda'(B+1)^2+\nu B)}{A^3 B\cdot(\mathfrak{B}\pi-\Phi)}\right)$$

7. Vt obiectum sine margine colorato appareat, si distantia O fuerit positiua, necesse est sit:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}=0$$

8. Sin autem capiatur $O=0$, fieri debet $\frac{(A+1)\mathfrak{B}\pi}{\Phi}=0$ seu $\frac{\pi}{\mathfrak{A}\Phi}=0$.

9. Omnis autem confusio a diuersa radiorum refrangibilitate oriunda penitus tolletur si insuper fuerit

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}=0$$

10. Praeterea autem erit distantia lentium

$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$ quae debet esse positiua.

11. Denique Semidiameter aperturae lentis ocularis est;

$$\frac{A\mathfrak{B}a\pi+x}{\mathfrak{B}\pi-\Phi}.\,\Phi$$

vbi signorum $+$ id capi debet, quod valorem praebet maximum siue positiuum siue negatiuum.

Coroll. 1.

348. Cum sit (n°. 4) $\Phi=\frac{-\pi b}{ma-b}$, et ob multiplicationem $B=\frac{ml}{Ab}$, hincque $\mathfrak{B}=\frac{ml}{ml+Ab}$; habebitur pro loco oculi $O=\frac{-Abl(ma-b)}{mmal+Abb}$.

Tum

Tum vero eſt $\mathfrak{B}\pi - \Phi = \frac{ml\pi}{ml+\mathrm{A}b} + \frac{\pi b}{ma-b} = \frac{\pi(mmal+\mathrm{A}bb)}{(ma-b)(ml+\mathrm{A}b)}$ vnde ſit lentium diſtantia $\frac{\mathrm{A}\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi} = \frac{m\mathrm{A}a(ma-b)}{mmal+\mathrm{A}bb}l$.

Coroll. 2.

349. Cum ſit $\frac{\mathrm{A}}{\mathrm{A}+1} = \mathfrak{A}$ et $\mathrm{A} = \frac{\mathfrak{A}}{1-\mathfrak{A}}$ ſemidiameter confuſionis etiam ita exprimi poteſt vt ſit

$$\frac{\mu m x^3}{4aab}\left(\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3} + \frac{\Phi(\lambda+\nu\mathfrak{B}(1-\mathfrak{B}))}{\mathrm{A}\mathfrak{B}^3(\mathfrak{B}\pi-\psi)}\right).$$

Coroll. 3.

350. Cum ſit lentium diſtantia $= \frac{\mathrm{A}\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$, obiectum ſine coloribus cerni nequit, niſi lentium diſtantia euaneſcat; ſiquidem oculus in eo loco, quem campus apparens poſtulat, teneatur. Et quoniam huic conditioni ſatisfieri nequit, multo minus tota confuſio ex n°. 9 tolli poterit.

Scholion.

351. Hic ſtatim ab inſtrumentis duabus lentibus inſtructis incepi, quoniam lens vnica facillime expeditur. Primum enim neceſſe eſt vt ſit $\mathrm{O} = l + \alpha = l + \mathrm{A}a$. Tum vero multiplicatio praebet $m = \frac{b}{a}$ pro ſitu erecto, et campus apparens non terminatur ſumendo $\mathrm{A}a + l = 0$, ſeu $\mathrm{O} = 0$, ita vt oculus lenti immediate debeat applicari. Semidiameter vero confuſionis erit

$$= \frac{\alpha}{4l}x^3.\mathrm{P} = \frac{-\mathrm{A}ax^3}{4l}.\mathrm{P} = -\frac{1}{4}x^3\,\mathrm{P} = \frac{-max^3}{4b}.\mathrm{P} \text{ ob } l = -\mathrm{A}a \text{ et } ma = b$$

Quare ſemidiameter confuſionis erit

$$\frac{\mu m x^3}{4aab}\left(\frac{(\mathrm{A}+1)(\lambda(\mathrm{A}+1)^2+\nu\mathrm{A})}{\mathrm{A}^3}\right) = \frac{\mu m x^3}{4aab}\cdot\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3},$$

Pro

Pro gradu claritatis autem habetur vt ſemper $y = \frac{hx}{ma}$.

Obiectum porro hoc caſu ſine margine colorato cernetur, tota autem confuſio tolli nequit, niſi ſit $\frac{1}{\mathfrak{A}} = 0$ ſeu $A = -1$, id quod ſcopo lentium repugnat. Quare ad conſiderationem plurium lentium progrediamur.

Problema 5.

352. Si inſtrumentum dioptricum tribus inſtructum ſit lentibus, quarum craſſities tam ſit parva, vt negligi queat, definire cuncta momenta ad conſtructionem dirigendam neceſſaria.

Solutio.

Hic igitur lens tertia erit vltima ſeu ocularis, ideoque momenta ſequenti modo ſe habebunt:

1. Vt oculo imago in diſtantia iuſta ſpectanda offeratur debet eſſe $O = \gamma + l$, ac loco γ valore ſubſtituto

$O = l + \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ et ex conditione campi apparentis

$O = \frac{\mathfrak{C}c\pi'}{\pi' - \pi + \Phi}$.

2. Exponens multiplicationis m ad diſtantiam h relatus praebet hanc aequationem:

$m = -\frac{ABCh}{l}$ pro ſitu erecto.

vnde ſi ſitus inuerſus deſideretur, numerum m negatiue accipi conuenit.

3. Pro gradu claritatis habemus, vt ſemper, $y = \frac{bx}{ma}$.

4. Pro campo autem apparente definiendo habemus hanc aequationem

$$\Phi = \frac{-\pi + \pi'}{ma - b} \cdot b.$$

5. At ſi diſtantia O hinc prodeat negatiua, vt capi oporteat O = o, ideoque $\frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi} = -l$ campus apparens definiri debet ex hac aequatione $\pi'\mathfrak{C}c = \omega$ ſeu $\frac{AB\mathfrak{C}a\Phi\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = \omega$ vel $(C+1)\omega = -\pi' l$.

6. Semidiameter confuſionis vero ita exprimetur:

$$\frac{Ccc'}{4ABal}x^3\left(\frac{AABBaa}{cc}P + \frac{BBb^4}{AAaacc}Q + \frac{cc}{AABBaa}R\right)$$

quae ob $\frac{1}{l} = \frac{-m}{ABCb}$ abit ſigno mutato in hanc:

$$\frac{\mu m x^3}{4aab}\left\{\begin{array}{l} + \frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^3} \\ + \frac{\Phi(B+1)(\lambda'(B+1)^2 + \nu B)}{A^3B^3(\mathfrak{B}\pi - \Phi)} \\ + \frac{\Phi(C+1)(\lambda''(C+1)^2 + \nu C)}{A^3B^3C^3(\mathfrak{C}\pi' - \pi + \Phi)} \end{array}\right.$$

7. Vt obiectum ſine margine colorato appareat, ſi quidem diſtantia O prodierit poſitiua, eſſe oportet.

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = 0$$

8. Sin autem ob iſtam diſtantiam prodeuntem negatiuam capiatur O = o, margo coloratus euaneſcet faciendo:

$$\frac{\pi'}{\mathfrak{A}\Phi} + \frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} = \frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi - \Phi)}$$

9. Quod

9. Quod ſi marginem coloratum tollere liceat, praeterea viſio ab omni confuſione liberabitur, ſi huic aequationi ſatisfiat.

$$\frac{1}{\mathfrak{B}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} = 0.$$

10. Denique effici debet, vt diſtantia lentium fiat poſitiua vnde habebitur:

Interuallum
Lentium

I et II $= \frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi - \Phi} > 0$

II et III $= \frac{ABa\Phi(\mathfrak{C}\pi' - (1 - \mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\mathfrak{C}\pi' - \pi + \Phi)} > 0$

11. Tandem numeros π et π' ita accipi oportet, vt aperturae lentium non fiant nimis magnae: eſt vero

Semidiameter
aperturae

Lentis ſecundae $= \frac{A\mathfrak{B}a\pi + x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$

Lentis tertiae $= \frac{AB\mathfrak{C}a\pi' + x}{\mathfrak{C}\pi' - \pi + \Phi} \cdot \Phi.$

Coroll. I.

353. Cum ſit $C = \frac{-ml}{ABb}$, ideoque $\mathfrak{C} = \frac{-ml}{ABb - ml}$; et $\Phi = \frac{-\pi + \pi'}{ma - b} \cdot b$, ſi hi valores ſubſtituantur erit pro loco oculi.

$$O = \frac{ABbl(ma - b)\pi'}{(mmal - ABbb)\pi' + ma(ABb - ml)\pi}.$$

Coroll. 2.

354. Semidiameter confuſionis etiam ita exprimi poteſt vt ſit

$$\frac{\mu m x^3}{4 a a b}\left(\frac{\lambda + \nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3} + \frac{\Phi(\lambda' + \nu\mathfrak{B}(1-\mathfrak{B}))}{A^3\mathfrak{B}^3(\mathfrak{B}\pi - \Phi)} + \frac{\Phi(\lambda'' + \nu\mathfrak{C}(1-\mathfrak{C}))}{A^3 B^3 \mathfrak{C}^3(\mathfrak{C}\pi' - \pi + \Phi)}\right)$$

cuius formae cohaerentia cum praecedentibus iam ita eſt manifeſta, vt ad plures lentes facile extendi queat.

Problema 6.

355. Si inſtrumentum dioptricum quatuor lentibus ſit inſtructum, quarum craſſities in computum duci non mereatur, definire omnia momenta ad conſtructionem dirigendam neceſſaria.

Solutio.

Quia hic lens quarta eſt vltima.

1. Vt oculo imago poſtrema in diſtantia iuſta offeratur debet eſſe $O = \delta + l$, ideoque

$O = l + \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$ et ex conditione campi apparentis

$O = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi}$.

2. Exponens multiplicationis m ad diſtantiam b relatus, ſi obiectum vt hactenus ſitu erecto exhiberi aſſumatur, erit

$$m = + ABCD. \frac{b}{l} \text{ pro ſitu erecto.}$$

ſin autem ſitus deſideretur inuerſus numerus m negatiue eſt accipiendus.

3. Pro gradu claritatis perpetuo habetur $y = \frac{bx}{ma}$.

4. Campus autem apparens definiri debet ex hac aequatione

$$\Phi = \frac{-\pi + \pi' - \pi''}{ma - h} h$$

hinc enim $a\Phi$ praebebit ſemidiametrum ſpatii in obiecto conſpicui; ſiquidem diſtantia O fuerit poſitiua.

5. Sin autem diſtantia O prodeat negatiua quo caſu capi oportet $O = 0$, ſeu $\frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = -l$ quantitas Φ determinari debet ex hac aequatione $\pi''\mathfrak{D}d = \omega$ ſeu hac

$$\frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = \omega \text{ ſiue } (D+1)\omega = -\pi'' l.$$

6. Semidiameter confuſionis ita reperietur expreſſus:

$$\frac{\mu m x^3}{4aah}\left\{\begin{aligned} &+\frac{(A+1)(\lambda(A+1)^2+\nu A)}{A^3} \\ &+\frac{\Phi(B+1)(\lambda'(B+1)^2+\nu B)}{A^3B^3(\mathfrak{B}\pi-\Phi)} \\ &+\frac{\Phi(C+1)(\lambda''(C+1)^2+\nu C)}{A^3B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)} \\ &+\frac{\Phi(D+1)(\lambda'''(D+1)^2+\nu D)}{A^3B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}.\end{aligned}\right.$$

7. Vt obiectum ſine margine colorato appareat, ſiquidem diſtantia O ſit poſitiua eſſe oportet:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = 0.$$

8. Verum ſi acceperimus $O = 0$, margo coloratus euaneſcet, huic aequationi ſatisfaciendo.

$$\frac{\pi''}{\mathfrak{A}\Phi} + \frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} = \frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi - \Phi)} + \frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi' - \pi + \Phi)}.$$

9. Omnis autem confuſio a diuerſa radiorum natura oriunda penitus tolletur, ſi ſatisfieri liceat huic aequationi:

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0.$$

10. Denique efficiendum eſt vt diſtantiae lentium ſint poſitiuae vnde oriuntur formulae ſequentes:

Interuallum
Lentium

I et II $=\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}>0$

II et III $=\frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$

III et IV $=\frac{ABCa\Phi(\mathfrak{D}\pi''-(\quad-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0.$

11. Aperturae autem ſingularum lentium ita ſunt capiendae vt ſit

Semid: apert:

Lentis ſecundae $=\frac{A\mathfrak{B}a\pi\pm x}{\mathfrak{B}\pi-\Phi}.\ \Phi$

Lentis tertiae $=\frac{AB\mathfrak{C}a\pi'\pm x}{\mathfrak{C}\pi'-\pi+\Phi}.\ \Phi$

Lentis quartae $=\frac{ABC\mathfrak{D}a\pi''\pm x}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}.\ \Phi.$

Coroll. 1.

356. Cum ſit ob multiplicationem $D=\frac{ml}{ABCb}$ ideoque $\mathfrak{D}=\frac{ml}{ABCb+ml}$ et $\Phi=\frac{-\pi+\pi'-\pi''}{ma-b}.\,b$; pro loco oculi habebitur haec expreſſio:

$$O=\frac{-ABCbl(ma-b)\pi''}{(mmal+ABCbb)\pi''-ma(ABCb+ml)(\pi'-\pi)}.$$

Coroll.

Coroll. 2.

357. Quia hic tot occurrunt litterae determinandae, dubium nullum eſſe videtur, quin formulis ſub n°. 7 vel 8 et 9 contentis ſatisfieri queat, et certum ſaltem eſt, id fieri poſſe, ſi plura adhibeantur mediorum refringentium genera.

Problema 7.

358. Si inſtrumentum dioptricum quinque lentibus ſit inſtructum, quarum craſſities ob paruitatem negligi queat, definire omnia momenta conſtructionem dirigentia.

Solutio.

Cum hic lens quinta vltimum locum teneat; habebimus.

1. Pro loco oculi, vt imago viſa ipſi in iuſta diſtantia offeratur debet eſſe $O = \varepsilon + l$ ideoque

$O = l + \frac{ABCDE\alpha\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$ et ex conditione campi apparentis

$O = \frac{\mathfrak{E}\, e\, \pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi}$.

2. Exponens multiplicationis m ad diſtantiam $= h$ relatus, ſi quidem obiectum ſitu erecto ſit ſpectandum

$$m = -ABCDE. \frac{h}{l}.$$

pro ſitu autem inuerſo numerus m negatiue capi debet.

3. Pro

3. Pro gradu claritatis eſt vt hactenus $y = \frac{bx}{ma}$.

4. Campus apparens definiri debet ex hac aequatione

$$\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{ma - b} \cdot b$$

ſiquidem diſtantia illa O prodierit poſitiua.

5. Sin autem haec diſtantia prodeat negatiua vt capiatur O = o, ſeu $\frac{ABCDEa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = -l$, campus apparens non ex formula n°. 4 ſed ex hac $\pi''' \mathfrak{E} e = \omega$ definiri debet, vnde fit

$$\frac{ABCD\mathfrak{E}a\pi'''\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = \omega \text{ ſeu } (E + 1)\omega = -\pi''' l$$

6. Semidiameter confuſionis ab apertura lentium oriundae ſequenti modo exprimitur,

$$\frac{\mu m x^3}{4aab} \left\{ \begin{array}{l} + \frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^3} \\ + \frac{\Phi(B+1)(\lambda'(B+1)^2 + \nu B)}{A^3 B^3 (\mathfrak{B}\pi - \Phi)} \\ + \frac{\Phi(C+1)(\lambda''(C+1)^2 + \nu C)}{A^3 B^3 C^3 (\mathfrak{C}\pi' - \pi + \Phi)} \\ + \frac{\Phi(D+1)(\lambda'''(D+1)^2 + \nu D)}{A^3 B^3 C^3 D^3 (\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} \\ + \frac{\Phi(E+1)(\lambda''''(E+1)^2 + \nu E)}{A^3 B^3 C^3 D^3 E^3 (\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)} \end{array} \right.$$

7. Vt obiectum ſine margine colorato appareat, ſiquidem diſtantia O fuerit poſitiua, huic aequationi erit ſatisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} + \frac{\pi'''}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = 0.$$

8. At

8. At ſi diſtantia O ſit negatiua, capiaturque O = o pro hoc ſcopo obtinendo oportet ſit

$$\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$=\frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right).$$

9. Omnis autem confuſio a diuerſa radiorum natura oriunda penitus tolletur, ſi fieri poſſit

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}=0$$

10. Tum vero efficiendum eſt, vt interualla lentium omnia ſint poſitiua, vnde oriuntur formulae ſequentes.

Interuallum
Lentium

$$\text{I. et II.}=\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}>0$$
$$\text{II. et III.}=\frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$$
$$\text{III. et IV.}=\frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0$$
$$\text{IV. et V.}=\frac{ABCDa\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}>0$$

11. Aperturae denique ſingularum lentium ita ſunt capiendae, vt ſit:

Semid. apert:

Lentis ſecundae $=\frac{A\mathfrak{B}a\pi\pm x}{\mathfrak{B}\pi-\Phi}\,\Phi$

Lentis tertiae $=\frac{AB\mathfrak{C}a\pi'\pm x}{\mathfrak{C}\pi'-\pi+\Phi}.\,\Phi$

Lentis quartae $=\frac{ABC\mathfrak{D}a\pi''\pm x}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}.\,\Phi$

Lentis quintae $=\frac{ABCD\mathfrak{E}a\pi'''\pm x}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}.\,\Phi.$

Corollarium.

359. Ob multiplicationem ergo erit:

$$\mathrm{E} = \frac{-ml}{\mathrm{ABCD}b} \text{ et } \mathfrak{E} = \frac{-ml}{\mathrm{ABCD}b - ml}$$

vnde oritur:

$$\mathrm{O} = \frac{\mathrm{ABCD}bl(ma - b)\pi'''}{(mmal - \mathrm{ABCD}bb)\pi''' + ma(\mathrm{ABCD}b - ml)(\pi'' - \pi' + \pi)}.$$

Scholion.

360. Hactenus ſuppoſui oculum vel in eo loco teneri, quem viſio campi apparentis poſtulat, vel lenti poſtremae immediate applicari; quorum vtroque caſu efficiendum eſt, vt imago vltima oculo ad diſtantiam iuſtam offeratur. Hic quidem locum oculi, quem campus praebet ſtatim coniunxi cum iuſta imaginis ſpectandae diſtantia; ſed etiam ſine reſpectu ſiue ad hanc diſtantiam ſiue ad multiplicationem habito definiri poteſt. Ex ſuperioribus enim §. 271. habemus

Pro caſu

vnius lentis $\mathrm{O} = 0$

duarum lentium $\mathrm{O} = \frac{\mathrm{A}\mathfrak{B}a\pi\Phi}{(\pi - \Phi)(\mathfrak{B}\pi - \Phi)}$

trium lentium $\mathrm{O} = \frac{\mathrm{AB}\mathfrak{C}a\pi'\Phi}{(\pi' - \pi + \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$

quatuor lentium $\mathrm{O} = \frac{\mathrm{ABC}\mathfrak{D}a\pi''\Phi}{(\pi'' - \pi' + \pi - \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$

quinque lentium $\mathrm{O} = \frac{\mathrm{ABCD}\mathfrak{E}a\pi'''\Phi}{(\pi''' - \pi'' + \pi' - \pi + \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$

etc.

Verum

Verum si nolimus ad campum apparentem respicere semper eiusmodi locum oculo assignare licet, vnde obiectum sine margine colorato cernatur. Haud abs re autem fore arbitror hunc locum determinasse, quia ex eius distantia a loco oculi ob rationem campi assumto facilius iudicare licebit, quanto margine cinctum apparere debeat obiectum si oculus in loco isto posteriori teneatur. Si enim aequationi *n*°. 7 appositae satisfieri nequeat, nil aliud intelligimus, nisi obiectum non sine margine colorato esse appariturum at si insuper constet ille locus, vnde obiectum sine huiusmodi margine spectari posset, facilius quantitatem istius marginis colligere poterimus.

Problema 8.

361. Si instrumentum dioptricum ex lentibus quotcunque constet, quarum crassitiem negligere liceat, eum pro oculo locum assignare, vnde obiectum sine margine colorato conspiciatur.

Solutio.

Ponatur haec oculi post lentem vltimam distantia $= \Omega$, atque ex aequationibus §. 325 datis, si pro O scribatur Ω haec ipsa distantia Ω, quam quaerimus, elici potest. Scilicet pro quouis lentium numero erit vt sequitur.

I. Pro vnica Lente.

Habetur haec aequatio $\frac{\Omega\, d\alpha}{\alpha(\alpha-\Omega)}=0$ vnde fit $\frac{\Omega}{\alpha(\alpha-\Omega)}=0$, ſeu $\frac{\Omega}{\alpha-\Omega}=0$. Verum etiam campus apparens exigit $O=0$, vnde hoc caſu ambo loca oculi O et Ω conveniunt, neque margo coloratus eſt pertimeſcendus.

II. Pro duabus Lentibus

Hoc caſu reperta eſt iſta aequatio

$$\frac{\Omega\, d\beta}{\beta(\beta-\Omega)}=d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)$$

vnde ſubſtitutis valoribus ſupra aſſignatis oritur

$$\frac{\Omega}{\beta(\beta-\Omega)}=\frac{\pi}{\mathfrak{A}\Phi}:(B+1)ABa\left(\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}\right)$$

ſeu ſi ponamus

$$Y=\frac{\pi}{\mathfrak{A}\Phi} \text{ et } Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}$$

erit

$$\frac{\Omega}{\beta-\Omega}=\frac{Y}{AB(B+1)aZ}.\ \beta=\frac{Y}{(B+1)Z}\cdot\frac{\Phi}{\mathfrak{B}\pi-\Phi}.$$

III. Pro tribus Lentibus.

Eodem modo ſi ponamus

$$Y=\frac{\pi'}{\mathfrak{A}\Phi}+\frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}-\frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi-\Phi)} \quad \text{et}$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}$$

erit pro caſu trium lentium

$$\frac{\Omega}{\gamma-\Omega}=\frac{Y}{ABC(C+1)aZ}.\ \gamma=\frac{Y}{(C+1)Z}\cdot\frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$

vbi

vbi notandum eſt $Y=0$ et $Z=0$ eſſe eas ipſas aequationes, quae in ſuperioribus problematibus ſub numeris 8 et 9 retulimus.

IV. Pro quatuor lentibus

Si iam ponatur:

$$Y=\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}-\frac{1}{ABC\mathfrak{D}}\left(\frac{\pi}{(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}\right)$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

erit:

$$\frac{\Omega}{\delta-\Omega}=\frac{Y}{ABCD(D+1)aZ}\delta$$

hincque $\dfrac{\Omega}{\delta-\Omega}=\dfrac{Y}{(D+1)Z}\cdot\dfrac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}.$

V. Pro quinque Lentibus.

Poſito ſecundum aequationes n^o. 8 et 9 exhibitas

$$Y=\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

$$-\frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

$$+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$$

habebimus

$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{ABCDE(E+1)aZ}\cdot\varepsilon \text{ vel}$$

$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{(E+1)Z}\cdot\frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}.$$

Lex harum formularum pro quouis maiori lentium numero hinc ſatis eſt perſpicua.

Coroll. 1.

362. Cum igitur supra pro quouis lentium numero exhibuerimus aequationes n°. 8. et n°. 9. formula n°. 8. dabit valorem Y et formula n°. 9. valorem ipsius Z, quibus cognitis locus oculi, vbi margo coloratus euanescit, facile innotescit.

Coroll. 2.

363. Si praeterea ex aequatione n°. 7. exposita ponatur

$$X = \frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} + \frac{\pi'''}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

erit quidem pro casu quinque lentium

$$Y = \frac{\pi'''}{\Phi} Z - \frac{1}{ABCD\mathfrak{E}} X$$

Pro quatuor lentibus $Y = \frac{\pi''}{\Phi} Z - \frac{1}{ABC\mathfrak{D}} X$

Pro tribus lentibus $Y = \frac{\pi'}{\Phi} Z - \frac{1}{AB\mathfrak{C}} X$

et pro duabus lentibus $Y = \frac{\pi}{\Phi} Z - \frac{1}{A\mathfrak{B}} X$.

Scholion.

364. Quo facilius haec oculi loca littera Ω designata cum praecedentibus littera O designatis comparare liceat, notandum est esse,

Pro casu

Vnius Lentis $\frac{O}{\alpha - O} = 0$

Duarum Lentium $\frac{O}{\beta - O} = \frac{\pi}{(B+1)(\mathfrak{B}\pi - \Phi)}$

Trium Lentium $\frac{O}{\gamma - O} = \frac{\pi'}{(C+1)(\mathfrak{C}\pi' - \pi + \Phi)}$

Quatuor Lentium $\frac{O}{\delta - O} = \frac{\pi''}{(D+1)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$

Quinque Lentium $\frac{O}{\varepsilon - O} = \frac{\pi'''}{(E+1)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$ etc.

Quare

Quare vt ambo loca congruant retentis pro quouis lentium numero litteris Y et Z modo adhibitis, oportet fit

Pro vna Lente $\eth = \eth$

Pro duabus Lentibus $\frac{Y}{Z} = \frac{\pi}{\Phi}$

Pro tribus Lentibus $\frac{Y}{Z} = \frac{\pi'}{\Phi}$

Pro quatuor Lentibus $\frac{Y}{Z} = \frac{\pi''}{\Phi}$

Pro quinque Lentibus $\frac{Y}{Z} = \frac{\pi'''}{\Phi}$ etc.

vnde generatim fit $X = 0$, quae eft ipfa aequatio n°. 7. allata. Haec igitur funt principia generalia, ex quibus conftructio inftrumentorum dioptricorum erit perficienda, quae quidem inftrumenta hic potiffimum ad vifionem accommodaui. Nihilo vero minus ad repraefentationem obiectorum in camera obfcura fuper Tabula alba adhiberi poffunt, vbi praeter iam data praecepta tenendum eft, has effigies etiam fine margine colorato effe apparituras, fi aequationibus n°. 7. fatisfiat, omnem vero confufionem a diuerfa radiorum natura oriundam penitus tolli, fi infuper aequationibus n°. 9 fatisfiat.

Supplementum VII.

Si ratio refractionis in fingulis lentibus fuerit diverfa et fecundum lentium ordinem litteris n, n', n'', n''' etc. exhibeatur, inde pro lentium conftructione

I°. Litterae refpondentes ϱ, σ, τ et pro confufione

fusione litterae μ et ν conuenienter definiri debent secundum formulas supra datas §. 55

$$\varrho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1 ; \sigma = \frac{1}{2(n-1)} - \frac{1}{n+2} + 1$$

$$\tau = \frac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right) \sqrt{(4n-1)}$$

ita, vt sit

$$\varrho + \sigma = \frac{1}{n-1} \text{ et } \sigma - \varrho = \frac{-2}{n+2} + 2 = \frac{2(n+1)}{n+2}.$$

Porro autem

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n-1)^2}$$

$$\nu = \frac{4(n-1)^2}{4n-1};$$

vnde intelligitur, quemadmodum ex ratione refractionis n' litterae respondentes ϱ', σ', τ', μ', ν' definiri debeant.

II°. Quod iam ad elementa occasione campi apparentis introducta π, π', π'' etc. attinet, distantiae determinatrices lentium inde eodem modo manent determinatae, vt supra, ita, vt non opus sit, eas formulas hic transscribere; interim tamen meminisse iuuabit; formulas primitiuas, vnde illae sunt natae, quae sunt

1°. $\frac{\mathfrak{B}\pi - \Phi}{\Phi} = \frac{Aa}{b}$

2°. $\frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = \frac{ABa}{c}$

3°. $\frac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = \frac{ABCa}{d}$

4°. $\frac{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}{\Phi} = \frac{ABCDa}{e}$

etc.

III.

III. Verum valores litterarum P, Q, R etc. ob diuerſas refractiones mutantur, vt ſequitur.

$$P = \frac{\mu}{A^3 a^3}(A+1)(\lambda(A+1)^2+\nu A)$$

$$Q = \frac{\mu'}{B^3 b^3}(B+1)(\lambda'(B+1)^2+\nu' B);$$

$$R = \frac{\mu''}{C^3 c^3}(C+1)(\lambda''(C+1)^2+\nu'' C);$$

$$S = \frac{\mu'''}{D^3 d^3}(D+1)(\lambda'''(D+1)^2+\nu''' D);$$

etc.

IV. Diſtantiae lentium etiam manent, vt ante, vna cum ſemidiametris aperturarum ſingularum lentium; at radii binarum facierum vtriusque lentis ita ſunt ad praeſentem caſum accommodandae, vt pro, lente cuius refractio eſt n', ei reſpondentes litterae ϱ', σ', et τ' vſurpari debeant, ſimilique modo etiam pro ſequentibus lentibus, quarum refractio litteris n'', n''' etc. indicatur.

His praemiſſis ſingula momenta, quae in conſtructione inſtrumentorum dioptricorum ſunt obſeruanda, ſequenti modo repraeſentabimus.

I. Pro loco oculi ſeu eius diſtantia poſt vltimam lentem $= O$

habebimus pro ſingulis lentium numeris ſequentes determinationes:

Num. lentium.	
I.	$O = o = \alpha + l$; ideoque $\alpha = -l = \mathrm{A}a$
II.	$O = \frac{\mathfrak{B}b\pi}{\pi - \Phi} = \beta + l$; ideoque $\beta = \frac{\mathfrak{B}b\pi}{\pi - \Phi} - l = \mathrm{B}b$; $l = \frac{-\mathrm{B}b(\mathfrak{B}\pi - \Phi)}{\pi - \Phi}$
III.	$O = \frac{\mathfrak{C}c\pi'}{\pi' - \pi + \Phi} = \gamma + l$; hinc $\gamma = \frac{\mathfrak{C}c\pi'}{\pi' - \pi + \Phi} - l = \mathrm{C}c$;
	hinc $l = -\frac{\mathrm{C}c(\mathfrak{C}\pi' - \pi + \Phi)}{\pi' - \pi + \Phi}$
IV.	$O = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi} = \delta + l$; hinc $\delta = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi} - l$;
	adeoque $l = \frac{-\mathrm{D}d(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}{\pi'' - \pi' + \pi - \Phi}$
V.	$O = \frac{\mathfrak{E}e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} = \varepsilon + l$; hinc $\varepsilon = \frac{\mathfrak{E}e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} - l$;
	adeoque $l = -\frac{\mathrm{E}e(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}{\pi''' - \pi'' + \pi' - \pi + \Phi}$
	etc.

II. Pro ratione multiplicationis ad diſtantiam *b* relata

habebimus pro quolibet lentium numero ſequentes valores litterae *m*.

I.	$m = -\mathrm{A}\frac{b}{l}$	Pro ſitu erecto, ſi *m* hinc prodierit poſitiuum; pro ſitu autem inuerſo, ſi *m* obtinuerit valorem negatiuum.
II.	$m = +\mathrm{AB}\frac{b}{l}$	
III.	$m = -\mathrm{ABC}\frac{b}{l}$	
IV.	$m = +\mathrm{ABCD}\frac{b}{l}$	
V.	$m = -\mathrm{ABCDE}\frac{b}{l}$	
	etc.	

III.

III. Pro gradu claritatis y

ex superioribus liquet, eum semper pari modo exprimi; quantuscunque fuerit lentium numerus; perpetuo enim erit $y = \frac{bx}{ma}$

IV. Pro Campo apparente si quidem O habeat valorem positiuum,

eius semidiameter Φ pro quouis lentium numero sequenti modo definietur:

Num. lentium.	
I.	$\Phi = \infty$ seu indefinitum
II.	$\Phi = \frac{-\pi b}{ma - b}$
III.	$\Phi = \frac{-\pi + \pi'}{ma - b} \cdot b$
IV.	$\Phi = \frac{-\pi + \pi' - \pi''}{ma - b} \cdot b$
V.	$\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{ma - b} \cdot b$ etc.

(IV.) Pro Campo apparente,

si distantia O fuerit negatiua, quo casu oculus lenti vltimae immediate adplicari debet, Φ definietur ex aequationibus sequentibus.

Num. lentium.	
I.	$\Phi = \infty$ indefinitum
II.	$\frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi - \Phi} = \omega$
III.	$\frac{ABC\mathfrak{C}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi} = \omega$
IV.	$\frac{ABC\mathfrak{D}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = \omega$
V.	$\frac{ABCD\mathfrak{E}.a\Phi}{\pi''' - \pi'' + \pi' - \pi + \Phi} = \omega$. etc.

V. Pro semidiametro confusionis habebuntur sequentes expressiones

I. $\frac{mx^3}{4aab} \cdot \frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}$

II. $\frac{mx^3}{4aab}\left(\frac{\mu(A+1)(\lambda(A+1)^2\nu A}{A^3}\right)+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B(\mathfrak{B}\pi-\Phi)}\Phi)$

III. $$\frac{mx^3}{4aab}\left\{\begin{array}{l}\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3(\mathfrak{B}\pi-\Phi)}\Phi \\ \qquad +\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi]\end{array}\right.$$

IV. $$\frac{mx^3}{4aab}\left\{\begin{array}{l}\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3(\mathfrak{B}\pi-\Phi)}\Phi \\ \frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)(\lambda'''(D+1)^2+\nu''' D)}{A^3 B^3 C^3 D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\Phi\end{array}\right.$$

V. $$\frac{mx^3}{4aab}\left\{\begin{array}{l}\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3(\mathfrak{B}\pi-\Phi)}\Phi \\ +\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)(\lambda'''(D+1)^2+\nu''' D)}{A^3 B^3 C^3 D^3(\mathfrak{D}\pi''-\pi'+\pi+\Phi)}\Phi \\ \qquad +\frac{\mu''''(E+1)(\lambda''''(E+1)^2+\nu'''' E)}{A^3 B^3 C^3 D^3 E^3(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}\Phi\end{array}\right.$$

etc.

VI. Pro tollendo margine colorato,

si distantia O prodierit positiua, obiectum sine margine colorato apparebit, satisfaciendo sequentibus aequationibus:

Num. lentium.	
I.	$0=0$
II.	$0=\frac{bdn'}{n'-1}\cdot\frac{\pi}{A\alpha\Phi}$
III.	$0=\frac{bdn'}{n'-1}\cdot\frac{\pi}{A\alpha\Phi}+\frac{cdn''}{n''-1}\cdot\frac{\pi'}{AB\alpha\Phi}$
VI.	$0=\frac{bdn'}{n'-1}\cdot\frac{\pi}{A\alpha\Phi}+\frac{cdn''}{n''-1}\cdot\frac{\pi'}{AB\alpha\Phi}+\frac{ddn'''}{n'''-1}\cdot\frac{\pi''}{ABC\alpha\Phi}$ etc.

(VI.)

(VI). Pro tollendo margine colorato.

Sin autem distantia O prodierit negatiua; margo coloratus euanescet, si sequentibus aequationibus satisfiat

I. $0 = 0$

II. $0 = \frac{adn}{n-1}(A+1)B\pi$

III. $0 = \frac{adn}{n-1}(A+1)BC\pi' + \frac{bdn'}{n'-1}\frac{(B+1)C\pi' - (C+1)\pi}{A}$

IV. $0 = \frac{adn}{n-1}(A+1)BCD\pi'' + \frac{bdn'}{n'-1}\left(\frac{(B+1)CD\pi'' - (D+1)\pi}{A}\right)$
$+ \frac{cdn''}{n''-1}\left(\frac{(C+1)D\pi'' - (D+1)\pi'}{AB}\right)$

V. $0 = \frac{adn}{n-1}(A+1)BCDE\pi''' + \frac{bdn'}{n'-1}\left(\frac{(B+1)CDE\pi''' - (E+1)\pi}{A}\right)$
$+ \frac{cdn''}{n''-1}\left(\frac{(C+1)DE\pi''' - (E+1)\pi'}{AB}\right)$
$+ \frac{ddn'''}{n'''-1}\left(\frac{(D+1)E\pi''' - (E+1)\pi''}{ABC}\right)$

etc.

VII. Pro tollenda confusione omni

insuper sequentibus aequationibus est satisfaciendum

I. $0 = \frac{adn}{n-1}\cdot\frac{A+1}{A}$

II. $0 = \frac{adn}{n-1}\cdot\frac{A+1}{A} + \frac{bdn'}{n'-1}\cdot\frac{B+1}{A^2B}$

III. $0 = \frac{adn}{n-1}\cdot\frac{A+1}{A} + \frac{bdn'}{n'-1}\cdot\frac{B+1}{A^2B} + \frac{cdn''}{n''-1}\cdot\frac{C+1}{A^2B^2C}$

IV. $0 = \frac{adn}{n-1}\cdot\frac{A+1}{A} + \frac{bdn'}{n'-1}\cdot\frac{B+1}{A^2B} + \frac{cdn''}{n''-1}\cdot\frac{C+1}{A^2B^2C}$
$+ \frac{ddn'''}{n'''-1}\cdot\frac{D+1}{A^2B^2C^2D}$

etc.

Diſtantiae autem lentium et ſemidiametri aperturarum perinde definiuntur, ac ſupra; tantum notetur inſuper, formulas pro ſemidiametro confuſionis exhibitas, niſi penitus ad nihilum redigi queant, aequales poni debere formulae $\frac{1}{4k^3}$, exiſtente circiter $k = 40$ (§. 193) vel adhuc minore, prout circumſtantiae poſtulauerint.

Supplementum VIII.

De Lentibus obiectiuis perfectis.

In Cap. III. nullas alias lentes obiectiuas commemorauimus, niſi quae ex eadem vitri ſpecie et ex principio minimi ſunt paratae, neque tales lentes ex diuerſis vitri ſpeciebus conficere conuenit, quoniam eae ad confuſionem ex diuerſa radiorum refrangibilitate oriundam, tollendam ſunt ineptae. Cum igitur iſta confuſio aliter tolli nequeat, niſi diuerſae materiae refringentes adhibeantur; hic adcuratius perpendamus, quemadmodum ex diuerſis mediis diaphanis eiusmodi lentes conſtrui queant, in quibus non ſolum prior confuſio ex apertura lentium, ſed etiam poſterior ex diuerſa radiorum refrangibilitate oriunda ad nihilum redigi queat, cuiusmodi lentes compoſitas merito *perfectas* appellare licebit.

I. Primo igitur examinemus, quomodo poſteriori confuſioni ſit occurrendum, quando quotcunque lentes inuicem iungantur, ita, vt earum diſtantiae quaſi euaneſcant. Quem in finem conſiderentur formulae ſupra

ſupra datae tam pro margine colorato vitando, quam pro confuſione penitus tollenda N°. VI, VII. At ex aequatione N°. VI. patet, marginem coloratum euaneſcere, ſi litterae π, π', π'' etc. ſint $=0$, quod quidem ſponte euenit, ſi lentes immediate coniungantur; quare in genere nullus margo coloratus eſt metuendus, ſtatim atque interualla omnia lentium euaneſcunt.

II. Vt autem aequationibus N°. VII. ſatisfiat, pro caſu duarum lentium immediate iunctarum habemus

$$0=\frac{adn}{n-1}\cdot\frac{A+1}{A}+\frac{bdn'}{n'-1}\cdot\frac{B+1}{A^2 B}$$

$$0=\frac{adn}{n-1}\cdot\frac{a+\alpha}{\alpha}+\frac{bdn'}{n'-1}\cdot\frac{(b+\beta)a^2}{\alpha\,\beta}$$

$$0=\frac{dn}{n-1}\cdot\frac{a+\alpha}{a\alpha}+\frac{bdn'}{n'-1}\cdot\frac{b+\beta}{\alpha^2\beta}\text{; ſeu ob } \alpha=-b$$

$$0=\frac{dn}{n-1}\cdot\frac{a+\alpha}{a\alpha}+\frac{dn'}{n'-1}\cdot\frac{b+\beta}{b\beta},$$

$$0=\frac{dn}{n-1}\cdot\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{dn'}{n'-1}\cdot\left(\frac{1}{b}+\frac{1}{\beta}\right)$$

vbi duo hi coëfficientes denotant reciproca diſtantiarum focalium primae et ſecundae lentis.

III. Nunc inſuper effici debet, vt prior confuſio ab apertura lentium oriunda ad nihilum redigatur; vbi obſeruare debemus, hoc caſu eſſe debere $\alpha+b=0$; $\beta+c=0$; $\gamma+d=0$ etc. pro numero lentium, quas coniungere velimus. Vnde ſemidiameter confu-

confuſionis ex lentibus iunctis ortae euaneſcet, ſi reddatur

$$0 = \frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3} - \frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3\,B^3}$$

$$+ \frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3\,B^3\,C^3} \text{ etc.}$$

quoniam ſupra iam vidimus, fore $\pi = 0$, $\pi' = 0$ etc. atque hinc pro quouis numero lentium inuicem iungendarum conſtructionem lentium perfectarum petere debemus; quas inueſtigationes hic ſuſcipiamus.

De lentibus perfectis ex duabus lentibus compoſitis.

Quoniam hoc caſu diſtantiae determinatrices ipſius lentis compoſitae ſunt a et β; ſtatuamus

$$\tfrac{1}{a}+\tfrac{1}{\alpha} = f\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right) \text{ et } \tfrac{1}{b}+\tfrac{1}{\beta} = g\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right) \text{ ſiue}$$

$$\frac{a\alpha}{a+\alpha} = \frac{1}{f}\left(\frac{a\beta}{a+\beta}\right);\ \frac{b\beta}{b+\beta} = \frac{1}{g}\cdot\frac{a\beta}{a+\beta};$$

vnde pro prima conditione implenda ſequitur:

$0 = \frac{dn}{n-1}\cdot f + \frac{dn'}{n'-1}\cdot g$. Porro ob $\frac{1}{\alpha} = \frac{f-1}{a} + \frac{f}{\beta}$ et $\frac{1}{b} = \frac{g}{a} + \frac{g-1}{\beta}$ et $\alpha + b = 0$, erit $\frac{f+g-1}{a} + \frac{f+g-1}{\beta} = 0$, ſeu $f+g = 1$.

Sit breuitatis gratia $\frac{dn}{n-1} = \zeta$ et $\frac{dn'}{n'-1} = \eta$, vt ſit $\zeta f + \eta g = 0$; ex qua aequatione cum altera $f+g = 1$ coniuncta vtraque littera f et g determinabitur, ita, vt ſit

$$f = \frac{\eta}{\eta-\zeta} \text{ et } g = -\frac{\zeta}{\eta-\zeta}$$

deinde

deinde cum fit

$$A = \frac{\alpha}{a} = \frac{\beta}{\beta(f-1)+af} = \frac{\beta}{af-\beta g} = \frac{(\eta-\zeta)\beta}{a\eta+\beta\zeta} \text{ et}$$

$$B = \frac{\beta}{b} = \frac{g\beta+(g-1)a}{a} = \frac{g\beta-fa}{a} = \frac{-\zeta\beta-\eta a}{(\eta-\zeta)a};$$

$$\text{ergo } A+1 = \frac{\eta(a+\beta)}{a\eta+\beta\zeta} \text{ et } B+1 = \frac{-\zeta(a+\beta)}{a(\eta-\zeta)};$$

vnde pro confusione priori tollenda habebimus hanc aequationem

$$0 = \frac{\mu\lambda.\eta^3(a+\beta)^3}{(\eta a+\zeta\beta)^3} + \frac{\mu\nu\eta(\eta-\zeta)(a+\beta)\beta}{(\eta a+\zeta b)^2}$$
$$-\frac{\mu'\lambda'.\zeta^3(a+\beta)^3}{(\eta a+\zeta\beta)^3} + \frac{\mu'\nu'\zeta(\eta-\zeta)(a+\beta)a}{(\eta a+\zeta b)^2}$$

quae reducitur ad hanc

$$0 = \frac{(a+\beta)^2}{\eta a+\zeta\beta}(\mu\lambda\eta^3-\mu'\lambda'\zeta^3)+(\eta-\zeta)(\mu\nu\eta\beta+\mu'\nu'\zeta a)$$

Breuius etiam ex §. 214 deducitur haec aequatio, cui fatisfieri debet $P+Q=0$ ob $\alpha=-b$, exiftente pro hoc cafu

$$P = \mu f\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu}{a}\left(\tfrac{f-1}{a}+\tfrac{f}{\beta}\right)\right) \text{ et}$$

$$Q = \mu'.g\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)\left(\lambda' g^2\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu'}{\beta}\left(\tfrac{g}{a}+\tfrac{g-1}{\beta}\right)\right).$$

ex qua aequatione fiue λ, fiue λ' inueniri poteft, dummodo hae litterae non minores prodeant vnitate; quamobrem quosdam eiusmodi cafus euoluamus et ad praxin accommodemus.

IV. Tria autem mediorum diaphanorum genera hic potiffimum contemplabimur, quorum primum fit aqua pluuia, pro qua eft $n=\frac{4}{3}=1,3333$; fecundum vitrum ordinarium, pro quo eft $n=\frac{31}{20}=1,5500$ et

tertium vitrum Cryſtallinum, anglice Flint-Glaſs, pro quo eſt $n = \frac{8}{5} = 1,6000$; factoque calculo inuenimus, pro litteris graecis inde pendentibus ſequentes valores:

	Aqua	Vitrum commune	Vitrum Cryſtall.
n	$=1,3333$	$=1,5500$	$=1,6000$
ϱ	$=0,8000$	$=0,1907$	$=0,1111$
σ	$=2,2000$	$=1,6274$	$=1,5555$
τ	$=1,2490$	$=0,9051$	$=0,8606$
μ	$=1,9500$	$=0,9381$	$=0,8333$
ν	$=0,1025$	$=0,2326$	$=0,2666$
$\mu\nu$	$=0,1999$	$=0,2184$	$=0,2221.$

V. Quod autem ad valores differentialium dn attinet, optandum eſſet, vt ii per experimenta accuratiſſime definirentur; interim tamen duae ſequentes hypotheſes ex theoria deductae perpendi merentur, quandoquidem *Newtoniana*, ex qua dn ipſi $n-1$ foret proportionale, ad confuſionem tollendam eſſet inepta. Prima autem hypotheſis, quam dudum propoſueram, facit dn ipſi $n.\ \log.\ n$ proportionale; altera autem ex theoria attractionis deducta dat dn ipſi $\frac{nn-1}{n}$ proportionale; atque hinc valores formulae $\frac{dn}{n-1}$ pro vtraque hypotheſi euoluamus:

I^ma Hypotheſis $dn = n \log. n$

	Aqua	Vitrum commune	Vitrum Cryſtall.
dn	0,1665	0,2950	0,3265
$\frac{dn}{n-1}$	0,4995	0,5364	0,5442

II^da

II[da] Hypothefis $dn = \frac{nn-1}{n}$

	Aqua	Vitrum commune	Vitrum Cryftall.
$\frac{dn}{n-1}$	1, 7500	1, 6452	1, 6250

VI. Verum neutra harum hypothefium cum iis experimentis, quae Celeb. *Dollondus* circa prismata inftituit, fubfiftere poteft. Vfus nempe eft duobus prismatibus, quorum alterum ex vitro communi, erat paratum, angulo refringente exiftente triginta graduum; alterum autem ex vitro chryftallino confectum continebat angulum 19°; hisque duobus prismatibus inuerfe iunctis obferuauit, in fpectro inde ad parietem proiecto iridis colores nullos adparere; vnde conclufit, (fi n pro ratione refractionis vitri communis, n' vero pro refractione vitri chryftallini affumatur) fore $dn : dn' = 2 : 3$, quam rationem vocat rationem difperfionis; quo experimento admiffo fequitur, $\frac{dn}{n-1} : \frac{dn'}{n'-1} = 8 : 11$; ita, vt pro noftro calculo foret $\zeta = 8$ et $\eta = 11$, quae ratio ab allatis hypothefibus enormiter aberrat. Prima enim dat $\zeta : \eta = 0,5364 : 0,5442$ hoc eft proxime, vt $68 : 69$. Altera autem praebet $\zeta : \eta = 1,6452 : 1,6250$ vel proxime, vti $81 : 80$, vnde patet, hypothefin pofteriorem non poffe fubfiftere, quia ex ea fequeretur $\zeta > \eta$; prior vero tantum adhuc difcrepat, vt cum experimento neutiquam conciliari poffit; ex quo merito ancipites haeremus, quomodo nos in calculo hoc gerere debeamus.

amus. Interim experimentum *Dollondi* nonnullis adhuc difficultatibus premi videtur, quas hic ob oculos ponere visum est.

Digressio de refractione vitri chrystallini secundum experimenta *Dollondi*.

Tab. III. Fig. 17. I. Quo clarius theoriam, cui haec experimenta innituntur, ob oculos ponamus, rem aliquanto generalius, quam *Dollondus* fecit, consideremus. Repraesentet ergo triangulum ABC prisma triangulare ex vitro communi paratum, cuius angulus ad C, quem *Dollondus* 30° assumit, vocetur $= f$; alterum vero prisma ex vitro chrystallino factum DBE ita inuerse priori sit adplicatum, vt sit angulus $CBD = g$, ipseque angulus huius prismatis $DBE = h$ vbi in experimento examinando erat $g = 0$ et $h = 19°$ tum vero refractio radiorum mediorum in prismate priori sit vt $m : 1$, in posteriori vero vt $n : 1$, ita vt facultates radios dispergendi nobis per differentialia dm et dn repraesententur. Quibus positis *Dollondus* se obseruasse affirmat, si radius solis per haec duo prismata transmittatur, spectrum inde in parietem proiectum nullis coloribus inquinari. Vnde conclusit, fore $dm : dn = 2 : 3$, quam conclusionem adcuratius examinemus.

II. Hic primum notari conuenit, angulum, sub quo radius solis Op per faciem prioris lentis AC immittitur, non definiri, quasi perinde sit, sub quocunque

cunque angulo haec immerſio fiat, id quod pleniorem expoſitionem poſtulat. Ponamus igitur angulum OPA $= a$, qui eſt complementum anguli incidentiae et anguli ſequentes quauis refractione orti ſint p, q, r, t, u, v, e et radius iterum emergens VZ; ex quo ſtatim perſpicitur, fore $q = p + f$, $r = g + t$ et $u = v + h$; tum vero ex ratione refractionis intelligimus I) Coſ. $a = m$. Coſ. p. II) Coſ. $r = m$. Coſ. q. III) Coſ. $t = n$. Coſ. u. IV) Coſ. $e = n$. Coſ. v.

III. Quia iam hoc tranſitu nulla colorum diſperſio obſeruata eſſe perhibetur, neceſſe eſt, vt in emerſione V omnes radii colorati redditi ſint inter ſe paralleli, ideoque angulus e conſtans perinde ac primus angulus a, etiamſi rationes refractionis m et n ſuis differentialibus dm et dn augeantur; quippe qua mutatione tantum anguli p, q, r, t, u, v, fiunt variabiles.

IV. Nunc igitur primo hanc angulorum mutationem in primo prismate euoluamus, differentiando ſcilicet aequationes ſupra notatas; vnde erit

$dq = dp$ deinde $o = dm.\ \text{coſ.}\ p - m\,dp.\ \text{ſin.}\ p$ ſeu

$dp = \frac{dm.\ \text{coſ.}\ p}{m.\ \text{ſin.}\ p} = dq$;

tum vero $-dr.\ \text{ſin.}\ r = dm.\ \text{coſ.}\ q - m\,dq.\ \text{ſin.}\ q$ ſeu

$-dr.\ \text{ſin.}\ r = dm.\ \text{coſ.}\ q - \frac{dm.\ \text{coſ.}\ p.\ \text{ſin.}\ q}{\text{ſin.}\ p}$

$= \frac{dm}{\text{ſin.}\ p}(\text{coſ.}\ q\ \text{ſin.}\ p - \text{coſ.}\ p.\ \text{ſin.}\ q) = \frac{dm.\ \text{ſin.}\ (p-q)}{\text{ſin.}\ p}$

hincque tandem $dr = \frac{dm.\ \text{ſin.}\ f}{\text{ſin.}\ p.\ \text{ſin.}\ r}$ ob $p - q = -f$

 Simili

Simili modo euoluantur refractiones per alterum prisma, incipiendo ab angulo conſtante e vbi aequatio

$$\text{coſ.}\ e = n.\ \text{coſ.}\ v \text{ dat } o = dn.\ \text{coſ.}\ v - n dv.\ \text{ſin.}\ v \text{ ſiue}$$

$$dv = \frac{dn.\ \text{coſ.}\ v}{n.\ \text{ſin.}\ v} = du \text{ ob } u = v + h;$$

deinde vero aequatio coſ. $t = n.$ coſ. u dabit

$$-dt.\ \text{ſin.}\ t = dn.\ \text{coſ.}\ u - ndu.\ \text{ſin.}\ u = dn.\ \text{coſ.}\ u - \frac{dn.\ \text{coſ.}\ v.\ \text{ſin.}\ u}{\text{ſin.}\ v} = \frac{dn}{\text{ſin.}\ v}$$

$$(\text{coſ.}\ u.\ \text{ſin.}\ v - \text{coſ.}\ v.\ \text{ſin.}\ u) = \frac{dn}{\text{ſin.}\ v}.\ \text{ſin.}\ (v - u)$$

ergo ob $u - v = h$ habebitur $dt = + \frac{dn.\ \text{ſin.}\ h}{\text{ſin.}\ v.\ \text{ſin.}\ t}$; Cum autem ſit

$$r = g + t,\ \text{erit}\ dr = dt;\ \text{ergo}\ \frac{dm.\ \text{ſin.}\ f}{\text{ſin.}\ p.\ \text{ſin.}\ r} = \frac{dn.\ \text{ſin.}\ h}{\text{ſin.}\ v.\ \text{ſin.}\ t}$$

ex qua aequatione haec relatio inter dm et dn concluditur

$$dm : dn = \frac{\text{ſin.}\ h}{\text{ſin.}\ v.\ \text{ſin.}\ t} : \frac{\text{ſin.}\ f}{\text{ſin.}\ p.\ \text{ſin.}\ r}$$

$$dm : dn = \frac{\text{ſin.}\ p.\ \text{ſin.}\ r}{\text{ſin.}\ f} : \frac{\text{ſin.}\ v.\ \text{ſin.}\ t}{\text{ſin.}\ h}$$

quae proportio generaliter locum habet, quoties angulus ad e prodit conſtans

V. Iam hanc concluſionem ad *Dollondi* experimentum adplicemus et quia ibi eſt $g = o$, hincque

$r = t$ erit ratio inter vtramque diſperſionem ſeu

$$dm : dn = \frac{\text{ſin.}\ p}{\text{ſin.}\ f} : \frac{\text{ſin.}\ v}{\text{ſin.}\ h} = \frac{CQ}{PQ} : \frac{BU}{VU};$$

ex qua aequatione iam ſatis clare elucet, rationem $\frac{dm}{dn}$ ab angulo incidentiae a neutiquam eſſe independentem, vti *Dollondus* ſuppoſuiſſe videtur; ita, vt etſi diffu-

diffusio colorum pro certo quodam angulo a euanescens sit deprehensa, hanc conclusionem neutiquam ad omnes angulos incidentiae extendere liceat. Quoniam igitur hic angulus a *Dollondo* non est assignatus, ex hoc experimento nihil certi determinari poterit.

VI. Vt autem hic nullum dubium relinquatur, aliquot casus praecipuos pro angulo a calculo euoluamus, vt adpareat, quanta diuersitas inde in rationem $\frac{dm}{dn}$ ingrediatur, dum tamen plus vna vera esse nequit. Quare cum *Dollondus* inuenisset pro vtraque vitri specie a se adhibita I. $m = 1,53$ et II) $n = 1,58$, tum vero angulos $f = 30^\circ$ et $h = 19^\circ$ sequentia exempla hinc expediamus:

Exemplum I.

Angulum a ita constitutum concipiamus, vt fiat q angulus rectus, ideoque etiam r, t et u recti; erit sin. $p =$ cos. f et sin. $v =$ cos. h, ex quo coligitur $dm : dn =$ cotang. f : cotang. h; siue $\frac{dm}{dn} = 0,59639$, quae iam minor est, quam $\frac{2}{3}$

Exemplum II.

Sit $a = 90^\circ$, qui forte est ipse casus *Dollondi*; atque hinc inuenientur sequentes anguli $p = 90^\circ$; $q = 120^\circ$ $r = 139^\circ, 53' = t$. $u = 118^\circ 57'$. $v = 99^\circ 57'$. $e = 105^\circ 50'$. Vnde sequitur $\frac{dm}{dn} = \frac{\sin. p . \sin. h.}{\sin. l . \sin. v.} = 0,6615$. ideoque proxime $dm : dn = 2 : 3$. vt habet *Dollondus*.

Cete-

Ceterum hinc iam euidens eſt, hanc rationem pendere etiam ab angulo incidentiae, quae tamen in hoc experimento non eſt commemorata.

Si ergo hanc rationem cum *Dollondo* aſſumamus, fiet porro $\frac{dm}{m-1} : \frac{dn}{n-1} = 116 : 159 = 7 : 10$, ita, vt futurum ſit pro lentibus obiectiuis duplicatis, quibus *Dollondus* vtitur, $\zeta = 7$, $\eta = 10$, quarum conſtructionem hic ex noſtris principiis inueſtigemus.

Lens Obiectiua Clariſſ. *Dollondi* duplicata prior, anteriore lente ex vitro communi, poſteriore vero ex vitro chryſtallino parata.

Cum igitur ſit $n = 1,53$ et $n' = 1,58$, $\zeta = 7$, $\eta = 10$, erit ſtatim $f = \frac{\eta}{\eta - \zeta} = \frac{10}{3}$ et $g = \frac{-\zeta}{\eta - \zeta} = -\frac{7}{3}$;

vnde diſtantiae determinatrices vtriusque lentis erunt

Pro prima a, et $\alpha = \frac{(\eta - \zeta) a \mathfrak{b}}{\eta a + \zeta \mathfrak{b}}$

Pro poſteriori b et $\beta = \frac{-(\eta - \zeta) a \mathfrak{b}}{\eta a + \zeta \mathfrak{b}} = -\alpha$.

Dum ipſius lentis duplicatae diſtantiae determinatrices ſunt a et $\mathfrak{b}$.

Tantum igitur reſtat, vt pro his binis lentibus numeri arbitrarii λ et λ' definiantur; et quoniam quaeſtio circa lentes obiectiuas verſatur, ſtatuamus ſtatim $a = \infty$, vt adhuc reſolui debeat haec aequatio

$$0 = \mu \lambda \eta^3 - \mu' \lambda' \zeta^3 + \eta (\eta - \zeta) \zeta \mu' \nu'.$$

ſeu $\mu' \lambda' \zeta^3 = \mu \lambda \eta^3 + \eta \zeta (\eta - \zeta) \mu' \nu'$. quae ob $\zeta = 7$ et $\eta = 10$ abit in hanc

$$343 \cdot \mu' \lambda' = 1000 \cdot \mu \lambda + 210 \cdot \mu' \nu'.$$

II. Vt igitur *Dollondum* sequamur, pro his valoribus $n = 1,53$ et $n' = 1,58$ litteras inde deriuatas quaerere debemus.

Eruitur autem ex formulis superioribus

$\mu = 0,9875$	$\mu' = 0,8724$
$l\mu = 9.9945449$	$l\mu' = 9.9407397$
$\nu = 0,2194$	$\nu' = 0,2529$
$l.\nu = 9.3413418$	$l.\nu' = 9.4030044$
$l.\mu\nu = 9.3358867$	$l.\mu'\nu' = 9.3437441$

Hinc coëfficientes trium terminorum nostrae aequationis computentur.

$l.\ 343 = 2,5352941$	$l.\ 1000 = 3,0000000$	$l.\ 210 = 2,3222193$
$l.\ \mu' = 9.9407397$	$l.\ \mu = 9.9945449$	$l.\ \mu'\nu' = 9.3437441$
2.4760338	2.9945449	1.6659634
Subtrah. - - - -	2.4760338	2.4760338
	0.5185111	9.1899296
	$3,3000$	$0,1548$

ita, vt sit

$$\lambda' = 3,3000\,\lambda + 0,1548$$

quare sumto

$$\lambda = 1, \text{ fit } \lambda' = 3,4548; \text{ ergo}$$

$$\lambda' - 1 = 2,4548, \text{ et log. } \sqrt{\lambda' - 1} = 0,1950080.$$

III. Iam ad radios facierum harum lentium definiendos, cum sit

$$a = \infty \text{ et } \alpha = \frac{(\eta - \zeta)\epsilon}{\eta} \text{ et } b = -\frac{(\eta - \zeta)\epsilon}{\eta}$$

erit pro lente priore radius faciei

$$\text{anterioris} = \frac{\alpha}{\sigma} = \frac{3\,\mathfrak{G}}{10 \cdot \sigma}\,; \quad \text{posterioris} = \frac{\alpha}{\varrho} = \frac{3\,\mathfrak{G}}{10 \cdot \varrho}.$$

Pro lente posteriore radius faciei

$$\text{anter.} = \frac{-3\,\mathfrak{G}}{10\,\varrho' - 3\,\sigma' \pm 7\,\tau' \surd(\lambda' - 1)}$$

$$\text{poster.} = \frac{-3\,\mathfrak{G}}{10\,\sigma' - 3\,\varrho' \mp 7\,\tau' \surd(\lambda' - 1)}$$

quare pro duplici refractione $n = 1,53$ et $n' = 1,58$ valores litterarum ϱ, σ, τ quaeri oportet, qui ita se habebunt

$n = 1,53$	$n' = 1,58$
$\varrho = 0,2266$	$\varrho' = 0,1413$
$\sigma = 1,6602$	$\sigma' = 1,5827$
$\tau = 0,9251$	$\tau' = 0,8775$.

IV. Cum igitur $\mathfrak{G}$ denotet distantiam focalem ipsius lentis duplicatae, quam deinceps littera P indicabimus, pro lente priori ex vitro communi subviridi, cuius refractio est $n = 1,53$, reperiemus vtramque faciem; erit radius faciei.

Pro prima lente

anterioris $= 0,1807 \cdot P$; posterioris $= 1,3239 \cdot P$.

Pro altera ex vitro chrystallino, cuius refractio $n = 1,58$ facies ita erunt comparatae.

Pro secunda lente

$$\text{anterioris} = \frac{-3\,P}{-3,3351 \pm 5 \cdot 6240}$$

$$\text{posterioris} = \frac{-3\,P}{15,4031 \mp 5 \cdot 6240}$$

quia

quia nunc radios curuaturae minores euitari conuenit, fignorum ambiguorum ea funt fumenda, quae denominatores minores producunt; id quod fit, fi figna fuperiora valeant; hinc ergo obtinebimus fequentes determinationes:

radius faciei anterioris $= \frac{-3P}{6,2889} = 0.4770. P$

pofterioris $= \frac{-3P}{5.7791} = -0,5191\ P$

quae lens obiectiua capax eft aperturae, cuius femidiameter eft $= 0.0452\ P$.

Lens obiectiua duplicata altera, anteriore lente ex vitro chryftallino, pofteriori ex vitro communi parata.

I. Cum igitur hic fit

$n = 1,58$ et $n' = 1,53$ erit $\zeta = 10$ et $\eta = 7$; hinc

$f = \frac{\eta}{\eta - \zeta} = -\frac{7}{3}$. et

$g = -\frac{\zeta}{\eta - \zeta} = \frac{10}{3}$; Hincque

$a = \frac{-3.\,a g}{7a + 10g} = -\frac{3g}{7}$ ob

$a = \infty$, et $b = \frac{3g}{7}$.

Confufio autem primi generis vt euanefcat, fieri debet

$0 = 343\ \mu\lambda - 1000\ \mu'\lambda' - 210\ \mu'\nu'$ feu

$343\ \mu\lambda = 1000\ \mu'\lambda' + 210\ \mu'\nu'$.

 II.

II. Huius aequationis terni coëfficientes iam quaerantur:

$l.\ 343 = 2{,}5352941.$	$l.\ 1000 = 3{,}0000000$	$l.\ 210 = 2{,}3222193$
$l.\ \mu = 9{,}9407397$	$l.\ \mu' = 9{,}9945449$	$l.\ \mu'\nu' = 9.3358867$
2.4760338	2.9945449	1.6581060
	2.4760338	2.4760338
	0.5185111	9.1820722
	$3{,}3000$	$0{,}1520$

ita, vt fit

$\lambda = 3{,}3000\,\lambda' + 0{,}1520$; quare fumto

$\lambda' = 1$ fiet $\lambda = 3.4520$ ac propterea

$\lambda - 1 = 2{,}4520$ et $\log. \sqrt{(\lambda - 1)} = 0{,}3895205.$

III. Cum nunc fit

$a = \infty$ et $\alpha = \frac{-3\mathfrak{B}}{7}$ et $b = \frac{3}{7}\mathfrak{B}$

erunt radii facierum vtriusque lentis

Pro lente priore radius faciei

anterioris $= \frac{-3\mathfrak{B}}{7(\sigma \pm \tau\sqrt{(\lambda-1)})}$; pofter. $= \frac{-3\mathfrak{B}}{7\varrho \pm 7\tau\sqrt{(\lambda-1)}}$

Pro lente pofteriore radius faciei

anterioris $= \frac{3\mathfrak{B}}{7\varrho' + 3\sigma'}$; pofterioris $= \frac{3\mathfrak{B}}{7\sigma' + 3\varrho'}$

eft autem

$\varrho = 0{,}1413$; $\sigma = 1{,}5827$; $\tau = 0{,}8775.$

$\varrho' = 0{,}2266$; $\sigma' = 1{,}6602$:

quibus

quibus ſubſtitutis erit pro priore lente radius faciei

anter. $\frac{-3\,\mathfrak{C}}{11.0789 \pm 9.6187}$; poſter. $= \frac{-3\,\mathfrak{C}}{0.9891 \mp 9.6187}$

valeant autem ob rationes ſupra allegatas ſigna inferiora, eritque radius faciei

anterioris $= \frac{-3\,\mathfrak{C}}{1.4602} = -2,0545\ P$

poſterioris $= \frac{-3\,\mathfrak{C}}{10.5078} = -0,2828\ P.$

Pro lente autem poſteriore erit radius faciei

anterioris $= \frac{3\,\mathfrak{C}}{6,5668} = 0,4568\ P$

poſterioris $= \frac{3\,\mathfrak{C}}{12.3012} = 0,2438\ P.$

Inter quos quatuor radios minimus eſt 0,2438 P vnde haec lens duplicata aperturam admittere poteſt, cuius ſemidiameter $= 0,0609.$ P ita, vt haec lens maiorem admittat aperturam, quam praecedens.

De Lentibus obiectiuis ex tribus lentibus compoſitis.

I. Sit pro prima lente ratio refractionis $= n$, pro ſecunda $= n'$, pro tertia $= n''$, deinde diſtantiae determinatrices primae a et α, ſecundae b et $\mathfrak{C}$ tertiae c et γ cum numeris arbitrariis λ, λ', λ'' eritque $\alpha + b = 0$, et $\mathfrak{C} + c = 0$, eruntque a et γ di-

ſtantiae determinatrices ipſius lentis triplicatae. Iam vero ſtatuatur

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right);$$
$$\frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right);$$
$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{a}+\frac{1}{\gamma}\right);$$

fietque

$$\frac{1}{\alpha}=\frac{f-1}{a}+\frac{f}{\gamma};\ \alpha=\frac{a\gamma}{\gamma(f-1)+af}=-b$$
$$\frac{1}{\beta}=\frac{g}{a}+\frac{g}{\gamma}+\frac{f-1}{a}+\frac{f}{\gamma}$$
$$\frac{1}{\beta}=\frac{g+f-1}{a}+\frac{g+f}{\gamma}=-\frac{1}{c}$$

ac proinde

$$\frac{-f-g+1}{a}-\frac{f-g+1}{\gamma}=\frac{h}{a}+\frac{h}{\gamma}$$

vnde ſequitur fore

$$\frac{-f-g-h+1}{a}-\frac{(f+g+h-1)}{\gamma}=0$$

hinc $f+g+h=1$.

II. Conditio vero confuſionem colorum penitus tollens poſtulat, vt ſupra vidimus, hanc aequationem

$$0=\frac{dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right);$$

quae ponendo

$$\frac{dn}{n-1}=\zeta;\ \frac{dn'}{n'-1}=\eta;\ \frac{dn''}{n''-1}=\vartheta$$

abit in hanc formam $0=\zeta f+\eta g+\vartheta h$.

III. Vt autem confuſio ab apertura pendens deſtruatur, huic aequationi ſatisfieri debet:

$$0=\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}-\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3B^3}$$
$$+\frac{\mu''.(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3B^3C^3}.$$

vbi

vbi notetur esse

$$\frac{A+1}{A}=af\left(\frac{1}{a}+\frac{1}{\gamma}\right) \text{ et}$$

$$\frac{B+1}{B}=bg\left(\frac{1}{a}+\frac{1}{\gamma}\right) \text{ et}$$

$$\frac{C+1}{C}=ch\left(\frac{1}{a}+\frac{1}{\gamma}\right);$$

vnde termini priores continentes λ erunt

$$a^3\left(\frac{1}{a}+\frac{1}{\gamma}\right)^3(\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' h^3)$$

termini vero tres posteriores fiunt

$$a^3\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\frac{\mu\nu f}{a\alpha}+\frac{\mu'\nu' g}{b\beta}+\frac{\mu''\nu'' h}{c\gamma}\right)$$

adeoque prodit haec aequatio

$$0=\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' h^3$$
$$+\frac{1}{\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2}\left(\frac{\mu\nu f}{a\alpha}+\frac{\mu'\nu' g}{b\beta}+\frac{\mu''\nu'' h}{c\gamma}\right)$$

vbi notandum est, ex praecedentibus esse

$$\frac{1}{a\alpha}=\frac{1}{a}\left(\frac{f-1}{a}+\frac{f}{\gamma}\right);$$

$$\frac{1}{b\beta}=-\left(\frac{f-1}{a}+\frac{f}{\gamma}\right)\left(\frac{g+f-1}{a}+\frac{f+g}{\gamma}\right)$$

$$\frac{1}{c\gamma}=-\frac{1}{\gamma}\left(\frac{f+g-1}{a}+\frac{f+g}{\gamma}\right)$$

IV. Quodsi nunc ponamus $a=\infty$, sequentes aequationes habebuntur resoluendae:

$$\alpha=\frac{\gamma}{f} \text{ et } b=\frac{-\gamma}{f};\ \frac{1}{\beta}=\frac{g+f}{\gamma} \text{ seu}$$

$$\beta=\frac{\gamma}{f+g} \text{ et } c=\frac{-\gamma}{f+g}.$$

Quibus

Quibus adiungi debent hae aequationes:

$f+g+h=1$, et $\zeta f+\eta g+\vartheta h=0$ et

pro omni confusione tollenda:

ob $\frac{1}{a\alpha}=0$; $\frac{1}{b\mathfrak{b}}=\frac{-f(f+g)}{\gamma^2}$; $\frac{1}{c\gamma}=-\frac{f+g}{\gamma^2}$

habebitur haec aequatio

$$0=\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' h^3$$
$$-(f+g)(\mu'\nu' fg+\mu''\nu''.h).$$

V. Hic quidem assumsimus, omnes tres materias, ex quibus hae lentes sunt confectae, esse diuersas; in praxi autem nulla ratio suadet, vt tres diversas vitri species adhibeamus; sed potius primam et tertiam lentem ex eadem materia parari conueniet; ex quo consequimur statim

$$n''=n;\ \mu''=\mu;\ \nu''=\nu \text{ et } \vartheta=\zeta,$$

ita, vt sit

$$f+h+\frac{\eta}{\zeta}\cdot g=0;$$

quae ab altera subtracta dat

$$g-\frac{\eta}{\zeta}\cdot g=1;\ \text{hincque } g=\frac{\zeta}{\zeta-\eta}$$

vnde porro fit

$$f+h=-\frac{\eta}{\zeta-\eta}.$$

Ponamus hunc in finem

$$f-h=\frac{x}{\zeta-\eta} \text{ eritque}$$

$$f=\frac{x-\eta}{2(\zeta-\eta)} \text{ et } h=\frac{-x-\eta}{2(\zeta-\eta)}$$

ex

ex quibus valoribus prodit

$$\alpha = \frac{2\gamma(\zeta-\eta)}{x-\eta}, b = \frac{-2\gamma(\zeta-\eta)}{x-\eta};$$

$$\mathfrak{G} = \frac{2\gamma(\zeta-\eta)}{x+2\zeta-\eta}; c = \frac{-2\gamma(\zeta-\eta)}{x+2\zeta-\eta}.$$

VI. Aequatio ergo, cui adhuc ſatisfieri oportet, erit

$$0 = \tfrac{1}{4}\mu\lambda(x-\eta)^3 + 2\mu'\lambda'\zeta^3 - \tfrac{1}{4}\mu\lambda''(x+\eta)^3$$
$$-\tfrac{1}{2}(x+2\zeta-\eta)(\mu'\nu'(x-\eta)\zeta-\mu\nu(x+\eta)(\zeta-\eta))$$

ex qua aequatione vel vna litterarum λ vel etiam numerus x definiri poteſt; quo poſteriori caſu hoc commodi conſequeremur, vt ſingulae litterae $\lambda = 1$ ſtatui poſſent ſicque ſingulae lentes vtpote ex principio minimi deductae facillime conſtrui poſſent; hanc autem inueſtigationem non aliter, niſi in exemplis particularibus ſuſcipere licebit.

Ponamus igitur

$$\lambda = 1, \lambda' = 1 : \lambda'' = 1 \text{ fietque}$$

$$0 = -\tfrac{1}{2}\mu(3\eta x^2+\eta^3)+2\mu'\zeta^3-\tfrac{1}{2}\mu'\nu'\zeta(x^2+2(\zeta-\eta)x-\eta(2\zeta-\eta))$$
$$+\tfrac{1}{2}\mu\nu(\zeta-\eta)(x^2+2\zeta x+\eta(2\zeta-\eta))$$

quae aequatio reducitur ad hanc

$$x^2(-3\mu\eta-\mu'\nu'\zeta+\mu\nu(\zeta-\eta))$$
$$+2x.(\zeta-\eta)\zeta(\mu\nu-\mu'\nu')-\mu\eta^3+4\mu'\zeta^3+\mu'\nu'\eta\zeta(2\zeta-\eta)+\mu\nu.\eta(\zeta-\eta)(2\zeta-\eta)=0.$$

VII. Adplicemus haec ad illas duas ſpecies vitri, quibus *Dollondus* vſus eſt, ac duo caſus euolvendi occurrunt. Primus igitur caſus eſto, quo tam prima, quam tertia lens ex vitro chryſtallino con-

ficitur; media autem ex vitro communi subuiridi; ita, vt sit $n = n'' = 1,58$ et $n' = 1,53$, ideoque $\zeta = 10$ et $\eta = 7$ tum vero $\mu = 0,8724$, $\nu = 0,2529$ et $l.\,\mu\nu = 9.3437441$ deinde $\mu' = 0,9875$; $\nu' = 0,2194$ et $l.\,\mu'\nu' = 9.3358867$ vnde superior aequatio reducetur ad hanc formam

$$+19,8255\,x^2 = 0,2340\,x + 4177,6081, \text{ siue}$$

$$x^2 = 0,0118\,x + 210,7190$$

cuius resolutio praebet

$$x = 0,0059 \pm \sqrt{210,7190}$$

$$= 0,0059 \pm 14,5161$$

vnde ambo valores ipsius x sunt

$$x = 14,5220$$

$$x = -14,5102$$

Lens obiectiua triplicata perfecta prior cuius lens prima et tertia ex vitro chrystallino, pro quo $n = 1,58$, media vero ex vitro communi subuiridi, $n = 1,53$, est confecta.

I. Ob duplicem valorem ipsius x, duas etiam eiusmodi lentes exhibere poterimus; sit igitur primo $x = 14,5220$ et quia est $\zeta = 10$ et $\eta = 7$, erit $f = 1,2537$; $g = 3,3333$, $h = -3.5870$

$$\frac{1}{\alpha} = \frac{1,2537}{\gamma};\quad \frac{1}{b} = \frac{-1,2537}{\gamma};\quad \frac{1}{\beta} = \frac{4.5870}{\gamma} \text{ et } \frac{1}{c} = \frac{-4.5870}{\gamma};$$

quodsi

quodſi iam harum trium lentium radios facierum anteriorum ponamus F, F', F'', poſteriorum vero G, G', G'' ob $\lambda = \lambda' = \lambda'' = 1$, habebimus

$$\frac{1}{F} = \frac{\varrho}{a} + \frac{\sigma}{\alpha} = \frac{\sigma}{\alpha} = \frac{1{,}2537 \cdot \sigma}{\gamma}; \quad \frac{1}{G} = \frac{\varrho}{\alpha} + \frac{\sigma}{a} = \frac{\varrho}{\alpha} = \frac{1{,}2537 \varrho}{\gamma} \text{ et}$$

$$\frac{1}{F'} = \frac{\varrho'}{b} + \frac{\sigma'}{\mathfrak{b}}; \quad \frac{1}{G'} = \frac{\varrho'}{\mathfrak{b}} + \frac{\sigma'}{b}$$

et ſubſtitutis valoribus

$$\frac{1}{F'} = \frac{-1{,}2537 \cdot \varrho'}{\gamma} + \frac{4{,}5870 \cdot \sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{4{,}5870 \cdot \varrho'}{\gamma} - \frac{1{,}2537 \cdot \sigma'}{\gamma}$$

ſimili modo pro lente tertia

$$\frac{1}{F''} = \frac{\varrho}{c} + \frac{\sigma}{\gamma}; \quad \frac{1}{G''} = \frac{\varrho}{\gamma} + \frac{\sigma}{c}$$

$$\frac{1}{F''} = \frac{-4{,}5870 \cdot \varrho + \sigma}{\gamma}; \quad \frac{1}{G''} = \frac{\varrho - 4{,}5870 \cdot \sigma}{\gamma}$$

exiſtente

$$\varrho = 0{,}\ 1413; \quad \sigma = 1{,}\ 5827 \text{ et}$$
$$\varrho' = 0{,}\ 2266; \quad \sigma' = 1{,}\ 6602$$

ac ſi ipſius lentis triplicatae diſtantia focalis ponatur $= P$, vt hic ſit $\gamma = P$, reperiuntur valores ſequentes

$\frac{1}{F} = \frac{1{,}0843}{P}$, ſiue $F = +0{,}\ 5039.\ P.$

$\frac{1}{G} = \frac{0{,}1771}{P}$, hincque $G = +5{,}\ 6450.\ P.$

$\frac{1}{F'} = \frac{7{,}3312}{P}$, ſiue $F' = +0{,}\ 1364\ P.$

$\frac{1}{G'} = \frac{-1{,}0420}{P}$, hincque $G' = -0{,}\ 9597.\ P.$

$\frac{1}{F''} = \frac{0{,}9346}{P}$, hincque $F'' = +1{,}\ 0699.\ P.$

$\frac{1}{G''} = \frac{-7{,}1186}{P}$, hincque $G'' = -0{,}\ 1404.\ P.$

II. Inter hos ſex radios cum minimus ſit o, 1364. P, eius pars quarta $= 0,0341$. P dat ſemidiametrum maximae aperturae, cuius haec lens triplicata eſt capax. Contra vero haec lens hac praerogatiua gaudet, quod eius conſtructio ob $\lambda = 1$; $\lambda' = 1$; $\lambda'' = 1$ in praxi minimae difficultati ſit obnoxia.

Altera lens triplicata, pro qua pariter, vt ante, lens prima et tertia ex vitro chryſtallino, media vero ex vitro communi ſubviridi paratur.

I. Sumatur iam

$$x = -14,5102\text{, ob } \zeta = 10\text{, et } \eta = 7 \text{ erit}$$

$$f = -3,5850,\ g = 3,3333;\ h = +1,2517.$$

$$\frac{1}{a} = \frac{-3.5850}{\gamma};\ \frac{1}{b} = \frac{3.5850}{\gamma}$$

$$\frac{1}{\beta} = \frac{-0.2517}{\gamma};\ \frac{1}{c} = +\frac{0.2517}{\gamma}$$

vnde pro radiis ſingularum facierum prodibunt ſequentes formulae:

$$\frac{1}{F} = \frac{\sigma}{a} = \frac{-3.5850.\sigma}{\gamma};\ \frac{1}{G} = \frac{\rho}{a} = \frac{-3.5850.\rho}{\gamma}$$

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{\beta} = \frac{3.5850.\rho' - 0.2517.\sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\rho'}{\beta} + \frac{\sigma'}{b} = \frac{-0.2517\rho' + 3.5850\sigma'}{\gamma}$$

$$\frac{1}{F''} = \frac{\rho}{c} + \frac{\sigma}{\gamma} = \frac{0.2517\rho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\rho}{\gamma} + \frac{\sigma}{c} = \frac{\rho + 0.2517.\sigma}{\gamma}$$

vnde ſequentes valores determinati pro his radiis inveniuntur:

$$\frac{1}{F} = -\frac{5,6739}{P} \text{ hincque } F = -0,1762.\ P.$$

$$\frac{1}{G} = -\frac{0,5065}{P} \text{ hincque } G = -1,9741.\ P.$$

$$\frac{1}{F'} = \frac{0.3045}{P} \text{ hincque } F' = +2,5349.\ P.$$

$$\frac{1}{G'} = \frac{5.8048}{P}, \text{ hincque } G' = +0,1696.\ P.$$

$$\frac{1}{F''} = \frac{1.6182}{P}, \text{ hincque } F'' = +0,6194.\ P.$$

$$\frac{1}{G''} = \frac{0.5396}{P} \text{ hincque } G'' = +1,8532.\ P.$$

quorum ſex radiorum minimus eſt 0,1696 P, cuius pars quarta 0,0424. P dat ſemidiametrum aperturae maximae, cuius haec lens triplicata eſt capax. Ideoque haec lens obiectiua praecedenti eſt anteferenda.

VIII. Secundum §. VII. adhuc alius caſus euolui debet, quo lens prima et tertia ex vitro communi ſubuiridi $n = n'' = 1,53$; media autem ex Chryſtallino $n = 1,58$ parari poteſt; ſicque erit $\zeta = 7$, $\eta = 10$. Vnde cum fiat

$$\mu = 0.9875,\ l.\ \mu\nu = 9.3358867$$

$$\mu' = 0.8724.\ l.\ \mu'\nu' = 9.3437441.$$

aequatio noſtra pro confuſione tollenda ſequentem induet formam:

$$31,8198\, x^2 = +0,1638\ x + 245,2121 \text{ ſiue}$$

$$x^2 = 0,0051.\, x + 7,7063$$

cuius resolutio dat

$$x = 0,0025 \pm 2,7760$$

vnde bini valores ipsius x erunt

$$\text{I. } x = 2,7785$$

$$\text{II. } x = -2.7735.$$

Lens triplicata perfecta prior, cuius lens prima et tertia ex vitro communi, $n = 1,53$. media ex chrystallino, $n = 1,58$ est confecta.

Cum igitur sit

$$x = 2,7785 \text{ et } \zeta = 7, \eta = 10 \text{ erit}$$

$$f = +1,2036;\ g = -2,3333.\ h = 2,1297;$$

hincque

$$\frac{1}{\alpha} = \frac{1.2036}{\gamma},\ \frac{1}{b} = \frac{-1.2036}{\gamma},\ \frac{1}{\beta} = \frac{-1.1297}{\gamma},\ \frac{1}{c} = \frac{1.1297}{\gamma}$$

ex quibus colligitur

$$\frac{1}{F} = \frac{\sigma}{\alpha} = \frac{1.2036.\sigma}{\gamma};\ \frac{1}{G} = \frac{\varrho}{\alpha} = \frac{1.2036\,\varrho}{\gamma}$$

$$\frac{1}{F'} = \frac{\varrho'}{b} + \frac{\sigma'}{\beta} = \frac{-1.2036\,\varrho' - 1.1297\,\sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\varrho'}{\beta} + \frac{\sigma'}{b} = \frac{-1.1297.\varrho' - 1.2036.\sigma'}{\gamma}$$

$$\frac{1}{F''} = \frac{\varrho}{c} + \frac{\sigma}{\gamma} = \frac{1.1297.\varrho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\sigma}{c} + \frac{\varrho}{\gamma} = \frac{1.1297.\sigma + \varrho}{\gamma}$$

vbi

vbi $\varrho = 0,2266$; $\sigma = 1,6602$

$\varrho' = 0,1413$; $\sigma' = 1,5827$

Facto igitur calculo obtinebimus:

$\frac{1}{F} = \frac{1,9982}{P}$ hincque $F = 0,5004.\ P.$

$\frac{1}{G} = \frac{0,2727}{P}$ hincque $G = 3,6665.\ P.$

$\frac{1}{F'} = \frac{-1,9580}{P}$ hincque $F' = -0,5107.\ P.$

$\frac{1}{G'} = \frac{-2,0645}{P}$ hincque $G' = -0,4843.\ P.$

$\frac{1}{F''} = \frac{1,9161}{P}$ hincque $F'' = 0,5219.\ P.$

$\frac{1}{G''} = \frac{2,1021}{P}$ hincque $G'' = 0,4757.\ P.$

inter quos radios cum minimus fit 0, 4757 P eius pars quarta 0, 1189. P dat femidiametrum aperturae, quam haec lens admittit; ideoque haec lens duabus praecedentibus est anteferenda.

Lens triplicata perfecta altera, cuius lens prima et tertia ex vitro communi $n = 1,53$ media ex chryftallino, $n = 1,58$ eft parata.

Sumatur iam ex binis ipfius x valoribus alter negatiuus $x = -2,7735$; et cum fit $\zeta = 7$, $\eta = 10$, inuenitur

$f = +2,1289$; $g = -2,3333$ $h = +1,2044$;

hincque

$\frac{1}{\alpha} = \frac{2,1289}{\gamma}$; $\frac{1}{b} = \frac{-2,1289}{\gamma}$

$\frac{1}{\beta} = \frac{-0,1044}{\gamma}$; $\frac{1}{c} = \frac{0,1044}{\gamma}$

ex

ex quibus consequimur

$$\frac{1}{F} = \frac{\sigma}{\alpha} = \frac{2.1289.\sigma}{\gamma};\quad \frac{1}{G} = \frac{\varrho}{\alpha} = \frac{2.1289.\varrho}{\gamma}$$

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{\text{ß}} = \frac{-2.1289.\varrho' - 0.1044\,\sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\varrho'}{\text{ß}} + \frac{\sigma'}{b} = \frac{-0.1044.\varrho' - 2.1289.\sigma'}{\gamma}$$

$$\frac{1}{F_{,,}} = \frac{\varrho}{c} + \frac{\sigma}{\gamma} = \frac{0.1044\,\varrho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\sigma}{c} + \frac{\varrho}{\gamma} = \frac{0.1044\,\sigma + \varrho}{\gamma}.$$

Vnde facto calculo pro radiis facierum singularum lentium sequentes inuenientur radii.

$$\frac{1}{F} = \frac{3,5344}{P};\ \text{seu } F = 0,2829.\ P.$$

$$\frac{1}{G} = \frac{0.4824}{P};\ \text{seu } G = 2,0729.\ P.$$

$$\frac{1}{F'} = \frac{-0.4660}{P};\ \text{seu } F' = -2,1459.\ P.$$

$$\frac{1}{G'} = \frac{-3.3841}{P};\ \text{seu } G' = -0,2955.\ P.$$

$$\frac{1}{F''} = \frac{1.6838}{P};\ \text{seu } F'' = 0,5938.\ P.$$

$$\frac{1}{G''} = \frac{0.3999}{P};\ \text{seu } G'' = 2,5006.\ P.$$

Inter quos radios minimus est 0, 2829. P, cuius pars quarta 0, 0707. P, dat semidiametrum aperturae maximae, cuius haec lens triplicata est capax.

IX. Hae quatuor lentes triplicatae ideo prae ceteris, quas proponere liceret, sunt commendandae, quod in iis assumsimus $\lambda = 1$; $\lambda' = 1$; $\lambda'' = 1$. Quam ob caussam in praxi facillime construi possunt, ex iisdem vero principiis etiam constructio eiusmodi lentium com-

compoſitarum deduci poſſet qua inter binas lentes vitreas aqua aliudue fluidum includeretur, ita, vt lens media ex fluido conſtaret: tum autem praeter conditiones ante tractatas duae nouae eſſent implendae; ſcilicet vt radii facierum mediae lentis aequales eſſent ſtatuendi et contrarii radiis facierum internarum primae ac tertiae lentis. Ob has igitur nouas conditiones non amplius liceret numeros λ, λ' et λ'' vnitati aequales aſſumere; qua poſitione conſtructioni practicae quam maxime conſulitur; deinde vero etiam radii facierum tam parui prodirent, vt tales lentes nimis exiguam aperturam eſſent admiſſurae; quam ob cauſſam operae pretium haud videtur, in earum conſtructionem adcuratius inquirere.

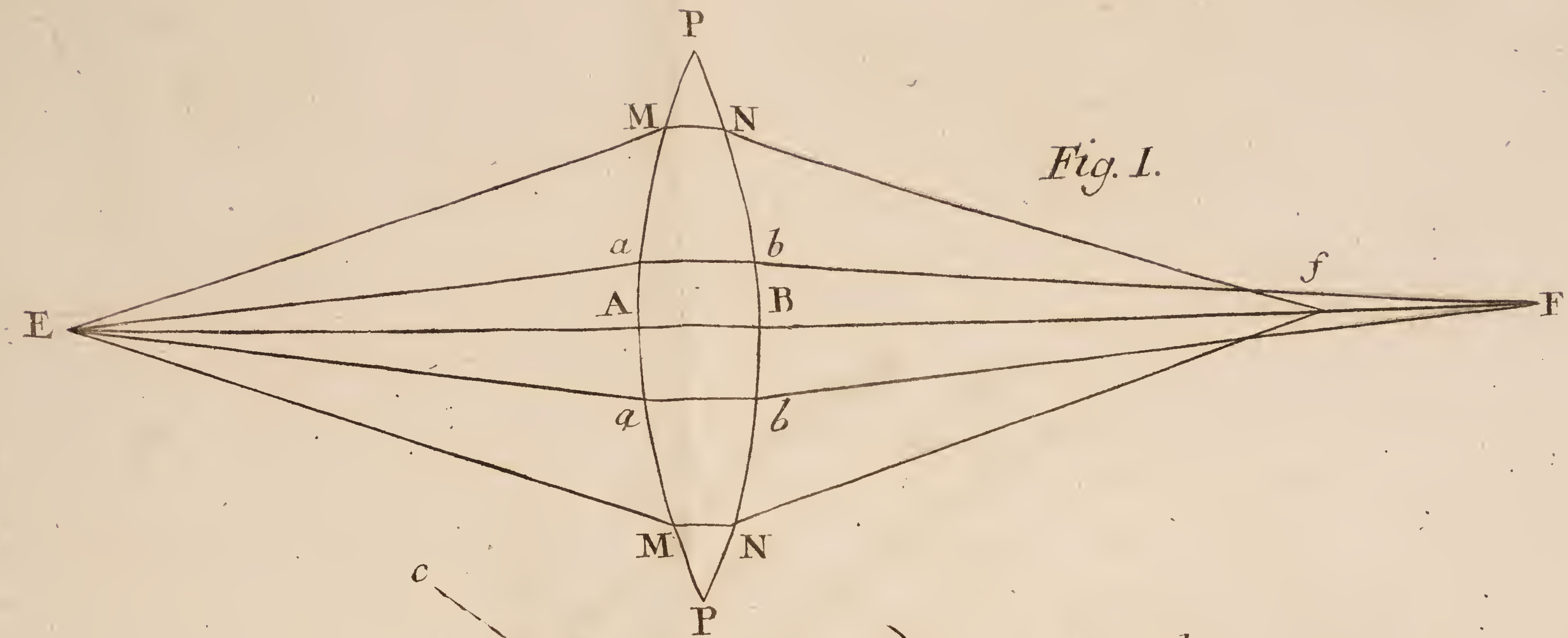

Fig. 1.

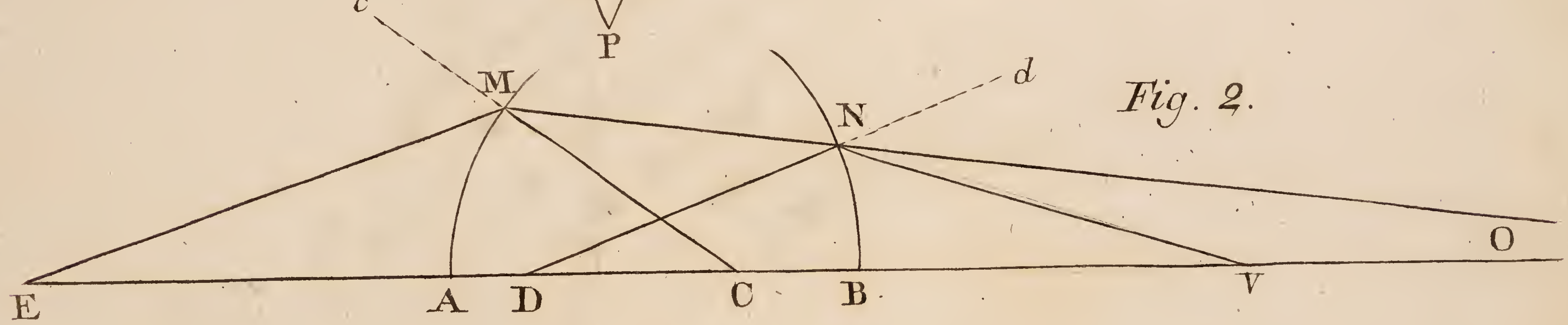

Fig. 2.

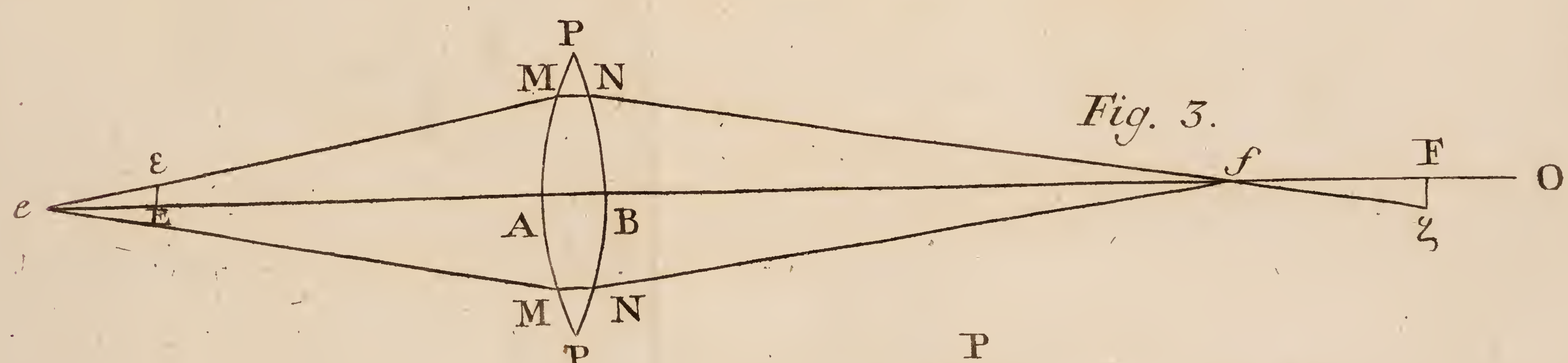

Fig. 3.

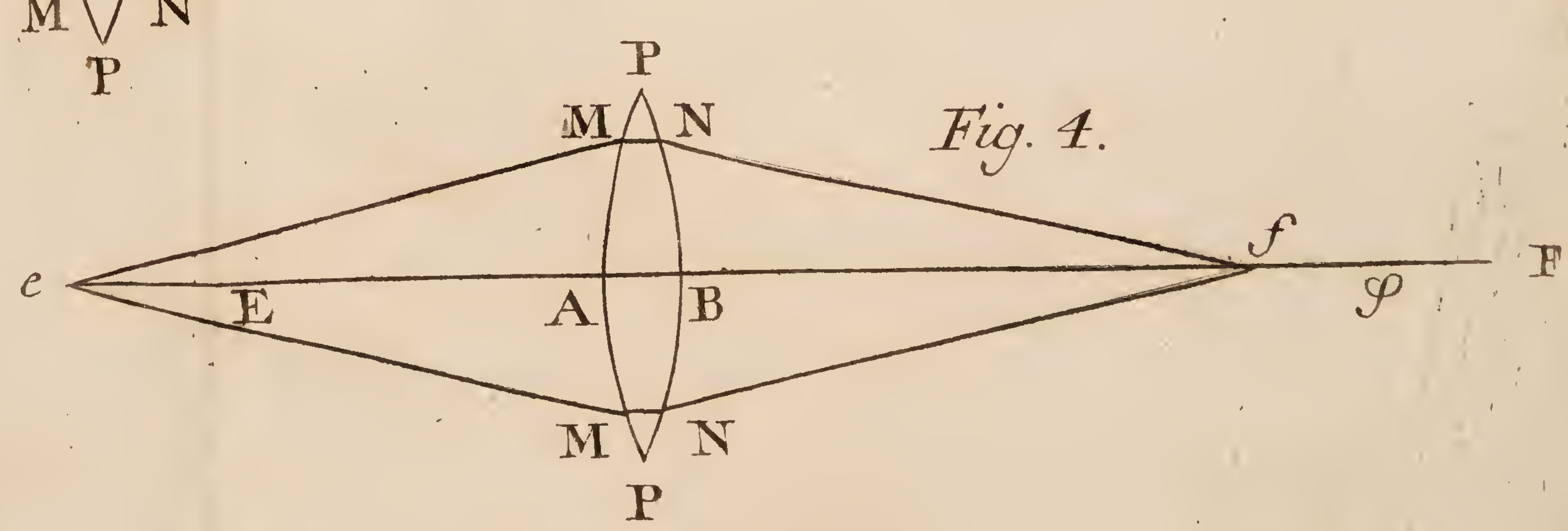

Fig. 4.

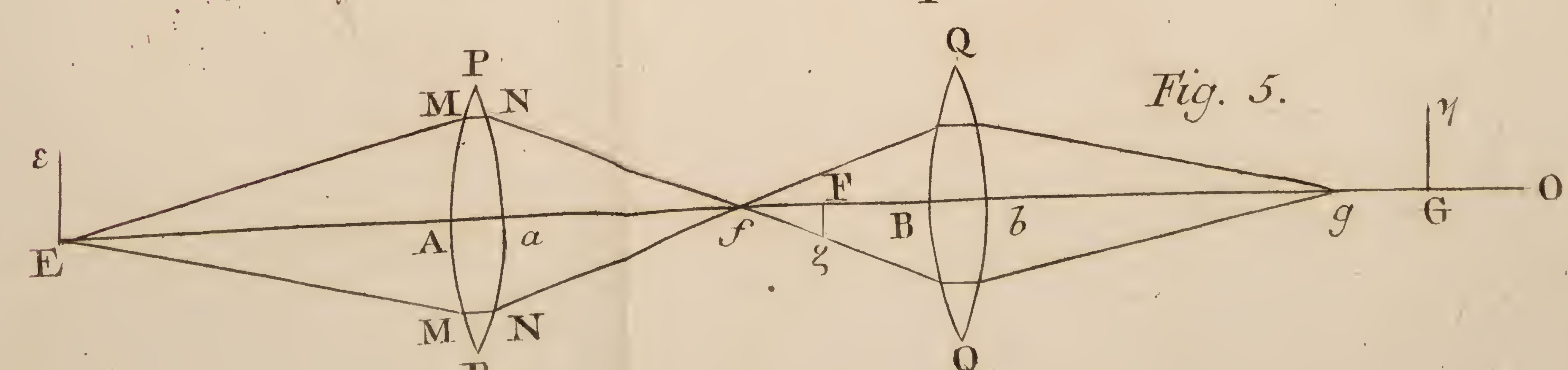

Fig. 5.

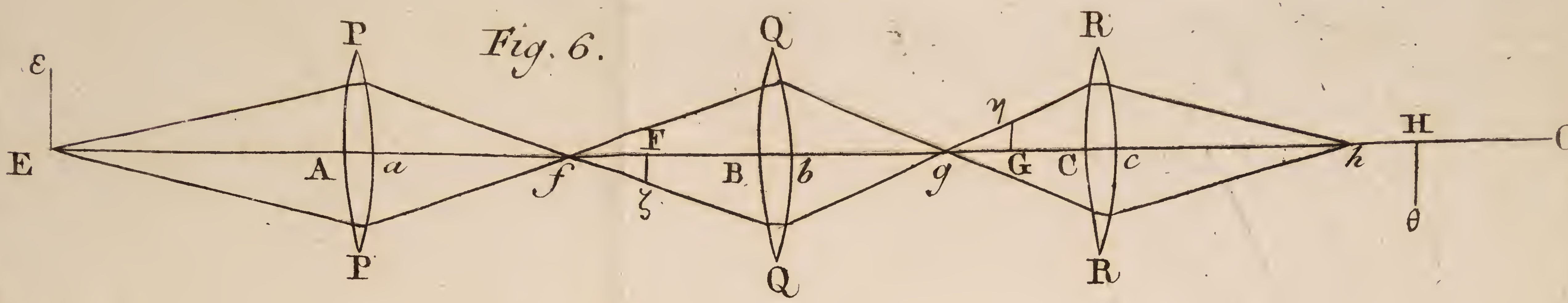

Fig. 6.

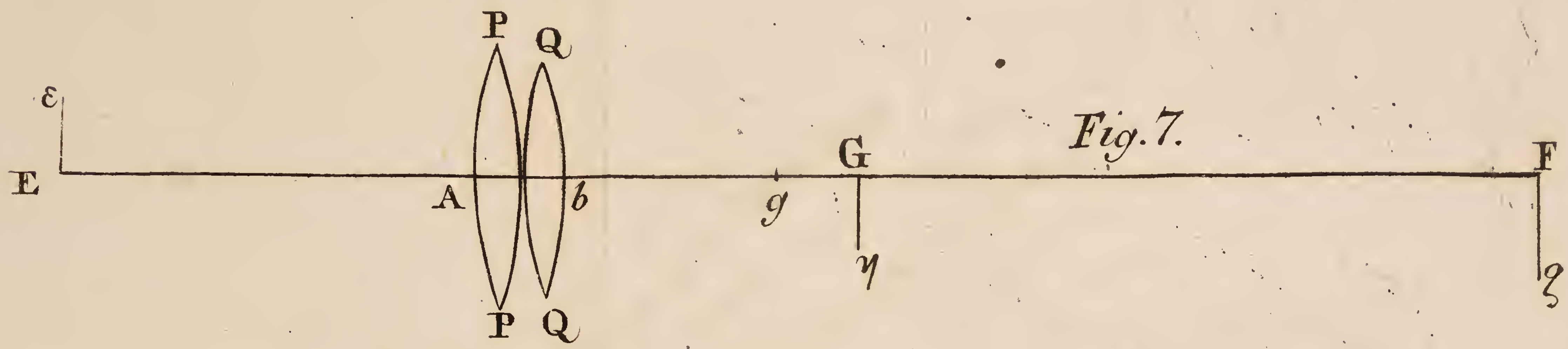

Fig. 7.

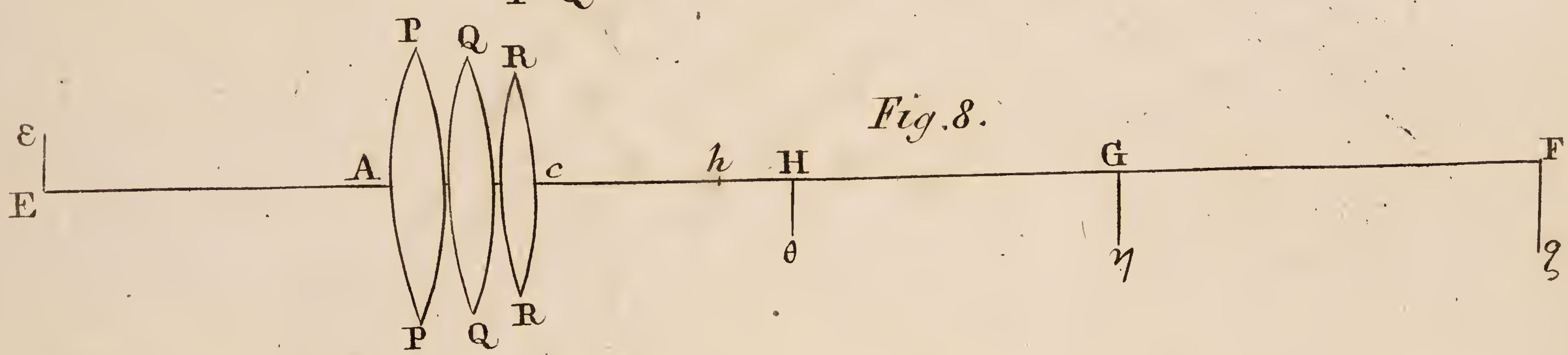

Fig. 8.

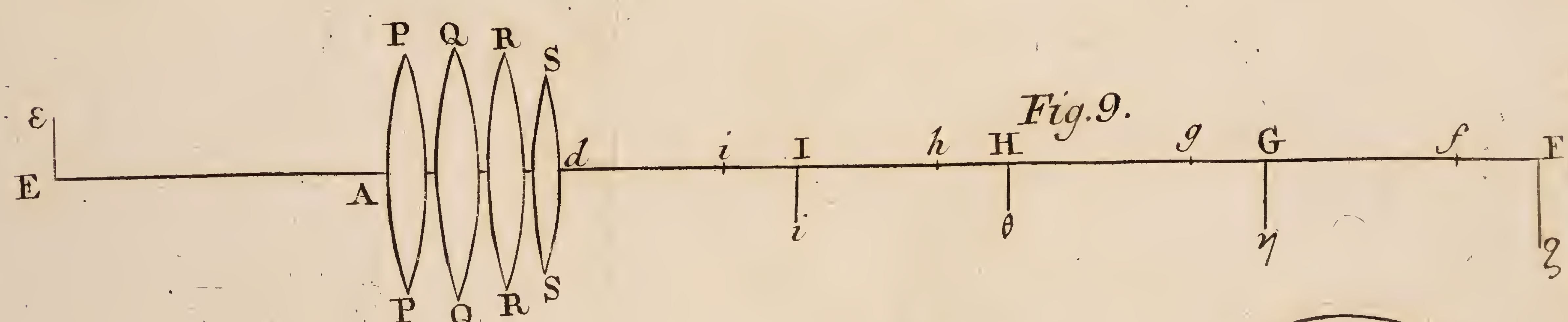

Fig. 9.

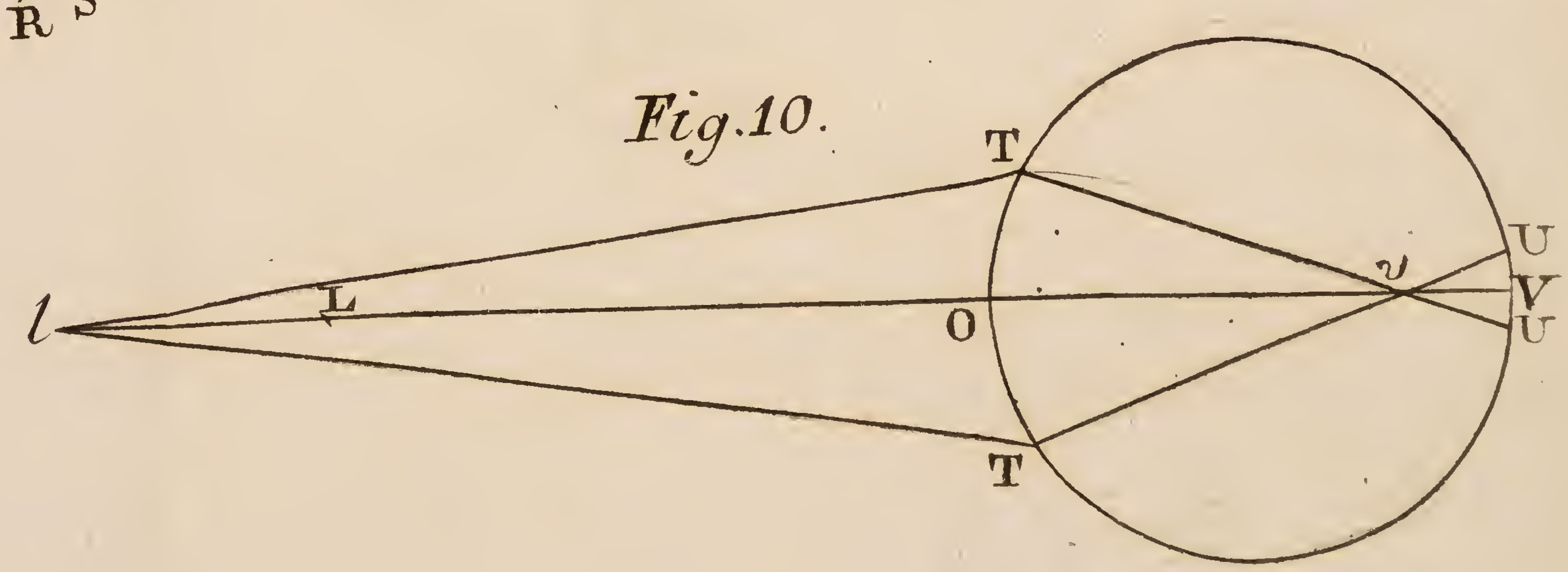

Fig. 10.

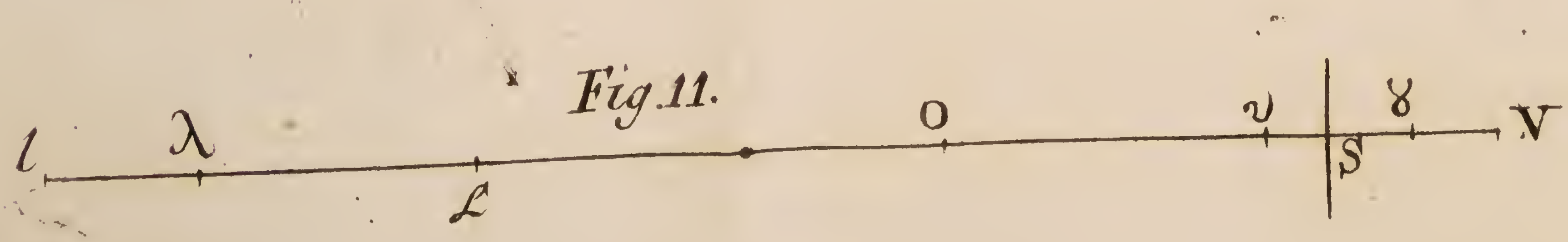

Fig. 11.

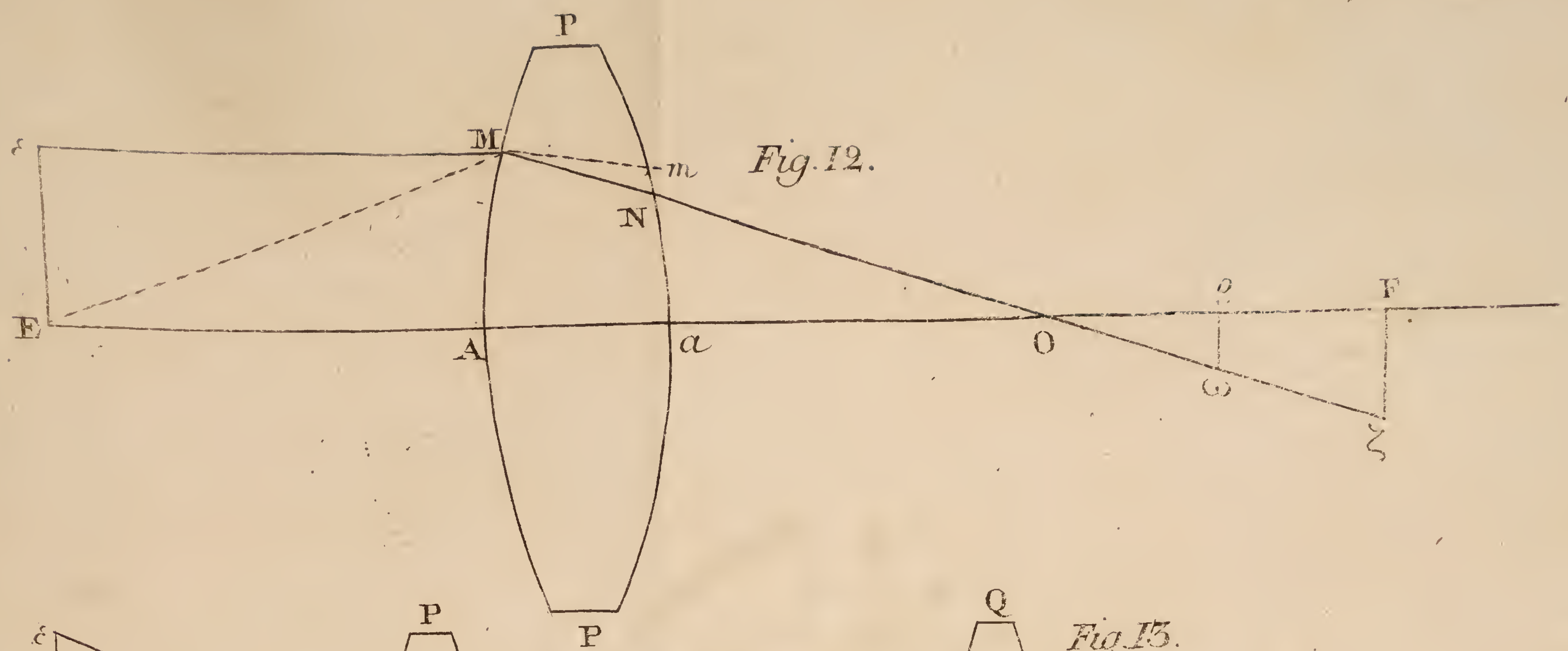

Fig. 12.

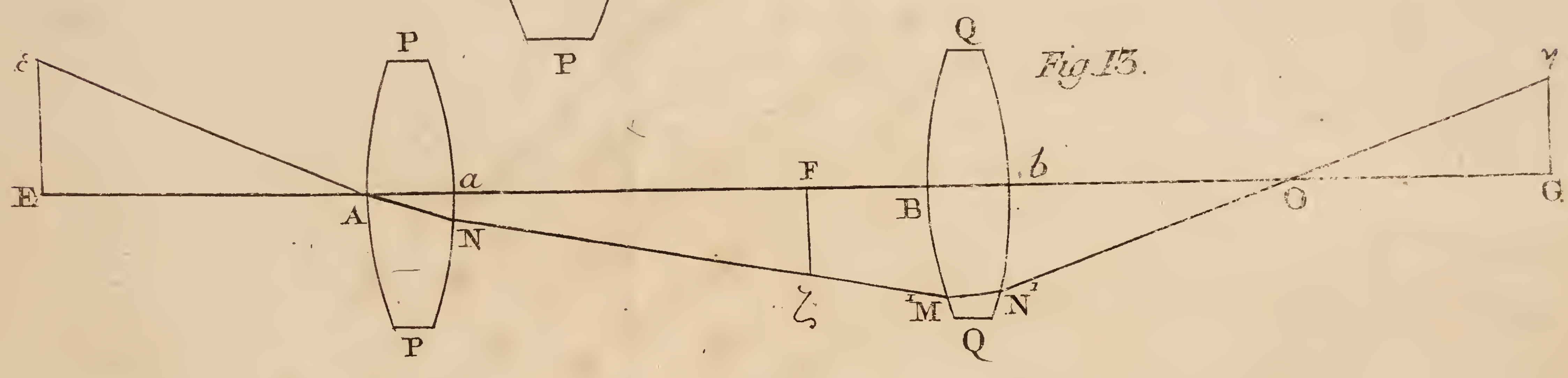

Fig. 13.

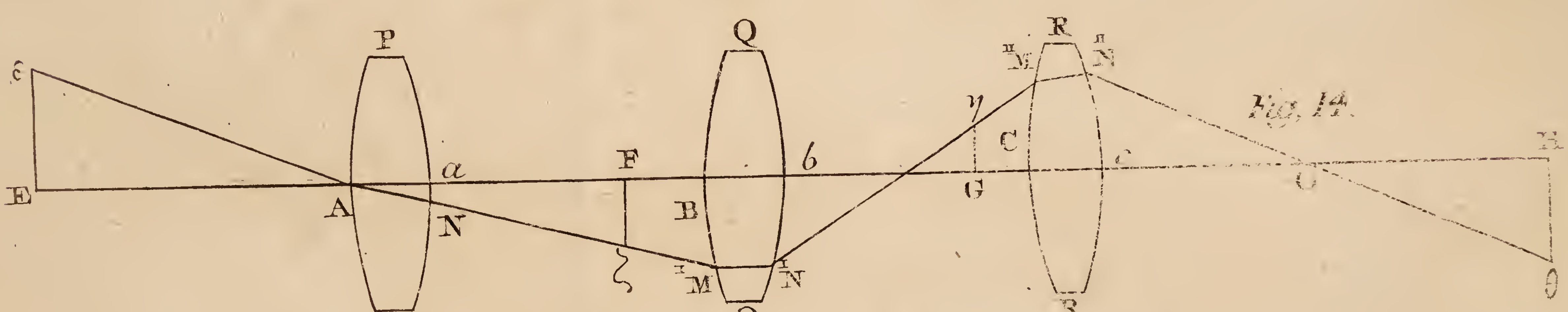

Fig. 14.

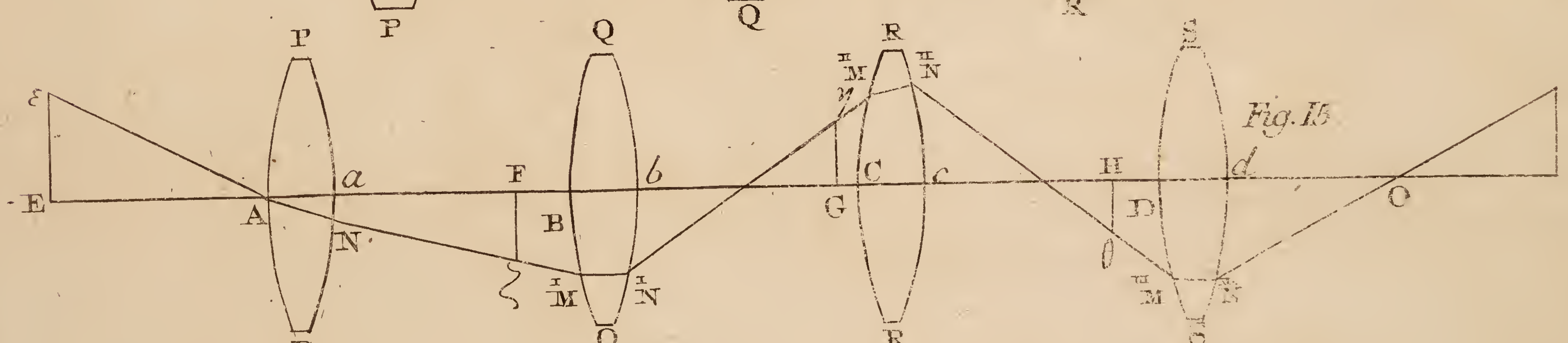

Fig. 15.

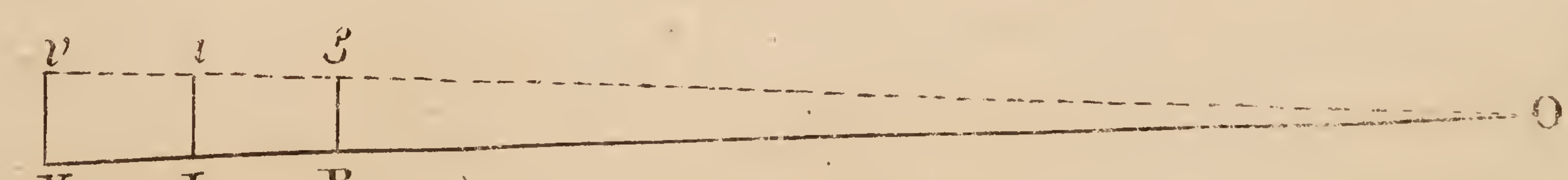

Fig. 16.

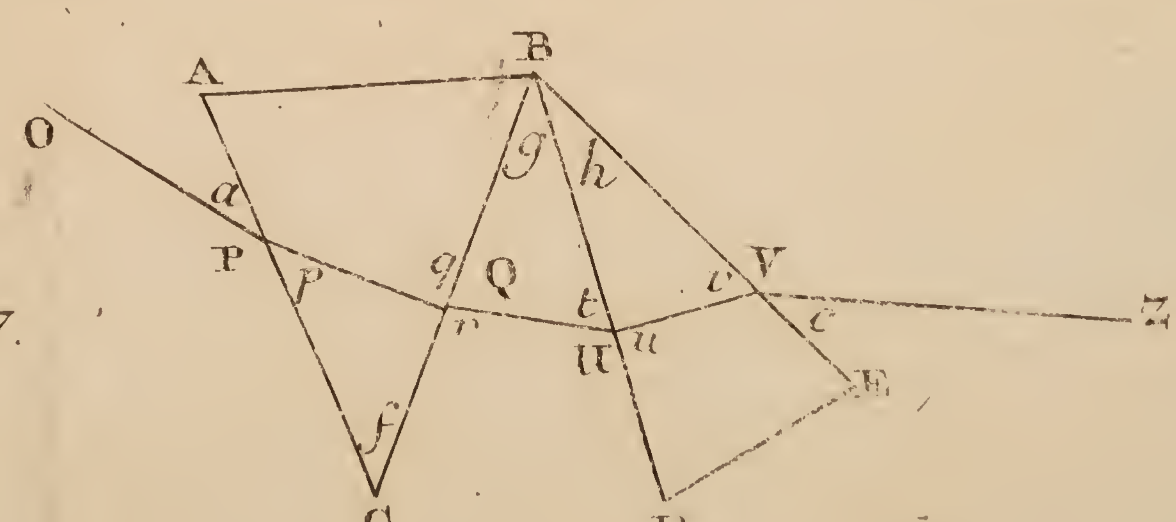

Fig. 17.